



Linear Modeling of Compressive Stress and Strain of EPS Packaging Material using Full Factorial Design of Experiment

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Abstract:

Effective good packaging is a need for Electronic products in order to reduce accidental transportation damage and therefore guarantee of the product safety from factory to customer. Researcher's works on increasing compressive strength of the packaging by studying the density effect on compressive strength and proposing the relation for compressive strength and density. This paper involves effect of density, height and force on compressive strength and strain of EPS material. For this short term compressive test has been done on various samples of EPS with variation in density, height and force. From test for various sample compressive stress and strain is recorded. This information is then use to find out linear model for compressive stress and strain with the help of MINITAB software. Different plots for the main effect as well as combine effect on the linear models were obtain. From the experimentation and from the linear models, for more compressive strength and less strain density and height need to be increase. Combine effect of density and force has more significant effect on the compressive model. For stain model individual effect is more as compare to combine effect of the parameters.

Keywords: EPS geform, Cmpressive Stress, Full factorial DOE, Plateau zone, Densification .

I. INTRODUCTION

The most important property of the expanded polystyrene is compressive stress when used as packaging or thermal insulating material [1]. When expanded polystyrene is under compression it gets deform. This deformation depends on the microstructure of closed cell of basic polymer of expanded polystyrene [1]. If this microstructure gets collapse, deformation leads to failure of the expanded polystyrene material. To understand the deformation of the expanded polystyrene we consider the following stress – strain curve obtain by compression test on 30 Kg/m³ density of EPS of size 100 mm X 100 mm X 50 mm.

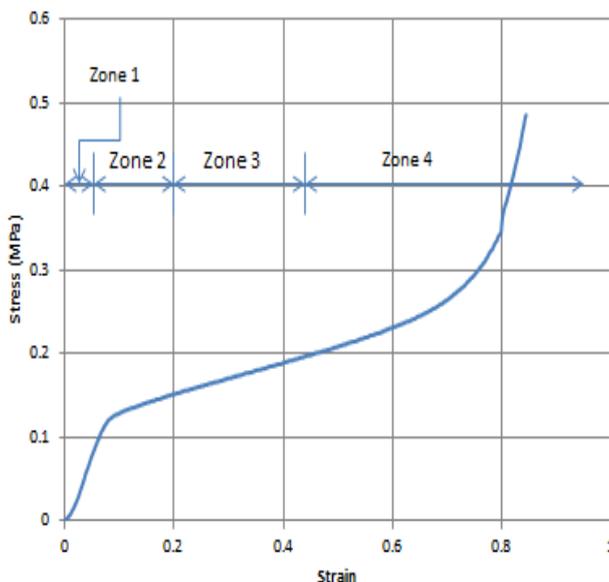


Figure. 1. Scheme of expanded polystyrene under short-term compression

Figure 1 the curve shows different zones which have different state of deformation. At the start of the compression we have zone – 1. In this zone the deformation is liner elastic and the energy is absorb by the cellular structure of EPS and when load is remove it is release [4]. The loading in this zone does not cause any failure in cellular structure. Zone – 2 corresponds to yielding of EPS material. In this state of EPS some part of deformation is elastic and some part of deformation is permanent [4]. That means in this zone some cellular cells start to get fail. Zone – 3 is the linear hardening of EPS material. When EPS material exceeds the yielding point more and more cellular cells gets fail under compression loading. In this deformation cause is permanent. Zone – 4 is the non- linear work hardening [4]. In this zone the cellular cells start to fail very rapidly. So to provide good packaging for the electronic products the compressive stresses coming on the package should fall in linear zone to ensure safety of the product. So for safety of the product design engineer has to decide the proper density and package thickness so that package will have good compressive strength and it can with stand with any load or any sudden shock coming on the product. This paper is mainly focus on finding out the model that will depend individual as well as combine effect of parameters like density, thickness and load. The linear model will help to find out the compressive stress and strain for the package. To find out the model we are going to use full factorial method in MINITAB software. The software also uses to find out the parameter effective plot. This plot helps to understand the individual and combine effect of the parameters on the compressive stress and strain of the EPS material.

II. FULL FACTORIAL DOE MODEL

DOE (design of experiments) helps us to investigate the effects of input variables (factors) on an output variable

(response) at the same time. These experiments consist of a series of runs, or tests, in which purposeful changes are made to the input variables. Data are collected at each run. We use DOE to identify the process conditions and product components that affect quality, and then determine the factor settings that optimize results. Minitab offers four types of designs factorial designs, response surface designs, mixture designs, and Taguchi designs (also called Taguchi robust designs). The steps we follow in Minitab to create, analyze, and visualize a designed experiment are similar for all types. After we perform the experiment and enter the results, Minitab provides several analytical tools and graph tools to help us to understand the results. We can apply these steps to any design that we like to create in Minitab.

