Design of Multi Loop PI/PID Controller for Interacting Multivariable Process with Effective Open Loop Transfer Function

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Abstract:
This paper represents details of multi loop processes, which are complicated for comparing with single loop processes due to the existence of interactions between manipulated and controlled variables. The interactions between input/output variables are a very common phenomenon encountered in the design of multi-loop controllers for interacting multivariable processes. The knowledge of an effective open-loop transfer function (EOTF) is first introduced to decompose a multi-loop control system into a set of equivalent independent single loops. With the help of a model reduction technique, the EOTF is further approximated to the reduced order form. The individual controller of each single loop is then independently designed based on the corresponding EOTF model by applying the internal model control (IMC) based PID tuning approach for single input single output (SISO) systems, while the main effects of the dynamic interactions are properly taken into account. Benchmark examples of multivariable industrial processes with different interaction characteristics are used to demonstrate and simplify effectiveness of the design method which is also simulated using MATLAB.

Keywords: Multi loop PI/PID controller, Effective open-loop transfer function (EOTF), Internal model control (IMC), Dynamic relative gain array (DRGA), Robustness.

I. INTRODUCTION

The PID controller is by far the most common control algorithm. Most feedback loops are controlled by this algorithm or minor variations. Basically, fig 1. shows the typical structure of multi input multi output (MIMO) systems. Despite the development of advanced multivariable controllers, the multi loop PI/PID control using multiple Single input single output (SISO) PI/PID controllers remains the standard for controlling MIMO systems with modest interaction because of its simple and failure tolerant structure and adequate Performance [1,2]. However, due to process and loop interactions, the design and tuning of multi-loop controllers is much more difficult compared with that of single loop controller; since the controllers interact with each other, the tuning of one loop cannot be done independently. Applying the tuning methods for a SISO system to multi-loop systems often leads to poor performance and stability.

Fig 1: Typical structure of multi-input/multi-output (MIMO) systems

Much research has been focused on how to efficiently take loop interactions into account in the multi-loop controller design. Many methods have been proposed, including the detuning method, sequential loop closing (SLC) method, relay auto-tuning method, and independent loop method. The biggest log modulus tuning (BLT) method proposed by Luyben [3] is a best example of the detuning method, wherein each individual controller is first designed based on the Ziegler–Nichols (Z–N) tuning rule [4] by ignoring process interactions from other loops. Then, the interactions are taken into account by detuning each controller until the multivariable Nyquist stability is satisfied. The attractiveness of this method is due to the simplicity in implementation and comprehensibleness for control engineers. However, a disadvantage is that the controller settings are made more conservative. The independent loop method is used to overcome the restriction of the relay auto-tuning. As discussed by a number of authors [5–7], the independent loop method has a potential advantage in that the failure tolerance of the overall control system is automatically guaranteed, wherein each controller is independently designed based on the corresponding open-loop and closed-loop transfer functions, thereby satisfying the inequality constraints on the process interactions [8]. A potential disadvantage of the method is When the interactions among different loops are modest a simple multi-loop PI/PID controller is normally adequate, however, the decentralized controller design method may fails to give acceptable responses if there exist severe loop interactions. As a result, many industrial decentralized controllers are tuned loosely to ensure system stability, which causes inefficient operation and higher energy costs. For this, the independent loop method for IMC type multi-loop controllers [9, 10] is used to reduce the conservatism. P. Jayachandra1 & dilip. K. Maghade [11] have compared two Methods for two by two processes with time delays. One is model order reduction and other is without reduction. Performance index and robustness has been used as the criterion for comparison. Several commonly used simulation examples are included for demonstrating effectiveness of the proposed methods and the results obtained are comparatively same. Multi-loop PI/PID Controller Design Based on Direct Synthesis for Multivariable Systems [12] is designed by Truong Nguyen Luan Vu and Monoyong Lee in which the method based on the direct synthesis approach is proposed for the designing controller. It is aimed to achieve a desired
closed-loop response with multiple time delays. The ideal multi-loop controller is firstly designed in terms of relative gain and desired closed-loop transfer function. Then the standard multi-loop PID controller is obtained by approximating the ideal multi-loop controller by the Macraulin series expansion. Simulation study demonstrates the effectiveness of the proposed method in the multi-loop PID controller design. The multi-loop PID controller designed by the proposed method shows a fast, well balanced, and robust response with the minimum integral absolute error (IAE). Again, the paper titled “Independent Design of Multi-loop PI/PID Controllers for Multi delay Processes” [13] represents the interactions between input/output variables are a very common phenomenon encountered in the design of multi-loop controllers for interacting multivariable processes, which can be a serious obstacle for achieving a good overall performance of multi loop control system. To overcome this impediment, the decomposed dynamic interaction analysis is proposed by decomposing the multi loop control system into more number of independent SISO systems with the corresponding effective open loop transfer function (EOTF) within the dynamic interactions embedded explicitly. For each EOTF, the reduced model is independently formulated by using the proposed reduction design strategy, and then the paired multi-loop proportional-integral-derivative (PID) controller is derived quite simply and straightforwardly by using internal model control (IMC) theory. This design method can easily be implemented for various industrial processes because of its effectiveness. Several case studies are considered to demonstrate the superior of the proposed method.

