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Infra Generalized b- Continuous Functions in Infra Topological Space

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Abstract:
The Purpose of this paper is to introduce new classes of functions called infra generalized b-continuous functions and infra generalized b-irresolute functions in infra topological spaces. Some properties and several Characterization of these types of functions are obtained. Also we investigate the relationship between these classes of functions.

Keywords: IGb-continuous functions and IGb-irresolute functions.

I. INTRODUCTION

The concept of generalization of closed mapping in topological spaces was introduced by Noiri [12] in 1973 and also he gave some characterization of such mappings. A new class of mapping called generalized continuous mapping was introduced by K.Balachandran, P.Sundram and H.Maki[4] in 1991, that contains the class of continuous mapping .Noiri [13] , Balachandran et al [5] and Donthchev.J and Ganster [8] introduced δ- continuity , generalized continuous function and δ -generalized continuous and δ - generalized irresolute functions respectively. Malghan [10] explained generalized closed functions in topological spaces. The notion of generalized continuous (g- continuous) which are called as g-irresolute functions was introduced by Munshy and Bassan [11] in 1981. Moreover, the notion of gs- irresolute [6] (resp. gp- irresolute [3], αg- irresolute [7], gb- irresolute [9], gsp-irresolute [15]) functions was introduced. In this paper, we introduce and investigate notions of new classes of functions namely infra generalized-continuous function, infra generalized semi- continuous function, infra semi generalized-continuous function, infra generalized pre- continuous function, infra generalized α - continuous function, infra α generalized- continuous function, infra generalized b-continuous function, infra weakly generalized- continuous function, infra generalized β - continuous function, infra generalized α b- continuous function, infra semi generalized b -continuous function, infra semi weakly generalized-continuous function in an infra topological spaces .Relations between these types of functions other classes of functions are obtained.

II. PRELIMINARIES

Definition 2.1: Let X be any arbitrary set. An Infra – topological space on X is a collection τ_{iX} subsets of X such that the following axioms are satisfying:
Ax-1: ϕ , X $\epsilon\tau_{iX}$.
Ax-2:The intersection of the elements of any sub collection of τ_{iX} in X. i.e) If Oi$\epsilon\tau_{iX}$,1 $\leq i \leq n \rightarrow \cap$ $O_i \epsilon\tau_{iX}$.
Terminology, the order pair (X,τ_{iX}) is called infra-topological space. We simply say X is an infra space.

Definition 2.2: Let (X,τ_{iX}) be an infra-topological space and A$\subset X$. A is called an infra open set (IOS) if A $\epsilon\tau_{iX}$.

Definition 2.3: Let (X,τ_{iX}) be an infra topological space. A subset C $\subset X$ is called infra-closed set (ICS) in X if X \C is infra-open set in X.
(i.e) C is infra-closed set (ICS) iff X \C $\epsilon\tau_{iX}$.

Definition 2.4: Let (X,τ_{iX}) be an infra topological space and A$\subset X$. The **Infra Closure Point (ICP)** of A is a set denoted by icp(A) and
given by : icp(A) = $\cap \{C_i : A \subseteq C_i , X - C_i \epsilon\tau_{iX} \}$
(i.e) icp(A) is the intersection of all infra closed set containing the set A.

Definition 2.5: Let (X,τ_{iX}) be an infra topological space and A$\subset X$. The **Infra Interior Point (IIP)** of A is a set denoted by iip(A) and given by: iip(A) = $\cup \{O_i : O_i \subseteq A, O_i \epsilon\tau_{iX} \}$
(i.e) iip(A) is the union of all infra open set contained in the set A.

Definition 2.6: Let (X, τ_X) be a topological space and Let (X, τ_{iX}) be an infra topological space. We say that τ_{iX} is an infra-topological space associated with τ_X , if $\tau_{iX} \subset \tau_X$.