Now for this paper, we have the parameters as follows,

1. Density = 22 Kg/m³ to 30 Kg/m³
2. Thickness of Contact (Height) = 50 mm to 100 mm
3. Force = 500 N to 1000 N

As per the full factorial DOE we have three factors and each factor has two levels, so the model can be referred as 2-level factorial model. Now to analyze this model minimum number of run require for this model can be calculated as,

$$\begin{aligned} \text{Number of experiment} &= (\text{level of factors})^{(\text{Total no. of factors})} \\ &= (2)^3 \\ &= 8 \end{aligned}$$

So the total minimum number of run (experiment) = 8
And this experiment can be carry out in order of

Table .1. Full Factorial Model

Ex no	Density(Kg/m ³)	Height(mm)	Force(N)
1	22	50	500
2	22	50	1000
3	22	100	500
4	22	100	1000
5	30	50	500
6	30	50	1000
7	30	100	500
8	30	100	1000

From the table No.1, it is clear that to analyze the effect of density, height and force on the response of compressive stress and strain we have to perform 8 experiments with reference to above table.

III. SAMPLE PREPARATION

The sample required for the test was prepared at S.R. Thermacol Company Ltd. Bhosri, Pune. The samples are cut from EPS bed using Hot Electric wire cutting tool. For the samples EPS bed of density 22 Kg/m³ and 30 Kg/m³ were use along with the height variation of 50 mm and 100mm. the cross section all samples are 100mm X 100mm.



Figure.2. 8 Sample of size 100mm X 100mm X 50mm and 100mm X 100mm X 100mm having density 22kg/ m³ and 30Kg/ m³.

IV. EXPERIMENTAL TESTING

Compression test is done at Praj testing laboratory. Star testing systems (UTM machine) were used for the tests in this research. Star testing systems use interconnected modules, power supplies and AD converters connected to a computer by a serial (COM) port. The computer exchanges reliable information (commands and data) with each module at high speeds, as high as 115200bits/sec. The sensors used in the tests include load cells, linear variable differential transducers (LVDTs), pressure transducers, and differential pressure transducers. All the sensors were calibrated and cross checked for accuracy and stability. LVDTs were used to measure the vertical deformation of the EPS geofoam sample. They have sensitivity of + 0.025 mm (0.001 in) and a working range of 76.2 mm (3in). The load cells were used to measure the vertical loads applied directly to the sample either using a loading platen alone or a piston rod attached with a platen. They can accurately measure a load as low as 0.5N (0.11lbf) and have capacities of 444.8 N (100lbf) to 8896.4 N (2000lbf). The samples are prepared using electric wire cutting having variation in density and height. The samples are place on bottom plate of the machine and load is applied from upper plate. The load is applied at the rate of 5mm/min. The compression load was assumed to be perpendicular to the surface of sample. As this UTM is equipped with load cell which directly give us the compressive stress data which depends on load.



Figure.3. Universal testing machine of capacity 9800N



Figure.4. Testing of 100mm X 100mm X 100mm having density 30 Kg/m³

With the help of Load cell and LVDT sensors, the force, deflection, stress and strain data is recorded. This data is then extracted in excel format which then help to plot the stress – strain curve for each sample.

