Mathematical Model to Show the Effect of No-Slip Liquid Holdup on Pressure Gradient for Multiphase Flow in Niger-Delta

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Abstract:
Multiphase fluid flow is a common phenomenon found in most hydrocarbon wells and surface pipes that transport produced fluids. Predicting the pressure gradient of multiphase flow is very essential especially to aid in the design of surface facilities. The major setback in these predictions however is that, most of the existing correlations are usually validated based on oilfield data gotten from a certain region which does not give a true representation of all situations and fields. In this study, mathematical models were developed to show the effect of no-slip liquid holdup on the pressure gradient for the various multiphase flow regimes for wells in the Niger Delta. The regression models developed were based on analysis from 11 different wells in the Niger Delta. The Aziz, Orkiszewski, and the Beggs & Brill’s correlations were utilized in estimation of the pressure gradients in the wells. Neglecting the frictional term accounts for not more than 10\% error, as this assumption helped to evade having a model that is an implicit function. A cubic model was obtained for slug flow with a correlation coefficient (\(R^2\)) of 0.861, while quadratic models were obtained for bubble, intermittent, and mist flows with correlation coefficients of 0.9747, 0.9277 and 0.9971 respectively. This study established the fact that the no-slip liquid holdup can give as high as 86.1\% minimum representation of data desired for slug flow, 97.47\% for bubble flow, 92.47\% for intermittent flow and 99.71\% for mist flow.

Keywords: Liquid Holdup, Modeling, Multiphase flow, No-slip, Niger Delta, Pressure Gradient.

I.INTRODUCTION AND BACKGROUND INFORMATION

Multiphase flow is found in many places, it occurs in almost all producing oil/gas wells and surface pipes that transport produced fluids. The significantly different densities and viscosities of these fluids make multiphase flow much more complicated than the single phase flow\(^1\). In the petroleum industry, multiphase flow occurs in oil/gas wells, gathering systems, many piping systems, and key pieces of equipment needed in refineries and petrochemical industries, including boilers, condensers, distillation towers and separators \(^2\). During multiphase flow in pipes, the flow pattern that exists depends on the relative magnitudes of forces that act on the fluids. Pressure drop is probably the quantity that one deals with most often in two-phase flow. Late investigators have recognized that improved understanding of multiphase flow in pipes requires a sophisticated combined experimental and theoretical approach \(^3\). This understanding was transformed into improved mechanistic models to better describe the physical phenomena occurring \(^4\), \(^5\). Among the empirical correlations, \(^6\) seems to have some theoretical justification and with some modification it could predict multiphase flow performance inside production pipelines more accurately \(^7\). The prediction of pressure gradients, liquid holdup and flow patterns occurring during the simultaneous flow of gas and liquid in pipes is necessary for design in the petroleum and chemical industries. Petroleum engineers encounter two-phase flow frequently in well tubing and in flow lines. The flow may be vertical, inclined or horizontal and methods must be available for predicting pressure drop in pipes at any inclination angle. Offshore production necessitated transporting both gas and liquid phases over long distance before separation \(^8\). Besides being able to size these lines from a pressure loss standpoint, the engineer must be able to calculate liquid content in the pipeline at various flow conditions in order to design separation and slug catching facilities \(^9\).

Liquid Holdup \((H_L, H_g) = \frac{\text{Volume of liquid in a pipe}}{\text{Volume of pipe segment}}\) \(^9\)
Gas Holdup, \(H_g = 1 - H_L\) \(^2\)
No-slip liquid holdup \((\lambda_L) = \frac{q_L}{q_L + q_g}\) \(^3\)
where \(q_L\) and \(q_g\) are the in-situ liquid and gas flow rates.
No-slip gas holdup, \(\lambda_g = 1 - \lambda_L = \frac{q_L}{q_L + q_g}\) \(^4\)

There is a wide range of possible flow regimes whenever two fluids with different physical properties flow simultaneously in a pipe. Flow pattern refers to distribution of each phase in the pipe relative to other phase \(^9\).