Multi loop controller synthesis and performance analysis by Vinay Kariwala [15] as over the past few decades, many algorithms have been proposed for controller design. In practice, an engineer needs to address the following issues before the actual controller can be designed: which variables should be measured, controlled and manipulated, and what links should be made between them. These decisions are often taken heuristically, which has an adverse effect on the safe and economic operation of the process. In this thesis, simple yet theoretically sound tools are developed for partitioning of the measurements and manipulations for control of complex systems. The task of controller design is much simplified by pre-stabilizing the system using a subset of variables. Selecting the subset of variables by minimization of the input energy required for stabilization reduces the likelihood of otherwise destabilizing input saturation. The achievable input performance for linear systems is characterized and an iterative method is presented for variable selection. Use of online performance monitoring tools is necessary to identify significant performance degradation and subsequent remedial steps. The existing methods are inadequate for performance monitoring of decentralized controllers and a sub-optimal, but explicit solution to the decentralized minimum variance benchmark problem is proposed. The tools presented in this thesis can be used individually or synthesized into a comprehensive design procedure with possible minor extensions.

In this paper section I represents overall introduction for system and basic literature used. In section II methodology is analyzed with relevant examples and mathematical modeling. The comparison in different methods are described in third session. The experimental results are obtained with help of MATLAB and SIMULINK in section IV. Finally, conclusion and future scope are drawn.

II. EFFECTIVE OPEN-LOOP TRANSFER FUNCTION:

Effective Open-loop Transfer Function method for design of multi-loop PI/PID controller for interacting multivariable processes is developed by A. Harsha Ragini, K. Rajesh, B. Hari Krishna [16]. In terms of control multi loop processes are very complicated when compared to single loop processes due to the existence of interactions between manipulated and controlled variables. During the last several decades this topic has drawn lot of research attention and many multivariable control approaches have been proposed. This Project proposes an Effective Open-loop Transfer Function (EOTF) method for PI/PID design of multi-loop control processes, by employing the concepts of Internal Model Control (IMC) with reduced EOTF and Dynamic Relative Gain Array (DRGA). With this method a multi loop control system is decomposed into a set of independent single input single output (SISO) loops. The advantages of the proposed method are: (i) the overall control system performance is better compared with existing methods, and (ii) the control system is robust. This technique works with satisfactory performance even under significant model mismatches that commonly occur in the process control industry. With the help of Effective Open-loop Transfer Function (EOTF) – based method and DRGA concept, dynamic interactions are satisfactorily taken in to account. Benchmark examples of multivariable industrial processes with different interaction characteristics are used in this paper to demonstrate the simplicity and effectiveness of the proposed design method. The proposed method is simulated using MATLAB/SIMULINK.

Consider the open-loop stable multi-loop system in Fig. 2 where $r^{-1}$, $u^{-1}$ and $y^{-1}$ are the set-point, manipulated, and controlled variable vectors, where $r$, $u$, and $y$ are discarded from $r$, $u$, and $y$, respectively. Let the EOTF of loop i be defined as the transfer function relating $u_i$ with $y_i$ where loop i is open while all other loops are closed. Fig. 1 shows the block diagram for the concept of the EOTF of loop i. The EOTF differs from the original open-loop transfer function (OTF) by transmission interaction through a path including other loops. It is clear that the EOTF corresponds to the actual open loop transfer function under multi loop situations and thus, tuning of the controller of loop i should be done based on the EOTF.

