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Paraxial Study of Self Focusing of Q- Gaussian Laser Beam in a Density Transition

Sukhdeep Kaur[1], Manpreet Kaur[2]
Assistant Professor[1], Ph.D Student[2]
Department of Physics
Guru Nanak Dev Univesity, Amritsar, India

Abstract:
This paper presents an investigation of relativistic and ponderomotive self-focusing of a q-Gaussian laser beam in a density transition. Non -linear parabolic partial differential equation governing the evolution of complex envelope in slowly varying approximation is solved in a modulated density profile. We have developed analytical theory of self focusing in paraxial ray approximation and derive nonlinear parabolic equation for the study of spot size of the laser beam with distance of propagation. We found that intensity distribution of q- Gaussian laser beam and density variation affects the phenomenon of self-focusing. Enhanced self focusing is achieved for a particular set of parameters.

Keywords: Beam width parameter, ponderomotive and relativistic nonlinearity q-gaussian laser beam, self-focusing.

I. INTRODUCTION

After the invention of chirped pulse amplification (CPA), the construction of ultra-intense short laser pulses has become feasible. The propagation of high power short laser pulse in non-linear media like plasma has a major area of research due to its application in laser ion accelerator [1], harmonic generation [2], super continuum generation [3], proton acceleration [4], inertial confinement fusion [5], X-ray generation [6] and fast ignition schemes of laser-driven fusion [7]. There are different types of nonlinearities such as thermal, relativistic and ponderomotive which modifies the propagation and nonlinear properties. One such non –linear phenomena is self-focusing of laser beams through plasma with density variation attracts the scientific community for the need of resonant harmonic generation. In non-linear media when laser pulse propagates, the refractive index changes, due to induced quiver motion of electrons which results from electric field [8]. Hence, the plasma initially behaves like a positive lens that decreases the spot size of laser beam [9]. Self focusing of the laser beam effects the non-linear effects, which are very sensitive to the radiation distribution along the wave front. Kaur et al [10] studied the effect of density ripple on self focusing of laser beam in plasma. A suitable wave number $m = k_n - nk_1$ of the rippled density in the direction of laser propagation provides uncompensated momentum and turns the process into resonant. Kuo et al [11] experimentally investigated third harmonic generation in such density profile. They found that under quasi-phase matching conditions, the yield of third harmonic generation increases by factor 5. By using Ti:sapphire laser beam even- and odd-order harmonics are generated [12], and harmonics up to the 75th order of Nd laser beam were obtained by the wavelength of 14.0 nm [13]. To study the self-focusing of the laser beam in a plasma there are several analytical methods such as paraxial ray approximation, variational approach, moment theory approach and source dependent expansion method. Out of these paraxial ray approximation given by Akhmanov *et al.* [14] which is further developed by Sodha et.al [15] used by most scientific community due to its mathematical simplifications. Theoretical and experimental investigations of self-focusing on different types of laser beams such as Gaussian beam [16], hollow Gaussian beams [17], super Gaussian beams [18], Hermite Cosh Gaussian beams [19] and dark hollow Gaussian beams [20] in nonlinear media has been studied recently. Currently, there is a vital interest in self-focusing of q-Gaussian laser beams in plasmas. The laser profile deviated from Gaussian distribution has been found by Vulcan Peta watt laser. Nakat-sutsumi et al. [21] studied that q-Gaussian distribution given by Vulcan laser is of the form, $f(r) = f(0)\left(1+\frac{r^2}{qr_0^2}\right)^{-q}$. By using the experimental data, the values of q and r_0 can be obtained. In this case, the magnitude of harmonic generation is found more than obtained from Gaussian profile and it is due to the higher value of electric field average square value in the wave front. Singh and Gupta [22] examined that with increase in q value or with increase in laser intensity self-focusing is enhanced and yield of harmonic generation of increases in a plasma. The aim of the paper is to investigate self-focusing of q-Gaussian laser beam due to relativistic and ponderomotive nonlinearity in a periodically modulated density profile. Non - linear parabolic partial differential equation governing the evolution of complex envelope in slowly varying approximation is solved. We have developed analytical theory of self focusing in paraxial ray approximation and derive nonlinear parabolic equation for the study of spot size of the laser beam with distance of propagation. In section II, we develop the analytical theory. In section III, the wave equation for the beam width parameter is solved numerically for relevant parameters and results were discussed.