V. EXPERIMENTAL RESULT

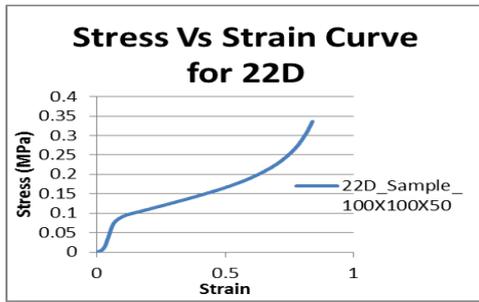


Figure.6. For 100mm X 100mm X 50mm and 22 Kg/mm³ density

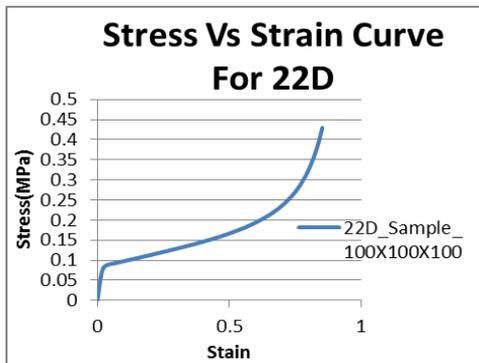


Figure.7. For 100mm X 100mm X 100mm and 22 Kg/mm³ density

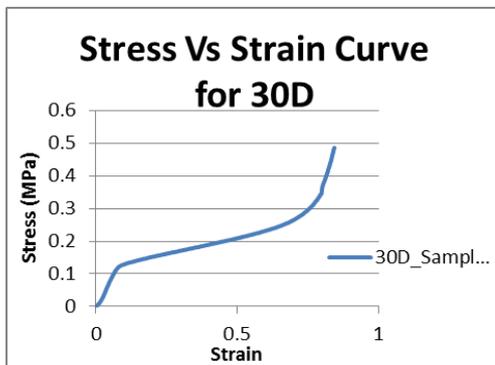


Figure.8. For 100mm X 100mm X 50mm and 30 Kg/mm³ density

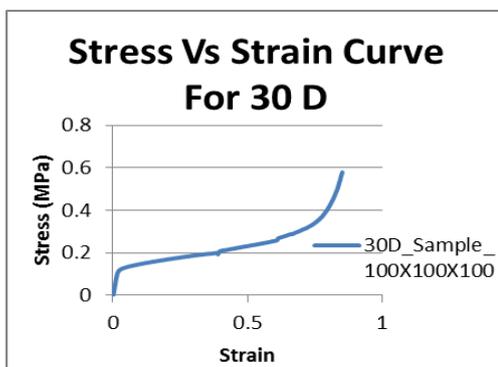


Figure. 9. for 100mm X 100mm X 100mm and 30Kg/mm³ Density

- 1) Samples of density 22Kg/m³ and 30Kg/ m³ having size of 100mm X100mm X 50mm and 100mm X 100mm X 100mm are tested on the UTM.
- 2) With the help of LVDT and Load cell sensors, the excel data was extracted. And the curve was plotted for the samples.

3) All the curve exhibit, all 4 zone namely linear elastic zone, yielding zone, linear work hardening zone and non-linear work hardening zone.

4) Linear elastic zone occurs at low strain value (<5%) due to uniform cellular wall elastic bending and stretching throughout the whole foam structure.

5) The Stress increases linearly with deformation and this deformation is recoverable.

6) The yielding zone is also called as Plateau zone. In this continuous load corresponds to permanent deformation at relatively constant stress. This stage provides majority of energy absorption capability of material. In this some cells gets crushed.

7) The work hardening zone is also called as Densification. In this stage all cells get collapse and all the voids are also get eliminated. And this results in formation of homogenous solid.

VI. EFFECT OF VARIATION IN HEIGHT

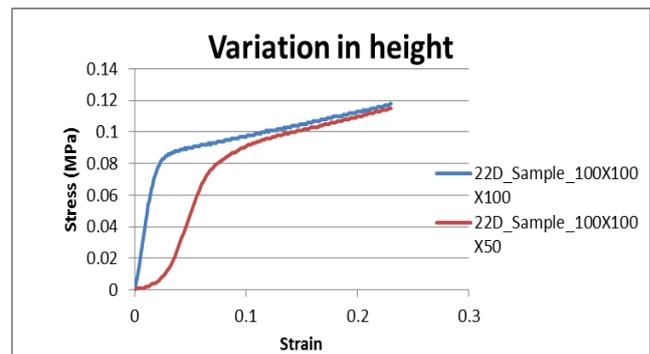


Figure. 10. Sample of Density 22Kg/m³

From above curves, it can be seen that for same density but variation in contact thickness (Height) have greater effect on compressive strength. The sample with height 100mm has good compressive strength as compare to the sample of height 50mm. This happens because in 100mm height sample there are more cells as compare to 50mm height sample which results in more strength. So while selecting package thickness we have to consider proper thickness which will ensure good compressive strength.

VII. VARIATION IN DENSITY

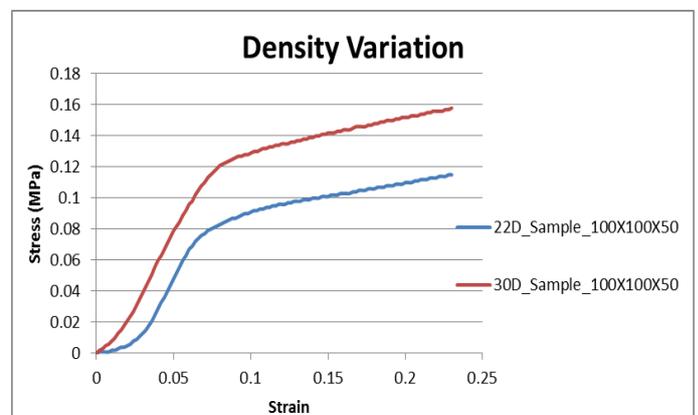


Figure. 11. Sample of size 100mm X 100mm X 50mm

From above curve with same sample size of 100mm X 100mm X 50mm, but variation in density shows the changes in compressive strength of the EPS material. Compressive strength is more for the sample of density 30Kg/m³ as compare to sample of 22 Kg/m³. So while considering the package design one has to consider the effect of density. So that packaged has good compressive strength.