![Fig. 2: Structure of an effective open loop transfer function in multi loop system](http://ijesc.org/)
Moreover, as shown an the open and i moved from G. The denotes the i (, consists of a process for this purpose. In this is discarded, respectively. = × − = 

III. REDUCED EFFECTIVE OPEN LOOP TRANSFER FUNCTION:
One of the most common approaches for controller design is to use a reduced-order model that simplifies the process dynamics. Since the EOTF is likely to show a complicated dynamic model form, the approach using a reduced-order model is generally required. Any conventional model reduction technique can be applied for this purpose. In this section, a simple model reduction technique is applied to approximate the EOTF to a reduced-order model, such as the first-order plus dead time (FOPDT) and the second-order plus dead time (SOPDT) models. A two-input, two-output (TITO) multi-delay process is one of the most commonly encountered multivariable processes in the process industry. A large number of previous studies focused on designing multi-loop control system of TITO processes. For a 2×2 system, the general stable square transfer function matrix is represented as

\[
G(s) = \begin{bmatrix}
    g_{11}(s) & g_{12}(s) \\
    g_{21}(s) & g_{22}(s)
\end{bmatrix}
\]

The DRGA obtained from (6) is

\[
\Lambda_{11}(s) = \Lambda_{22}(s) = \frac{g_{11}(s)g_{22}(s)}{g_{12}(s)g_{21}(s)}
\]

Therefore, the EOTFs for the first and second loops are found using (5), respectively:

\[
g_{11}^{\text{eff}}(s) = g_{11}(s) - \frac{g_{12}(s)g_{22}(s)}{g_{12}(s)g_{21}(s)}
\]

\[
g_{22}^{\text{eff}}(s) = g_{22}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{12}(s)g_{22}(s)}
\]

For a 3×3 system, the derivation of EOTF is substantiated in a similar manner as that of the 2×2 system. The diagonal elements of the DRGA are established from (6) as follows

Thus, the EOTFs for the first, second and third loops are constituted as,

\[
g_{11}^{\text{eff}} = g_{11} - \left[ \frac{g_{12}g_{21}g_{31} + g_{13}g_{32}g_{23} + g_{23}g_{32}g_{13}}{2g_{12}g_{21}g_{31}} \right]
\]

\[
g_{22}^{\text{eff}} = g_{22} - \left[ \frac{g_{23}g_{32}g_{13} + g_{12}g_{21}g_{31} + g_{13}g_{32}g_{23}}{2g_{12}g_{21}g_{31}} \right]
\]

\[
g_{33}^{\text{eff}} = g_{33} - \left[ \frac{g_{31}g_{13}g_{32} + g_{12}g_{21}g_{31} + g_{13}g_{32}g_{23}}{2g_{12}g_{21}g_{31}} \right]
\]

As seen from the equations above, the resulting EOTFs are usually too complicate to be directly utilized for the controller design. To overcome this difficulty, the EOTFs have to be simplified to low-order models, such as FOPDT and SOPDT. A lot of model reduction techniques are available, including, but not limited to, the east squares algorithm, polynomial approximation, Laguerre expansion, and the Gaussian

frequency domain approach. Any technique can be applied toward fitting the EOTFs into a low-order model.

In this work, for the purpose of evaluating the proposed EOTF, a simple model reduction technique was proposed based on the coefficient matching method. Expanding $g_{i;i}^{eff}$ in a Maclaurin series in $s$ gives

$$g_{i;i}^{eff}(s) = a_{ii} + b_{ii}s + c_{ii}s^2 + d_{ii}s^3 + e_{ii}s^4 + O(s^5) \quad (14)$$

Where,

$$a_{ii} = g_{i;i}^{eff}(0)$$

$$b_{ii} = \left. \frac{d g_{i;i}^{eff}(s)}{d s} \right|_{s = 0} = 0$$

$$c_{ii} = \frac{1}{2} \left. \frac{d^2 g_{i;i}^{eff}(s)}{d s^2} \right|_{s = 0} = 0$$

$$d_{ii} = \frac{1}{6} \left. \frac{d^3 g_{i;i}^{eff}(s)}{d s^3} \right|_{s = 0} = 0$$

$$e_{ii} = \frac{1}{24} \left. \frac{d^4 g_{i;i}^{eff}(s)}{d s^4} \right|_{s = 0} = 0$$