II. SELF FOCUSING

Let the equilibrium electron density n_0 be sinusoidal,

$$n_0 = n_0^0(1+\alpha_2 cosmz). \quad (1)$$

where α_2 is the depth of density modulation.

A q-Gaussian laser beam launched into the plasma propagates through the plasma along $\hat{z}$.

$$\vec{E} = \hat{x}A_0(r,z)e^{-i(\omega t - kz)}. \quad (2)$$

Where, $A_0^2|_{z=0} = A_{00}^2\left(1+\frac{r^2}{qr_0^2}\right)^{-q}$, and $A_0(r,z)$ is the complex amplitude of the electric field of the laser beam. A_{00} is the axial amplitude of the laser beam, r_0 is the spot size of the laser beam at $z = 0$ and q is a parameter associated with the deviation of intensity profile of laser beam from Gaussian distribution.

The laser imparts oscillatory velocity to electrons,

$$\overrightarrow{v_{\omega,k}} = \frac{e\vec{E}}{mi\omega\gamma}. \tag{3}$$

Where, $\gamma = \left(1+{a^2}/{2}\right)^{1/2}$ and $a = \frac{e|A|}{m\omega c}$.

The ponderomotive force on electron will be,

$$\vec{F}_p = -e\nabla\varphi_p\ . \tag{4}$$

Where, $\varphi_p = -\frac{mc^2}{e}(\gamma-1)$.

In quasi steady state with $\vec{F}_p - e\vec{E}_s = 0$ i.e $\varphi_s = -\varphi_p$, following Tripathi et al [23], modified electron density is written as:

$$n_e = n_0^0(1+\alpha_2 cosmz) + \frac{1}{4\pi e}\nabla^2\varphi_s. \tag{5}$$

$$n_e = \quad n_0^0(1+\alpha_2 Cosmz)\left[1 - \frac{c^2}{\omega_p^2 r_0^2 f^2}\frac{a^2}{\left(1+\frac{a^2}{2}\right)^{1/2}}\left(1 - \right.\right.$$

r2r022q+1+a241+a22
(6)

The effective permittivity of the plasma is given by:

$$\varepsilon = \epsilon_0 - \varphi\frac{r^2}{r_0^2} = 1 - \frac{1}{\gamma}\frac{\omega_p^2}{\omega^2}\frac{n_e}{n_0^0}. \tag{7}$$

with $\epsilon_0 = 1 - \frac{1}{\gamma}\frac{\omega_p^2}{\omega^2}\left[(1+\alpha_2 Cosmz) - \frac{c^2}{r_0^2 f^2\omega_p^2}\frac{a^2}{\left(1+\frac{a^2}{2}\right)^{1/2}}\right]$

$$\varphi = \frac{\omega_p^2}{\gamma_0^3\omega^2}\frac{a^2}{f^4}\left((1+\alpha_2 Cosmz) + \frac{8}{q}\frac{c^2\gamma_0}{\omega_{p0}^2 r_0^2 f^2} + \right.$$

c2ωp02r02f28+a2f2γ0.

The wave equation governing the laser field in the WKB approximation, given by

$$2ik_0\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega^2}{c^2}\varphi = 0. \tag{8}$$

Expressing $A = A_0(r,z)e^{ikS(r,z)}$, where A_0 and S are the real and separating out real and imaginary parts of upper equation. Real part is:

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 + \frac{2S}{k}\left(\frac{\partial k}{\partial z}\right) - \frac{1}{2k^2A_0^2}\left(\frac{\partial^2 A_0^2}{\partial r^2}\right) + \frac{1}{4k^2A_0^2}\left(\frac{\partial A_0^2}{\partial r}\right)^2 - \frac{1}{2rk^2A_0^2}\left(\frac{\partial A_0^2}{\partial r}\right) - \frac{\omega^2}{c^2}\varphi = 0\ . \tag{9}$$