VIII. MODELING METHOD

Full Factorial DOE:

To analyzed the effect of density, contact thickness and force variation on the compressive stress of the EPS material, we use the concept of Full factorial DOE. In this we first decide the no of parameter and their working levels. The parameters can be selected using the past experience or by studying literature. To analyze the effect of these parameters under the full factorial DOE we perform no of experiments.

In this paper we have decided the parameters

- 1) Density
- 2) Contact thickness
- 3) Force

To carry out the full factorial DOE we used MINITAB software which will directly provide the relation as well as the effect of parameters on the response (compressive stress and strain) that we require. The General equation that we could find out using this is

$$y = \mu + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \beta_{23} X_2 X_3 + \beta_{31} X_3 X_1 + \beta_{123} X_1 X_2 X_3$$

Where,

- y = response (Compressive stress)
- μ = mean of the response,
- X_1 = density parameter,
- X_2 = Height parameter,
- X_3 = Force parameter
- β_1 = coefficient of effect of parameter X_1
- β_2 = coefficient of effect of parameter X_2
- β_3 = coefficient of effect of parameter X_3 β_{12} = coefficient of combine effect of parameter X_1 and X_2
- β_{23} = coefficient of combine effect of parameter X_2 and X_3
- β_{31} = coefficient of combine effect of parameter X_3 and X_1
- β_{123} = coefficient of combine effect of parameter X_1 , X_2 and X_3

In MINITAB software at first we gives the inputs of all parameters and their levels. At first we create the factorial model by choosing the required level of factorial and selecting no of factors to 3, after that in Display Available design panel we select the no of experiment that we are going to perform as 8, in design panel we provides the parameter and their upper and lower levels, by selecting this finally we get the Full factorial DOE as

C1	C2	C3	C4	C5	C6	C7	C8
StdOrder	RunOrder	CenterPt	Blocks	Density	Height	Force	Stress
2	1	1	1	30	50	500	0.049
8	2	1	1	30	100	1000	0.104
6	3	1	1	30	50	1000	0.102
1	4	1	1	22	50	500	0.052
4	5	1	1	30	100	500	0.052
7	6	1	1	22	100	1000	0.101
3	7	1	1	22	100	500	0.054
5	8	1	1	22	50	1000	0.103

Figure.12. Factorial Model for compressive stress in MINITAB

Now by the Full factorial DOE model, the compressive stress for each experiment is recorded and it is then added to the DOE model as shown in figure 12. Now to calculate the response, we solve this table using the Full factorial DOE method which results in following data.

Full Factorial Design

Factors: 3 Base Design: 3, 8
 Runs: 8 Replicates: 1
 Blocks: 1 Center pts (total): 0

All terms are free from aliasing.

Factorial Fit: Stress versus Density, Height, Force

Estimated Effects and Coefficients for Stress (coded units)

Term	Effect	Coef
Constant		0.076750
Density	-0.000000	-0.000000
Height	0.002000	0.001000
Force	0.050000	0.025000
Density*Height	0.000500	0.000250
Density*Force	0.002500	0.001250
Height*Force	-0.000500	-0.000250
Density*Height*Force	-0.000000	-0.000000

Figure.13. Data obtain for Full Factorial DOE of compressive stress model

The data obtain after the full factor DOE gives us the coefficients of parameters. Which will give the relation for the response of compressive stress? And the relation is given as,

$$\sigma_c = 0.07675 + 0.001X_2 + 0.025X_3 + 0.00025X_1X_2 - 0.00025X_2X_3 + 0.00125X_1X_3$$

The above equation gives the relation between compressive stress and all the depending parameters.

IX. FACTORS EFFECT ON THE COMPRESSIVE STRESS

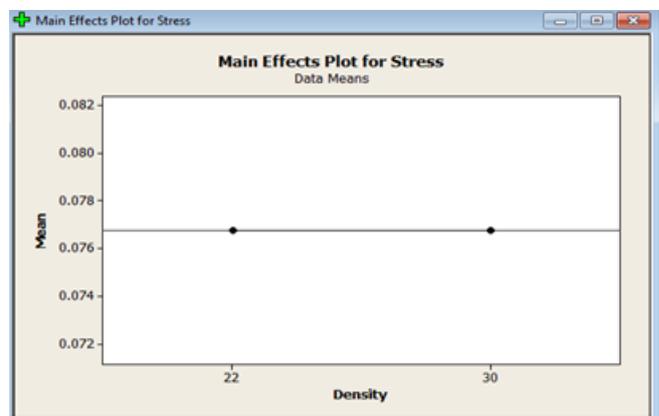


Figure.14. Main effect plot for density

From the above graphs we can understand individual the effect of these parameters on compressive stress. From the density graph we have the line plot between the average compressive stress at lower level of density to the average compressive stress at upper level of density. The curve has zero slope which indicates that as per individual factor is concern it does not have any effect on the compressive stress value.