IV. MULTI-LOOP PID CONTROLLER DESIGN

Once a reduced EOTF is obtained, any PID tuning method for a SISO system can be applied for the design of each individual PID controller. The IMC-PID design approach is commonly used for the PID controller tuning in the process industry because of its many advantages, including simplicity, robust performance and analytical form. The overall procedure for driving the tuning rules of loop $i$ is as follows:

First, the reduced EOTF, $g_{i;i}^{eff}$, is decomposed to $g_{i;i}^{eff} = p_{Ai}p_{Mi}$, where $p_{Ai}$ and $p_{Mi}$ are the non-minimum portion with an all-pass form and the minimum phase portion, respectively. The conventional IMC filter, $f$, is selected as: $f_i(s) = \frac{1}{\lambda_i^s+1}$, in which $\lambda_i$ is a design parameter that provides the tradeoff between performance and robustness. It is the desired closed-loop time constant for the set-point tracking. The filter order $m_i$ is selected as a positive integer so that the controller is proper and realizable. Then, the ideal feedback controller to yield the desired closed loop response perfectly is given by

$$g_{ci}(s) = k_{ci} \left( \frac{1}{\tau_{li} s + 1} + \frac{1}{\tau_{di} s + 1} \right)$$

Where,

$$k_{ci} = \frac{f'(0)}{f(0)}$$

$$\tau_{li} = \frac{f''(0)}{f'(0)}$$

$$\tau_{di} = \frac{f''(0)}{2f'(0)}$$

The derivative and integral time constants computed from (17) could have negative values when the reduced EOTF model has a strong lead term. In this case, a PID controller in series with the first-order lag filter structure is recommended for use

$$g_i(s) = k_{ci} \left( \frac{1}{\tau_{li} s + 1} + \frac{1}{\tau_{di} s + 1} \right)$$

Where

$$k_{ci} = \frac{f'(0)}{f''(0)}$$

$$\tau_{li} = \frac{f''(0)}{f'(0)}$$

$$\tau_{di} = \frac{f''(0)}{2f'(0)}$$

V. RESULT AND DISCUSSION

In this section, examples are considered to demonstrate the performance of the proposed method in comparison with those of other well-known methods. To ensure a fair comparison, the performance and robustness of the control system are measured by the following evaluation criteria.

1. Performance index
To evaluate closed-loop performance, the integral absolute error (IAE) criterion is considered, which is defined as:

$$IAE = \int_0^\infty |e(t)| \, dt$$

Where, $e(t) = r(t) - y(t)$.

2. Robustness index
In this study, a well-known method for robust stability is utilized for a fair comparison with other comparative methods. The multiple sources of uncertainty are lumped into a single complex perturbation (multiplicative input/output form). Since the output uncertainty is often less restrictive than input uncertainty in terms of control performance, the robust stability of multi-loop control systems is examined under...
output multiplication uncertainty. For a process with an output uncertainty of $[I + \Delta_0(s)] G(s)$, the upper bound of the robust stability is given as

$$\gamma = \delta(\Delta_0) < 1/\lambda \left[ \frac{G(j\omega) G_c(j\omega)}{1 + G(j\omega) G_c(j\omega)} \right] < \delta(\Delta_0(j\omega)), \forall \omega > 0$$

Where $\gamma$ represents the degree of robust stability, $\Delta_0$ perturbation as a multiplicative output, and $\lambda$ and $\delta$ maximum and minimum singular values, respectively. It should be noted that: $G(j\omega)$ is invertible. For a fair comparison, all of the controllers being compared were designed to have the same degree of robust stability in terms of the $\gamma$ value throughout all simulation examples. For the proposed control system, the $\gamma$ value was kept the same as or larger than those of the other methods.

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**Fig 4:** Nyquist magnitude plots of reduced EOTFs

**Fig 5:** Time response of reduced EOTFs

**Fig 6:** Closed loop responses to sequential step changes in the set point

Note that a control system with a larger $\gamma$ value implies more robust stability. Basically to design multi loop PID controller with reduced effective open loop transfer function three algorithm are implemented and compare with each other with the help of Vinante and Luyben (VL) column algorithm as bellow A 24 tray tower separating a mixture of methanol and water, examined by Luyben (3), has the following transfer function

$$G(s) = \begin{bmatrix} -2.42e^{-1.1s} & 1.43e^{-0.335s} \\ 7.7s + 1.1 & 7.7s + 1.1 \\ -3.08e^{-1.98s} & 4.73e^{-0.335s} \\ 10.45s + 1.1 & 10.12s + 1.1 \end{bmatrix}$$