Definition 2.7: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi-open if A $\subseteq icp\ (iip\ (A))$and infra semi- closed set if $iip\ (icp\ (A)) \subseteq$ A.

Definition 2.8: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra pre-open if A $\subseteq iip\ (icp\ (A))$ and infra pre-closed set if $icp\ (iip\ (A)) \subseteq A.$

Definition 2.9: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra α-open if A $\subseteq iip\ (icp(iip(A)))$ and infra α - closed set if $icp\ (iip\ (icp(A))) \subseteq A$.

Definition 2.10: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra b-open if
A $\subseteq iip(icp\ (A)) \cup icp\ (iip\ (A))$ and infra b- closed set if $iip(icp\ (A)) \cup icp\ (iip\ (A)) \subseteq A.$

Definition 2.11: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra β -open if A $\subseteq icp\ (iip\ (icp(A)))$ and infra β - closed set if $iip(icp(iip(A))) \subseteq A$.

Definition 2.12: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized- closed set if (IG-Closed set) if icp(A) ⊆ U whenever A⊆U and U is infra open.
The complement of an infra generalized- closed set is an infra generalized –open set.

Definition 2.13: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized semi- closed set if (IGS-Closed set) if sicp (A) ⊆ U whenever A⊆U and U is infra open. The complement of an infra generalized semi- closed set is an infra generalized semi–open set.

Definition 2.14: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi generalized closed if (ISG -Closed set) if sicp (A) ⊆ U whenever A⊆U and U is infra semi-open.
The complement of an infra semi generalized closed set is an infra semi generalized open set.

Definition 2.15: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized α- closed if (IGα -Closed set) if αicp (A) ⊆ U whenever A⊆U and U is infra α-open.
The complement of an infra generalized α - closed set is an infra generalized α –open set.

Definition 2.16: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra α generalized closed if (IαG-Closed set) if αicp (A) ⊆ U whenever A⊆U and U is infra -open.
The complement of an infra α generalized closed set is an infra α generalized open set.

Definition 2.17: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized b- closed set (IGB- closed set) if $b$icp (A) ⊆ U whenever A⊆U and U is infra open.
The complement of an infra generalized b - closed set is an infra generalized b –open set.

Definition 2.18: Let (X, τ_X) and (Y, τ_Y) be represent two topological spaces and τ_{iX} be associated infra topological space with τ_X. A function f: X ⟶Y is called I- continuous function at x ϵX , if ∀ open set O containing f(x) in Y, then ∃ Infra open set U containing x in τ_{iX} such that f(U) ⊂ O.

Definition 2.19: Let (X, τ_X) and (Y, τ_Y) be represent two topological spaces. Let τ_{iX} and τ_{iY} be associated infra topological space with τ_X and τ_Y respectively. A function f: X ⟶Y is called I^*- continuous function, if the inverse image of each infra open set in τ_{iY} is an τ_{iX} infra open set in X.

III. INFRA GENERALIZED b-CONTINUOUS FUNCTIONS IN INFRA TOPOLOGICAL SPACE

In this section, we devote the concept of an infra generalized continuous function. The relationship between IGB-continuous function and other defined infra continuous functions are deliberated.

Definition 3.1:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized continuous function (IG- continuous function) if the inverse image of each infra closed set in Y is an infra generalized closed set in X.

Definition 3.2:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized semi-continuous function (IGS- continuous function) if the inverse image of each infra closed set in Y is an infra generalized semi-closed set in X.

Definition 3.3:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra semi generalized continuous function (ISG- continuous function) if the inverse image of each infra closed set in Y is an infra semi generalized closed set in X.

Definition 3.4:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized pre- continuous function (IGP- continuous function) if the inverse image of each infra closed set in Y is an infra generalized pre- closed set in X.

Definition 3.5:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized α -continuous function(IG α - continuous function) if the inverse image of each infra closed set in Y is an infra generalized α- closed set in X.