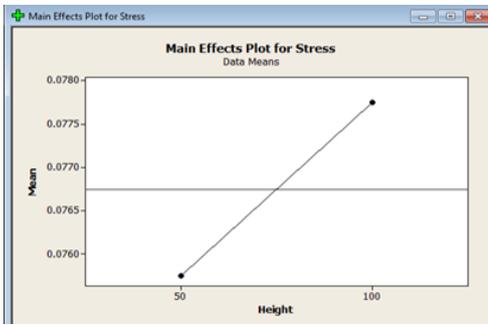


Figure.15. Main effect plot for Height

From the height graph, the plot is in between average compressive stress at lower level of height and the average compressive stress at upper level of height. The slope of this plot is positive and which shows that the higher the height more will be the compressive stress.

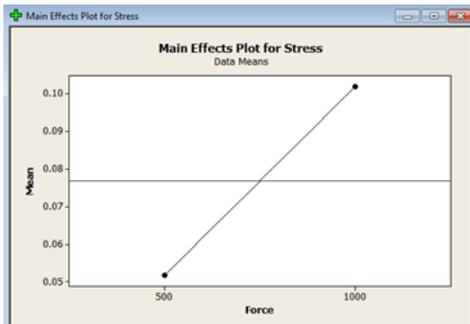


Figure.16 . Main effect plot for Force

From the force graph, the plot is in between average compressive stress at lower level of force and the average compressive stress at upper level of force. The slope of this graph is also positive which shows that at higher force level the compressive stress value will higher.

X. INTERACTION FACTOR EFFECT ON THE STRESS

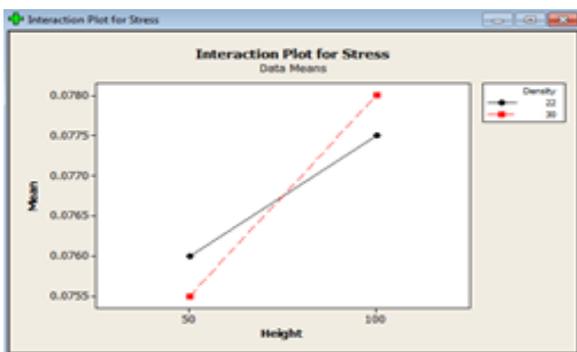


Figure. 17. Interaction Plot for Density and height

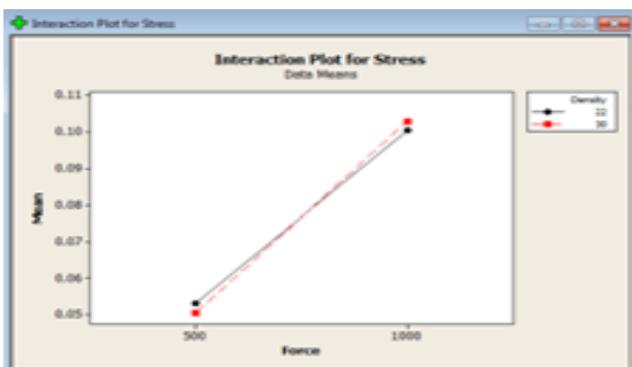


Figure. 18. Interaction Plot for Density and Force

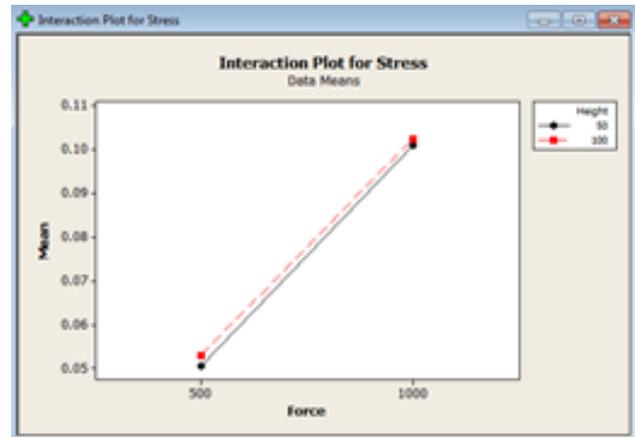


Figure. 19. Interaction Plot for Force and height

From the combine effect plot of density and height, there is interaction between the two straight lines. So if we see the plot there is no main effect of the individual parameter but their exist a combine effect of these parameter on the response. From the combine effect of density and force, there is interaction between the two lines. That means individually these parameter have less main effect on the output response, however because of the interaction there is significant combine effect of these parameters. For the combine effect of height and force, the two lines are parallel to each other, so there is no interaction which indicates that there is more main effect of these parameter if consider individually as compare to there combine effect on response.