Expanding S in powers of r^2 as $S = \frac{r^2}{2}\beta(z) + \varphi(z)$, where $\beta(z) = \frac{1}{f(z)}\frac{df}{dz}$, where $\phi(z)$ is an arbitrary function of z. on equating the coefficient of r^2 on both sides, we obtain the equation governing the beam width parameter,

$$\frac{d^2 f}{d\xi^2} = \frac{1}{f^3}\left(1+\frac{4}{q}\right) - \frac{r^2}{\epsilon_0}\varphi f. \tag{10}$$

where $\xi = {z}/{R_{d0}}$ and $d = mR_d$ is the normalized ripple wave number. The first term on RHS is due to diffraction, while the second term is due to non-linear self-focusing. For an initially plane wave front, the boundary conditions are at $\xi = 0$, $f = 1$ and $\frac{df}{d\xi} = 0$.

III. RESULTS

We have solved equation (10) numerically for q- Gaussian laser beam for following parameters: $\frac{\omega_p^2}{\omega^2} = 0.02, a = 1, \frac{r_0\omega}{c} = 50$, $\alpha2 = 0.1$. In Figure 1, at a fixed value of normalized density parameter d = 40, and varying the q- value viz, q=5, 10, 25 (red, dashed blue, dashed black color) in laser beam profile, we found that self-focusing enhanced for higher value of q i.e. , q = 25.

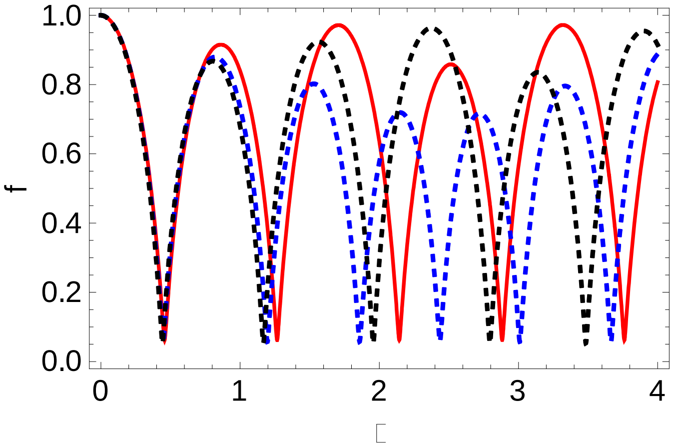

Figure.1. Variation of beam width parameter *f* against the normalized distance of propagation ξ for different values of q viz., q = 5(red line), 10 (dashed blue), 25 (dashed black) and at fixed values of parameters, d=40, α_2=0.1, $\frac{\omega_{p0}^2}{\omega^2} = 0.02$, $\frac{\omega r_0}{c} = 50$, $a = 1$.

Figure 2 demonstrates variation of beam width parameter with the normalized distance of propagation for different value of normalized density wave no. The plasma is underdense and other parameters are α_2=0.1, $\frac{\omega_{p0}^2}{\omega^2} = 0.02$, $\frac{\omega r_0}{c} = 100$, $a = 1$. When we fix the value of q and vary the periodically density parameter, we found that for less density wave no, wiggle in beam width parameter is large. As we increase the value of density wave number, the beam shows strong focusing.

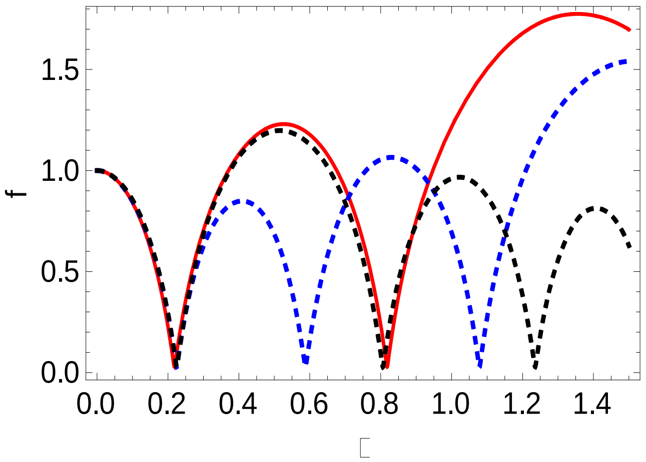

Figure.2. Variation of beam width parameter *f* against the normalized distance of propagation ξ for different values of m viz., d = 10 (red line), 25(dashed blue), 60 (dashed blacks) and at fixed values of parameters, q = 15, α_2=0.1, $\frac{\omega_{p0}^2}{\omega^2} = 0.02$, $\frac{\omega r_0}{c} = 100$, $a = 1$.