Definition 3.6:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra α - generalized continuous function (I α G- continuous function) if the inverse image of each infra closed set in Y is an infra α- generalized closed set in X.

Definition 3.7:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized b-continuous (IGB- continuous function) function if the inverse image of each infra closed set in Y is an infra generalized b- closed set in X.

Definition 3.8:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized $\beta -$ continuous function (IG β - continuous function) if the inverse image of each infra closed set in Y is an infra generalized $\beta -$ closed set in X.

Definition 3.9:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra weakly generalized continuous function (IWG- continuous function) if the inverse image of each infra closed set in Y is an infra weakly generalized closed set in X.

Definition 3.10:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra generalized α b-continuous function(IG αb - continuous function) if the inverse image of each infra closed set in Y is an infra generalized α b- closed set in X.

Definition 3.11:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra semi generalized b-continuous function(ISGB- continuous function) if the inverse image of each infra closed set in Y is an infra semi generalized b- closed set in X.

Definition 3.12:
A function f: (X, τ_{iX}) → (Y, τ_{iY}) is called an infra semi weakly generalized continuous function(ISWG- continuous function) if the inverse image of each infra closed set in Y is an infra semi weakly generalized closed set in X.

Theorem 3.13:
For an infra topological space (X, τ_{iX}) we have
1.Every infra continuous function is IGB- continuous.
2.Every infra b- continuous function is IGB- continuous.
3.Every infra G- continuous function is IGB- continuous.
4.Every infra SG- continuous function is IGB- continuous.
5.Every infra GS-continuous function is IGB- continuous.
Proof:
Let V be infra closed set in (Y, τ_{iY}). Since f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}) is infra continuous, $f^{-1}(V)$ is infra closed in (X, τ_{iX}).But every infra closed set is IGB-closed. Then $f^{-1}(V)$ is IGB-closed in (X, τ_{iX}). Hence f is IGB-continuous function.
The proof of 2,3,4 and 5 are similar to 1.

Theorem 3.14:
For an infra topological space (X, τ_{iX}) we have
1.Every infra αG -continuous function is IGB- continuous.
2.Every infra $G\alpha$ - continuous function is IGB- continuous.
Proof: It is Obvious from the definition.
Remark 3.15: The converse of the above theorem is not true as shown by the following example.
Example 3.16: Let X=Y= $\{a,b,c\}$, $\tau_{iX} = \{\phi , X , \{b\} , \{c\}\}$ and $\tau_{iY} = \{\phi , X , \{a\} , \{b\}, \{b,c\}\}$.
Let f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}) be the function defined by f(a)=c , f(b)=b and f(c)=a. Then f is IGB-continuous. But $f^{-1}(a)=\{c\}$ is not IαG-closed and IGα-closed(resp. not ISG-closed and IGS-closed). Hence f is neither IαG-continuous function and IGα-continuous function (resp. f is neither ISG-continuous function and IGS-continuous function).
Example 3.17: Let X=Y= $\{a,b,c\}$, $\tau_{iX} = \{\phi , X , \{b\} , \{c\}\}$ and $\tau_{iY} = \{\phi , X , \{a\} , \{b\}, \{b,c\}\}$.
Let f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}) be the function defined by f(a)=c , f(b)=b and f(c)=a. Then f is IGB-continuous. But $f^{-1}(a)=\{c\}$ is not Ig-closed. Therefore, f is not IG-continuous function.