Many investigators have attempted to predict the flow pattern that will exist for various sets of conditions, and many different names have been given to the various pattern. Of even more significance, some of the more reliable pressure loss correlations rely on knowledge of the existing flow pattern \(^10\). In two-phase gas-liquid flow in pipes or channels, an interface exists between the phases. The phase boundary can take a variety of configurations, known as the flow pattern. The existing flow pattern in a given two-phase flow system depends on the operational parameters (gas and liquid flow rates), the geometrical variables (pipe diameter and pipe inclination angle), and the physical properties of both phases (gas and liquid...
densities, viscosities and surface tension). The flow patterns usually categorized in a vertical two phase flow as follows: Bubble, slug, transition and annular- mist flow patterns[10]. A major challenge in multiphase flow is that most of the existing correlations were validated based on data got from a certain region which does not give the true result in all situation and fields [10].

This study therefore aims to develop a mathematical model (linear regression) to show the effect of no-slip liquid hold up on pressure gradient for the various multiphase flow regimes for wells in the Niger Delta province.

An accurate model is very necessary in selecting correct tubing sizes and predicting time for artificial lift.

### II. METHODOLOGY

Regression models will be developed by analyzing data got from 11 wells in the Niger Delta province. The linear regression models will show the effect of the no-slip liquid hold up $\lambda$ on the pressure gradient, $dp/dz$ and for the various flow regimes.

#### Data Collection

Measured field data (such as oil FVF, gas FVF, oil gravity, gas gravity, liquid surface tension, viscosities of oil/gas, flowrate of oil/gas, BHFP, WHP, solution GOR, flowing temperature and pressure well depth, tubing size, measured pressure gradient etc.) were obtained from eleven producing oil wells in the Niger-Delta and displayed in Table 1.

<table>
<thead>
<tr>
<th>FP</th>
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<th>2</th>
<th>3</th>
<th>4</th>
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<th>8</th>
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<td>0.5394</td>
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<td>0.7779</td>
<td>0.9175</td>
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<tr>
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<td>0.7616</td>
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<td>0.1248</td>
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<td>40</td>
<td>25</td>
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<tr>
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<td>1.015</td>
<td>1.017</td>
<td>1.002</td>
<td>1.191</td>
<td>1.2324</td>
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<td>0.0091</td>
<td>0.0087</td>
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<td>$D$</td>
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<tr>
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<td>0.012</td>
<td>0.016</td>
<td>0.017</td>
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<td>0.019</td>
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<td>323.05</td>
<td>323.01</td>
<td>322.94</td>
<td>281</td>
<td>320</td>
<td>336.78</td>
<td>336.83</td>
<td>11363</td>
<td>10785</td>
<td>2894</td>
</tr>
</tbody>
</table>

The Aziz Govier correlation, Beggs and Brill correlation and the Orkiszewski correlation were utilized in computing the multiphase flow pressure gradient from the given data with the input and output Graphic User Interface (GUI) of the visual basic software used.

As shown in Figure 1.

#### III. RESULTS

The results for the analyses for the various wells conditions and flow types are as displayed in figures 2-13.
<table>
<thead>
<tr>
<th>Figure.3. Analysis of well 1 with the Beggs and Brill model (for intermittent flow)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure.4. Analysis of well 1 with the Orkiszewski model (for Slug flow)</td>
</tr>
<tr>
<td>Figure.5. Analysis of well 2 with the Aziz et al model (for mist flow)</td>
</tr>
<tr>
<td>Figure.6. Analysis of well 2 with the Orkiszewski model (for Slug flow)</td>
</tr>
<tr>
<td>Figure.7. Analysis of well 3 with the Aziz et al model (for Bubble flow)</td>
</tr>
<tr>
<td>Figure.8. Analysis of well 3 with the Orkiszewski model (for Bubble flow)</td>
</tr>
</tbody>
</table>
Figure 9. Analysis of Well 4 with the Aziz Et Al Model (for Mist Flow)

Figure 10. Analysis of well 4 with the Orkiszewski model (for slug flow)

Figure 11. Analysis of well 5 with the Aziz et al model (for mist flow)

Figure 12. Analysis of well 5 with the Beggs and Brill model (for intermittent flow)

Figure 13. Analysis of well 5 with the Orkiszewski model (for slug flow)

The same analysis carried out for well 1 to well 5 as seen above, was also done for wells 6 to 11.