REFERENCES

[1]. B. M. Hegelich, I. Pomerantz, L. Yin, H. C. Wu, D. Jung, B. J. Albright, D. C. Gautier, S. Letzring, S. Palaniyappan, R. Shah, K. Allinger, R. Hörlein, J. Schreiber, D. Habs, J. Blakeney, G. Dyer, L. Fuller, E. Gaul, E. Mccary, A. R. Meadows, C. Wang, T. Ditmire, and J. C. Fernandez. (2013, August). "Laser-driven ion acceleration from relativistically transparent nanotargets." New Journal of physics.Vol.15, no 0. pp. 085015.

[2]. F. Dollar, P. Cummings, V. Chvykov, L. Willingale, M. Vargas, V.Yanovsky, C. Zulick, A. Maksimchuk, A. G. R. Thomas, and K. Krushelnick. (2013, April). "Scaling High-Order Harmonic Generation from Laser-Solid Interactions to Ultrahigh Intensity." Physical Review Letters. Vol.110, no 17. pp 175002-5.

[3]. V.P. Kandidov, O.G. Kosareva, I. S. Golubtsov, W. Liu, A. Becker, N. Akozbek, C. M. Bowden, and S. L. Chin. (2003, September). "Self-transformation of a powerful femtosecond laser pulse into a white-light laser pulse in bulk optical media (or supercontinuum generation)." Applied Physics B. Vol. 77. no 2-3. pp.149-165

[4]. H. W. Powell, M. King, R. J. Gray, D. A. MacLellan, B. Gonzalez-Izquierdo, L. C. Stockhausen, G.Hicks, N. P. Dover, D. R. Rusby, D. C. Carroll, H. Padda, R. Torres, S. Kar, R. J. Clarke, I. O. Musgrave, Z. Najmudin, M. Borghesi, D.Neely, and P. McKenna. (2015, October). "Proton acceleration enhanced by a plasma jet in expanding foils undergoing relativistic transparency." New Journal of Physics. Vol.17, no..pp. 103033.

[5]. F. H. Seguin, N. Sinenian, M. Rosenberg, A. Zylstra, M. J.-E. Manuel, H. Sio, C. Waugh, H. G. Rinderknecht, M. Gatu Johnson, J. Frenje, C. K. Li, R. Petrasso, T. C. Sangster, and S. Roberts. (2012, July). "Advances in compact proton spectrometers for inertial-confinement fusion and plasma nuclear science." Review of Scientific Instruments. Vol. 83, no 10. pp. 10D908 -3

[6]. J. Weisshaupt, V. Juvé, M. Holtz, M. Woerner, and T. Elsaesser. (2015, March). "Theoretical analysis of hard x-ray generation by nonperturbative interaction of ultrashort light pulses with a metal." Structural Dynamic. Vol. 2,no 2. pp. 024102-19.

[7]. J. J. Honrubia, J. C. Fernández, M. Temporal, B. M. Hegelich, and J. Meyer-ter-Vehn. (2009, October). "Fast ignition of inertial fusion targets by laser-driven carbon beams." Physics of Plasmas.Vol. 16.no 10. pp.1027017.

[8]. H. Hora. (1975) . "Theory of relativistic self-focusing of laser radiation in plasmas." Journal of the Optical Society of America. Vol. 65, no 8. pp. 882-886

[9]. J. Faure,V. Malka, J.-R.Marquès, P.-G. David, F.Amiranoff, K. Ta Phuoc, and A. Rousse, (2002, March). "Effects of pulse duration on self-focusing of ultra-short lasers in underdense plasmas." Physics of Plasmas. Vol. 9, no 3. pp. 756-759

[10]. S. Kaur and A. K. Sharma. (2009, November). "Self Focusing of a Laser Pulse in Plasma with Periodic Density Ripple." Laser and Particle Beams. Vol. 27.no 2. pp.193-199