XI. SIGNIFICANT FACTORS

To Understand, which parameters have more effect on the output response we Pareto Chart of the effect. This chart helps to understand more significant parameters. Which then use to find out more accurate model. Thus the compressive model equation can be further reduce to more simpler form. From this chart, it is clear height, force and combine density and force have greater effect on the compressive stress as their value is exceeding the average line of compressive stress as shown in Figure20

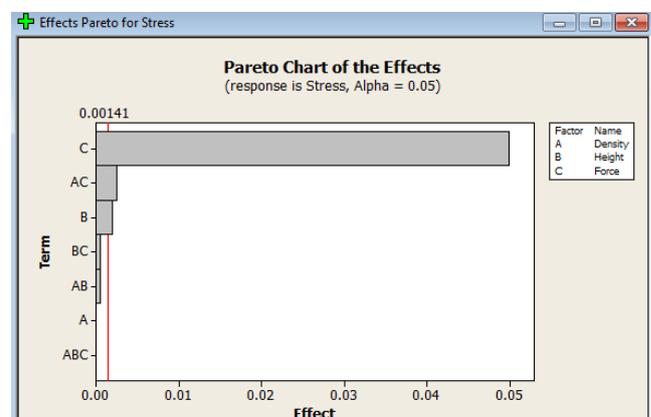


Figure.20. Pareto Chart

To find out the more accurate model depending on the significant parameter we again perform the Full factorial analysis with height, force and combine density and force parameter. So the reduce form of the linear model obtain is,

$$\sigma_c = 0.023125 - 9.38 \times 10^{-4} X_1 + 4 \times 10^{-5} X_2 + 6.75 \times 10^{-5} X_3 + 1.25 \times 10^{-6} X_1 X_3$$

XII. STRAIN MODEL

C1	C2	C3	C4	C5	C6	C7	C8
StdOrder	RunOrder	CenterPt	Blocks	Density	Height	Force	Strain
2	1	1	1	30	50	500	0.035
8	2	1	1	30	100	1000	0.016
6	3	1	1	30	50	1000	0.064
1	4	1	1	22	50	500	0.052
4	5	1	1	30	100	500	0.008
7	6	1	1	22	100	1000	0.118
3	7	1	1	22	100	500	0.012
5	8	1	1	22	50	1000	0.142

Figure. 21. Factorial Model for Strain in MINITAB

For the Strain model the strain for each experiment was recorded and it is then added to the strain model as shown in figure 21. After solving this model for full factorial DOE we get following results.

Full Factorial Design

Factors: 3 Base Design: 3, 8
 Runs: 8 Replicates: 1
 Blocks: 1 Center pts (total): 0

All terms are free from aliasing.

Factorial Fit: Strain versus Density, Height, Force

Estimated Coefficients for Strain using data (coded units)

Term	Coef
Constant	0.0345000
Density	-7.50000E-04
Height	-0.00387000
Force	0.000280000
Density*Height	0.000125000
Density*Force	-6.00000E-06
Height*Force	4.71000E-06
Density*Height*Force	-1.85000E-07

Figure.22. Data obtain for Full Factorial DOE of strain model

From the data obtain for full factorial DOE model for Strain we get the coefficients of parameters. And the linear model for the strain can be written as,

$$\begin{aligned} \epsilon_c = & 0.0345 - 7.7 \times 10^{-4} X_1 - 0.00387 X_2 + 0.00028 X_3 \\ & + 0.000125 X_1 X_2 - 6 \times 10^{-6} X_1 X_3 + 4.71 \times 10^{-6} X_2 X_3 \\ & - 1.85 \times 10^{-7} X_1 X_2 X_3 \end{aligned}$$