Theorem 3.18:
Suppose IGBO(X) is infra closed under arbitrary union. Then the following are equivalent for a function f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}).
1.f is IGB-continuous
2.For every infra open subset F of Y, $f^{-1}(F)$ is IGB-open in (X, τ_{iX})
3.For each xϵX and each infra open set F in Y containing f(x), there exists a IGB-open set U in (X, τ_{iX}) containing x such that f(U) $\subseteq$ F.
Proof:1$\Rightarrow$2. Let F be infra open in (Y, τ_Y), then Y-F is infra closed in (Y, τ_{iY}).
By (1), $f^{-1}(Y-F)$=X-$f^{-1}(F)$ is IGB-closed in (X, τ_{iX}) .This implies $f^{-1}(F)\epsilon$IGB-open. Therefore $f^{-1}(F)$ is IGB-open in (X, τ_{iX}).
2$\Rightarrow$ 1 It is Obvious.
2$\Rightarrow$3 Let F be infra open set in Y containing f(x). By (2), $f^{-1}(F)$ is IGB-open in (X, τ_X) and x $\epsilon f^{-1}(F)$. Take U=$f^{-1}(F)$ then f(U) $\subseteq$ F.
3$\Rightarrow$2 Let F be any infra open set of (Y, τ_Y) and x$\epsilon f^{-1}(F)$. From (3), there exists a IGB-open set U_x in X containing x such that $U_x \subseteq f^{-1}(F)$.
We have $f^{-1}(F)$ =$\cup_{x\epsilon f^{-1}(F)} U_x$ .Thus $f^{-1}(F)$ is IGB-open.
Remark 3.19: The composition of two infra continuous functions (I-continuous function) is also an infra continuous.
Remark 3.20: The composition of two infra generalized b- continuous functions (IGB-continuous function) need not be an infra generalized b- continuous as shown by the following example.
Example 3.21: Let X=Y=$\{a,b,c\}$,
$\tau_{iX} = \{\phi , X , \{a\} , \{b\}, \{c\}, \{a,c\}\}$,
$\tau_{iY} = \{\phi , X , \{a\} , \{b\}, \{b,c\}\}$ And
$\tau_{iZ} = \{\phi , X , \{b\}, \{c\}, \{a,c\}\}$. Let f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}) and g: (Y, τ_{iY}) $\rightarrow$ (Z, τ_{iZ}) are the identity functions. Then both f and g are IGB-continuous.
But $(g \circ f)^{-1}\{a\}$ ={a} is not IGB-closed in (X, τ_{iX}). Hence$g \circ f$: (X, τ_{iX}) $\rightarrow$ (Z, τ_{iZ})
Is not IGB-continuous function.

Theorem 3.22: Let (X, τ_{iX}) , (Y, τ_{iY}) and (Z, τ_{iZ}) be infra topological spaces. For any IGB-continuous function
f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}) and infra continuous function
g: (Y, τ_{iY}) $\rightarrow$ (Z, τ_{iZ}) then the composition $g \circ f$: (X, τ_{iX}) $\rightarrow$ (Z, τ_{iZ})
is IGB **-** continuous function.
Proof: The proof is immediate.

IV. INFRA GENERALIZED b-IRRESOLUTE FUNCTIONS IN INFRA TOPOLOGICAL SPACE

In this section, we use the new concept of infra generalized irresolute function and its characterizations are obtained.

Definition 4.1:
A function f: (X, τ_{iX}) $\rightarrow$ (Y, τ_{iY}) is called
(i) An infra generalized irresolute function (IG- irresolute function) if the inverse image of each infra generalized closed set in Y is an infra generalized closed set in X.

(ii) An infra generalized semi- irresolute function (IGS-irresolute function) if the inverse image of each infra generalized semi- closed set in Y is an infra generalized semi-closed set in X.

(iii) An infra semi generalized irresolute function (ISG-irresolute function) if the inverse image of each infra semi generalized closed set in Y is an infra semi generalized closed set in X.

(iv) An infra generalized pre- irresolute function (IGP-irresolute function) if the inverse image of each infra generalized pre- closed set in Y is an infra generalized pre-closed set in X.

(v) An infra generalized α - irresolute function (IG α -irresolute function) if the inverse image of each infra generalized α-closed set in Y is an infra generalized α- closed set in X.

(vi) An infra α -generalized irresolute function (I α G-irresolute function) if the inverse image of each infra α-generalized closed set in Y is an infra α- generalized closed set in X.