[11]. C. C. Kuo, C. H. Pai, M. W. Lin, K. H. Lee, J.Y. Lin, J. Wang, and S. Y. Chen. (2007, January). "Enhancement of Relativistic Harmonic Generation by an Optically Preformed Periodic PlasmaWaveguide." Physical Review Letters. Vol. 98, no 3. pp. 033901-4

[12]. S . Kohlweyer, G.D. Tsakiris, C.-G. Wahlstrijm, C. Tillman , and I. Mercer . (1995, March). "Harmonic generation from solid-vacuum interface irradiated at high laser intensities." Optics Communications. Vol. 117, no 5-6,. pp. 431-438

[13]. P. A. Norreys, M. Zepf, S. Moustaizis, A. P. Fews, J. Zhang, P. Lee, M. Bakarezos, C. N. Danson, A. Dyson, P. Gibbon, P. Loukakos, D. Neely, F. N. Walsh, J. S. Wark, And A. E. Dangor. (1996, March). "Efficient Extreme UV Harmonics Generated from Picosecond Laser Pulse Interactions with Solid Targets."Physical Review Letters. Vol. 76, no 11. pp. 1832-1835

[14]. S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov. (June, 1966). "Self-Focusing and Self-Trapping of Intense Light Beams in a Nonlinear Medium." Soviet Physics JETP. Vol. 23, no 6. pp. 1537-1549

[15]. M. S. Sodha, A. K. Ghatak, and V. K. Tripathi. (1976). "V Self –Focusing of Laser Beams in Plasmas and Semiconductors." Progress in Optics. Vol. 13, no 0. pp. 169-265

[16]. S. Zare, S. Rezaee, E. Yazdani, A. Anvari, and R. Sadighi-Bonabi. (2015, September). "Relativistic Gaussian Laser Beam Self-Focusing in Collisional Quantum Plasmas." Laser and Particle Beams. Vol. 33, no 3. pp. 397–403

[17]. M. S. Sodha, S. K. Mishra and S. Misra (2009, March). "Focusing of dark hollow Gaussian electromagnetic beams in a plasma" Laser and Particle Beams. Vol. no 27, no 1, pp. 57-68.

[18]. T. S. Gill, R. Kaur, and R. Mahajan. (2015, September). "Self-focusing of super-Gaussian laser beam inmagnetized plasma under relativistic and ponderomotive regime." Optik - International Journal for Light and Electron Optics. Vol. 126, no 18. pp 1683-1690,

[19]. S.D. Patil, M.V. Takale, S.T. Navare, and M.B. Dongare. (2010, June). "Focusing of Hermite-cosh-Gaussian laser beams in collisionless magnetoplasma." Laser and Particle Beams. Vol. 28, no 2. pp. 343-349

[20]. M. S. Sodha, S. K. Mishra, and S. Misra. (2008, March). "Focusing of Dark Hollow Gaussian Electromagnetic Beams in a Plasma." Laser and Particle Beams. Vol. 27, no 1. pp. 57-68

[21]. M. Nakatsutsumi, J. R. Davies, R. Kodama, J. S. Green, K. L. Lancaster, K. U. Akli, F. N. Beg, S. N. Chen, D. Clark, R. R. Freeman, C. D. Gregory, H. Habara, R. Heathcote, D. S. Hey, K. Highbarger, P. Jaanimagi, M. H. Key, K. Krushelnick, T. Ma, A. MacPhee, A. J. MacKinnon, H. Nakamura, R. B. Stephens, M. Storm, M. Tampo, W. Theobald, L. Van Woerkom, R. L. Weber, M. S. Wei, N. C. Woolsey, and P. A. Norreys. (2008, April). "Space and time resolved measurements of the heating of solids to ten million kelvin by

a petawatt laser.” New Journal of Physics. Vol. 10, no. pp. 043046.

[22]. A. Singh, and N. Gupta. (2015, January). “Second harmonic generation by relativistic self-focusing of q-Gaussian laser beam in preformed parabolic plasma channel.” Physics of Plasmas. Vol. 22, no 1. pp. 013102-9

[23]. V. K. Tripathi, T. Taguchi, and C. S. Liu. (2005, March). “Plasma channel charging by an intense short pulse laser and ion Coulomb explosion.” Physics of Plasmas. Vol. 12, no 4. pp.043106-7