XIII. FACTORS EFFECT ON THE COMPRESSIVE STRAIN

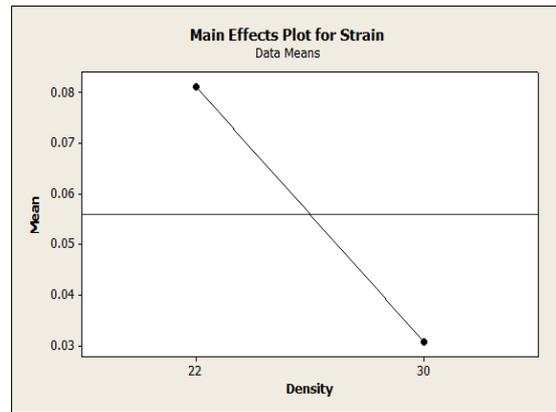


Figure. 23. Main effect plot for density

From the above graphs we can understand individual effect of these parameters on Strain. From the above plot the line joining the average strain value at lower level of density to higher level density have negative slope, so that if density is increase the strain rate gets decreased and if density is lower the strain will be more.

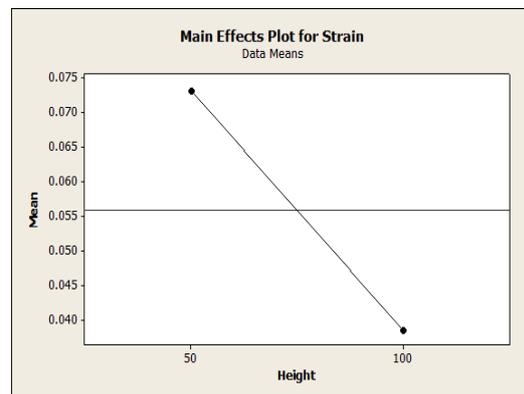


Figure. 24. Main effect plot for Height

From the above plot for the height parameter, the line joining average strain value at lower height and the average strain value at higher height have negative slope. That means if height is kept lower, will result in more strain and if height is kept at higher level the strain is less.

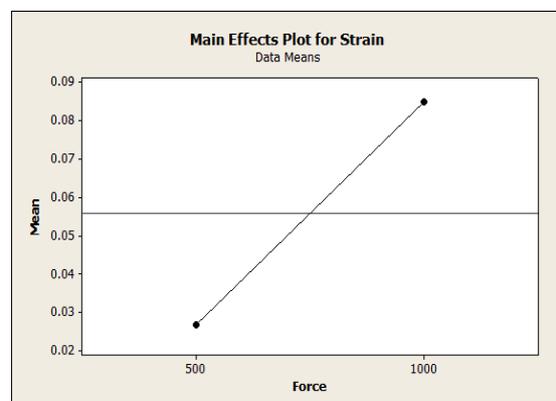


Figure. 25. Main effect plot for Force

From the above plot for the force parameter, the line joining the average strain value at lower level of force and average strain value at higher level of force have positive slope. That means for lower level of force we have less strain and for higher level of force we have more strain.