Models Developed

From the pressure gradient computed for the 11 wells, the following linear regression equations were developed for various flow regimes based on the liquid hold up, λ.

Mist flow: \[ \frac{dp}{dz} \text{ (psi/ft)} = 0.0595\lambda^2 + 0.3096\lambda + 0.0186 \] \[ (R^2 = 0.9971) \] 5

Slug flow: \[ \frac{dp}{dz} \text{ (psi/ft)} = 24.72\lambda^3 - 38.277\lambda^2 + 19.39\lambda - 2.991 \] \[ (R^2 = 0.861) \] 6

Intermittent flow: \[ \frac{dp}{dz} \text{ (psi/ft)} = -0.5065\lambda^2 + 0.8891\lambda - 0.1168 \] \[ (R^2 = 0.9277) \] 7

Bubble flow: \[ \frac{dp}{dz} \text{ (psi/ft)} = -1.7746\lambda^2 + 2.8401\lambda - 0.7829 \] \[ (R^2 = 0.9747) \] 8
The plots of the pressure gradients in psi/ft against the no-slip liquid holdup for the four considered flow patterns; the mist flow, intermittent flow, Bubble flow, and slug flow are displayed in figures 14-17.

Figure 14. Plot of pressure gradient against no-slip liquid holdup (Mist flow)

Figure 15. Plot of pressure gradient against no-slip liquid holdup (Intermittent flow)

Figure 16. Plot of pressure gradient against no-slip liquid holdup (Bubble flow)

Figure 17. Plot of pressure gradient against no-slip liquid holdup (Slug flow)

IV. DISCUSSION OF RESULTS

The pressure gradient was assumed to be a function of the no-slip liquid holdup since the flow rates of liquid and gas produced at any instant is known.

\[
\frac{dp}{dz} = f(\lambda)
\]

where \( \lambda = \text{no} - \text{slip liquid hold up (dimensionless)} \)

A cubic model was obtained for slug flow with \( R^2 = 0.861 \) while bubble, intermittent, and mist flows yield quadratic models with \( R^2 \) values 0.9747, 0.9277, and 0.9971 respectively. An almost linearly varying relationship was observed between the pressure gradient and \( \lambda_L \) in Figure 14, that is also apparent if the coefficient of first term in its model is neglected, 0.3096\( \lambda_L \) +0.0186 is left behind which is a linear equation which is in good agreement with actual observation practically while a nonlinear relationship between the pressure gradient and \( \lambda_L \) was observed in Figure 15. Figure 16 looks like a sinusoidal curve when rotated 90° to the right or left which indicates an increase in Pressure Gradient as the \( \lambda_L \) increases and vice-versa. Figure 17 shows that an oscillatory relationship was observed between pressure gradient and \( \lambda_L \) which is expected of slug flow because the liquid “slugs” are separated by gas phase which exists as large bubbles which results in pressure oscillation within the tubing. In summary, for slug flow, the no-slip liquid hold up represents a minimum of 86.1% of the data desired. For bubble flow, the no-slip liquid hold up represents a minimum of 97.47% of the data desired. For intermittent flow, the no-slip liquid hold up represents a minimum of 92.77% of the data desired and finally for the mist flow, the no-slip liquid hold up represents a minimum of 99.71% of the data desired.

V. CONCLUSION

Based on this study, a simple regression model was developed which can predict the pressure gradient in multiphase flow for the diverse flow regimes based on the no-slip liquid hold up, \( \lambda \) for fields in the Niger Delta. The no-slip liquid hold up has a great contribution in the multiphase pressure gradient estimation. The developed model is a good representation of the other parameters in the pressure drop computation equations.
VI. REFERENCES


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