(vii) An infra generalized b- irresolute (IGB- irresolute function) function if the inverse image of each infra generalized b- closed set in Y is an infra generalized b- closed set in X.

(viii) An infra generalized $\beta -$ irresolute function (IG β -irresolute function) if the inverse image of each infra generalized $\beta -$ closed set in Y is an infra generalized $\beta -$ closed set in X.

(ix) An infra weakly generalized irresolute function (IWG-irresolute function) if the inverse image of each infra weakly generalized closed set in Y is an infra weakly generalized closed set in X.

(x) An infra generalized α b- irresolute function (IG αb -irresolute function) if the inverse image of each infra generalized α b-closed set in Y is an infra generalized α b-closed set in X.

(xi) An infra semi generalized b- irresolute function (ISGB-irresolute function) if the inverse image of each infra semi generalized b- closed set in Y is an infra semi generalized b-closed set in X.

(xii) An infra semi weakly generalized irresolute function (ISWG- irresolute function) if the inverse image of each infra semi weakly generalized closed set in Y is an infra semi weakly generalized closed set in X.

Theorem 4.2: Every IGB-irresolute function is IGB-continuous function.

Proof: It is obvious.

Remark 4.3: The converse of the above theorem is not true as shown by the following example.

Example 4.4: Let X=Y= $\{a, b, c\}$, $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}\}$ and $\tau_{iY} = \{\phi, X, \{a\}, \{b\}\}$.

Let f: (X, τ_{iX}) → (Y, τ_{iY}) be the function defined by f(a)=b , f(b)=a and f(c)=c. Then f is IGB- continuous. But $f^{-1}\{a\}=\{b\}$ is not IGB-closed. Therefore, f is not IGB- irresolute function.

Theorem 4.5: If a function f: (X, τ_{iX}) → (Y, τ_{iY}) is IGB-irresolute, then we have

(i) f(IGB-icp (A)) ⊂ IGB-icp (f(A)) for every subset A of X.

(ii)IGB-icp $(f^{-1}(A)) \subset f^{-1}(IGB-icp(A))$ for every subset B of Y.

(iii) f (bicp(A)) ⊂ bicp(f(A)) for every subset A of X.

(iv) bicp $(f^{-1}(A)) \subset f^{-1}(bicp(A))$ for every subset B of Y.

Proof: (i) For every A ⊂ X, IGB-icp (f(A)) is a IGB-closed set in (Y, τ_{iY}).

By hypothesis, f^{-1}(IGB-icp (f(A))) is IGB-closed in (X, τ_{iX}).

As A ⊂ f^{-1}(f(A)) ⊂ f^{-1}(IGB-icp (f(A))) , then IGB-icp (A)) ⊂ f^{-1}(IGB-icp (f(A))).

i.e.) f(IGB-icp (A)) ⊂ IGB-icp (f(A)).

Proof of (ii), (iii) and (iv) are similar.

Theorem 4.6: Let f: (X, τ_{iX}) → (Y, τ_{iY}) and g: (Y, τ_{iY}) → (Z, τ_{iZ}) be two functions such that

g ∘ f: (X, τ_{iX}) → (Z, τ_{iZ}) then the following properties hold:

(i)If f is IGB-irresolute and g is IGB-irresolute function, then (g ∘ f) is also IGB-irresolute.

(ii) If f is IGB-irresolute and g is IGB-continuous function, then (g ∘ f) is also IGB-continuous.

(iii) If f is IGB-continuous and g is IGB-irresolute function, then (g ∘ f) is also IGB-irresolute.

Proof: (i) Let V be IGB-closed in (Z, τ_{iZ}). Since g is IGB-irresolute, $g^{-1}(V)$ is IGB-closed in (Y, τ_{iY}). Then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}$(V) is IGB-closed in (X, τ_{iX}).Hence $g \circ$ f is IGB-irresolute.

Proof of (ii) and (iii) are obvious.

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