XIV. INTERACTION FACTOR EFFECT ON THE STRESS

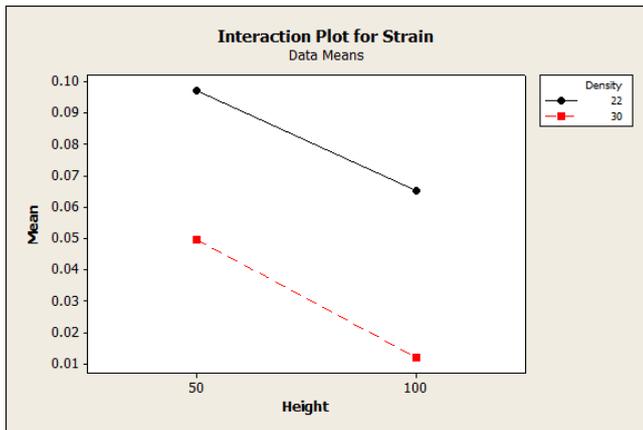


Figure. 26. Interaction Plot for Density and height

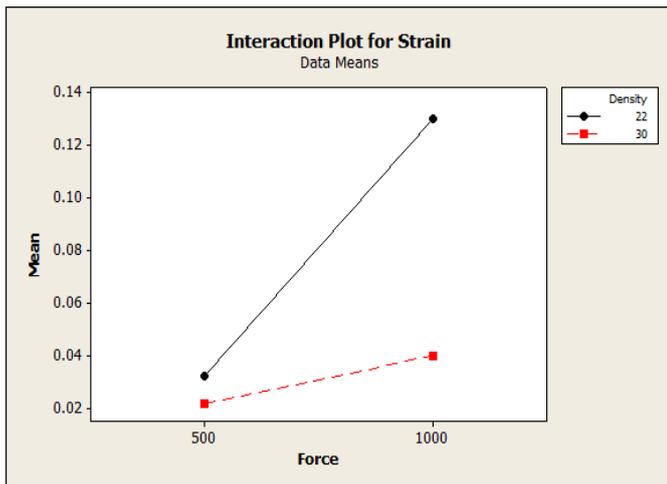


Figure. 27. Interaction Plot for Density and Force

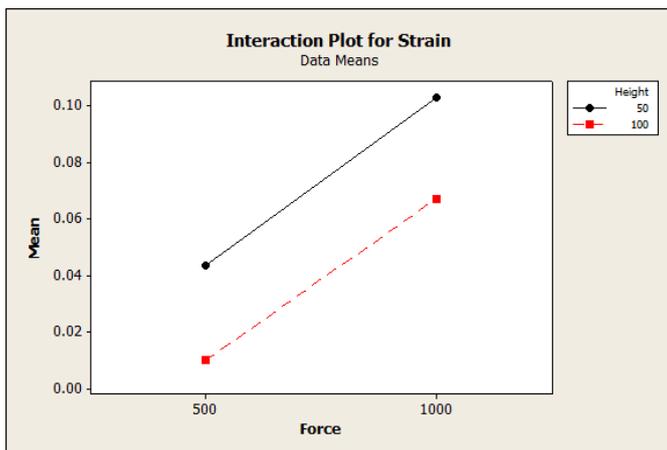


Figure. 28. Interaction Plot for Force and height

To understand the combine effect of all parameters we have these interaction plots. Now from the combine effect plot for density and height(Figure26) shows that there is no interaction between the lin. These line are plotted form the average strain at lower level to average strain at higher level. From density and height plot(Figure26) there is no any ineration so the combine effect of these two parameter is less on the strain. From combine effect plot for density and force(Figure27), the two line joining the average strain at lower and higher level are not interacting so the combine effect of density and force have less effect on strain output. From figure 28, plot for the

combine effect of force and height, the two lines are also not interacting, which results in less effect on strain

XV. SIGNIFICANT FACTORS

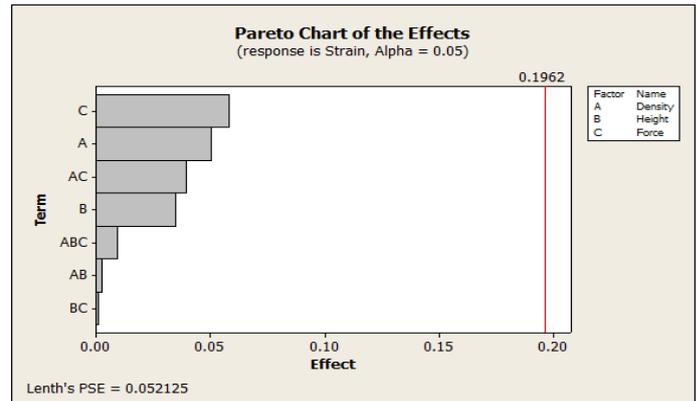


Figure.29. Pareto chart of the effect

From the Pareto chart for the Stain model, all the parameters are below the average stain line as shown in figure29. There is no any parameters that have greater effect on the strain. So the strain model obtain is final linear model.

XVI. CONCLUSION

- 1) The main parameters that have significant effect on the compressive stress and strain of EPS material are Density, Height (thickness of the packaging), and Force on packaged product.
- 2) To have more compressive strength and less strain, from the experimental results (figure10 and 11), it is clear that if we increase the density and height of the packaged product we get more compressive stress and reduction in strain.
- 3) The final relation obtain from full factorial DOE for compressive stress is,

$$\sigma_c = 0.023125 - 9.38 \times 10^{-4} X_1 + 4 \times 10^{-5} X_2 + 6.75 \times 10^{-5} X_3 + 1.25 \times 10^{-6} X_1 X_3$$

- 4) The strain model obtain is,

$$\epsilon_c = 0.0345 - 7.7 \times 10^{-4} X_1 - 0.00387 X_2 + 0.00028 X_3 + 0.000125 X_1 X_2 - 6 \times 10^{-6} X_1 X_3 + 4.71 \times 10^{-6} X_2 X_3 - 1.85 \times 10^{-7} X_1 X_2 X_3$$

- 5) From the relation for compressive stress as well as the plot for the main effect (figure 15 and 16) suggest that, there is direct effect of height and force on the compressive stress. So to have more compressive strength height and force must be increase.

- 6) From the strain model and from main effect of plot (figure 24,25) , with increase in density and height strain rate gets reduce.

- 7) From the combine effect plot for the compressive stress (Figure17,18 and 19) and strain (figure 267,27 and 28) , the combine effect of density and height is more on compressive strength model as average lines have interaction. Where as in

strain model , there is no interaction between the line. So on strain model the combine effect of parameters are less.

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