For this TITO system, it follows from (9) and (10) that the EOTFs for the first and second loops are obtained as

$$G_{11}^{\text{eff}}(s) = \frac{-2.42e^{-1.1s}}{7.7s + 1.1} - \frac{1.43e^{-0.335s}}{7.7s + 1.1} - \frac{-3.08e^{-1.98s}}{4.73e^{-0.335s}} \frac{10.45s + 1.1}{10.12s + 1.1}$$

$$G_{22}^{\text{eff}}(s) = \frac{4.73e^{-0.335s}}{10.12s + 1.1} - \frac{1.43e^{-0.335s}}{7.7s + 1.1} - \frac{-3.08e^{-1.98s}}{7.7s + 1.1} \frac{-2.42e^{-1.35}}{7.7s + 1.1}$$

The reduced EOTFs for the corresponding EOTFs are constituted using (21a)–(21c) as follows

$$g_{11}^{\text{eff}} = \frac{-1.29e^{-0.652s}}{6.641s + 1} ; \quad g_{22}^{\text{eff}} = \frac{-2.64e^{-0.552s}}{0.91s + 1}$$
To evaluate how closely the proposed reduced EOTF approximates the actual EOTF, the Bode diagrams and the time responses are drawn for several cases. Figs. 4 and 5 compare the Bode diagrams and the time responses of the reduced EOTF, EOTF, original OTF, and actual EOTF. From Fig. 4, both the reduced EOTF and EOTF show a fairly good coincidence with the actual EOTFs over the control relevant low and middle frequency ranges, becoming more conservative as the frequency increases. Note that the response of the actual EOTF is the actual response based on, and thus depends on the controller of other loops. Fig. 5 compares the time responses of the reduced EOTF, EOTF, original OTF, and actual EOTF. As seen from Fig. 5, the time response of the reduced EOTF is closely approximated to the actual EOTF. These Close approximations to the actual EOTF in the frequency and time responses illustrate the validity of the EOTF and the reduced EOTF, and also essentially lead to satisfactory control performance of the multi-loop controller designed based on the EOTF. Furthermore, the significant difference between the original OTF, \( g_{ii} \), and the other EOTFs confirm that the tuning of the individual multi-loop controller should be done based on the EOTF rather than the original OTF. Fig. 6 shows the closed-loop responses by several tuning methods. In the simulation study, the unit step set-point changes were sequentially introduced into the individual loops. For both the proposed method and Lee et al.’s method, \( \lambda_i \) was adjusted to have a degree of robust stability as \( \gamma = 0.53 \), which is the same as that obtained by Lee et al.’s method. Note that since the method of He et al. utilized the SIMC PID tuning rule by setting \( \lambda_i \) equal to \( \Theta_i \), their respective tuning values were employed in the simulation without adjusting the \( \gamma \) value. Fig. 6 compares the output responses afforded by the proposed method with those given by others where loop 1 is closed and loop 2 is open. The resulting controller parameters, together with the performance indices calculated using the abovementioned methods. It is apparent the proposed PI/PID controller provides a good performance with fast and well-balanced responses in comparison with those of the existing methods. The robustness of the controller is evaluated by inserting a perturbation uncertainty of \( \pm40\% \) in the process gain into the actual process, whereas the controller settings are those provided for the nominal process. The proposed controller gives best robust performance consistently.

**VI. Conclusion**

A novel analytical design method is proposed for independent design of a multi-loop PID controller. The proposed method is straightforward and easy to implement on the multi-loop control systems based on the concept of EOTF. The robust stability and performance can be efficiently fixed by adjusting a single parameter, i.e., the closed-loop time constant. The EOTF concept was successfully applied to decompose the complex multi-loop control systems into a number of independent SISO loops in which dynamic interaction is taken into account. Therefore, the multi-loop PID controller should be accomplished by designing the SISO PI/PID controllers for each loop based on the corresponding EOTF model. The time-domain simulation illustrates that the proposed control system provides a fast and well-balanced closed-loop time responses. A model reduction technique may be proposed to further simplify the EOTF to the reduce order form. The IMC-PID approach will apply to the reduce EOTF to design the individual PI/PID controller in each loop.

**VII. References**


