



Mathematical Model using MATLAB tool for Glucose-Insulin Regulatory System of Diabetes Mellitus

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Abstract:

This paper works on approach to regulate the blood glucose level of diabetes. A new mathematical model for the study of diabetes mellitus was proposed by us. The model takes into account all plasma glucose concentration, generalized insulin and plasma insulin concentration. The numerical solution presents the complex situation of diabetic patients. MATLAB is used to evaluate the effectiveness of the proposed work. Our model extends towards extra parameters that effect the concentration of glucose and insulin in our body like food habits, life style and most of them stress. In particular we include concentrations for glucose, and sinusoidal function of extra parameter. A mathematical model governed by first order differential equations is presented. The numerical resolution of these equations is done by using Matlab packages. The discussed results show the curves of the model variables against time identifying the variation of insulin concentration in human body adjusted for a diabetic person.

Keywords: diabetes mellitus, glucose-insulin regulatory system, mathematical model

INTRODUCTION

Diabetes is a metabolic disease that growing encouraged by decreasing levels of activity and increasing prevalence of obesity and bad food habits and heredity is also cause of diabetes. It can affect other organs of body. Its essential feature is excessive sugar in the blood and urine. Diabetes is diagnosed by glucose tolerance test(GTT).In which patient Is called after overnight fasting .on his arrival given a large dose of glucose and then several measurements of the concentration of glucose in the patient blood is taken during next 3 to 5 hours.

There are most common two types of diabetes due to either a diminished production of insulin (TYPE1) or diminished response by the body to insulin (type2). Both lead to hyperglycaemia.

Diabetes will remain as elevated blood sugar levels if treated properly. But if it starts reach other organs or other processes in the body then its time to be more cautious.

For a normal person without diabetes blood sugar level as high 175mg/dl after meal is taken this level will return to normal rate after some time. The time consumed for return to normal level is faster in normal healthy person.

To construct a mathematical model it is necessary to follow the following well know facts from the elementary biology-

1. Glucose is a source of energy for all tissue and organs and has an important role in the metabolism of vertebrate. The blood glucose concentration has an optimal level for each individual.

2. The blood glucose levels tend to be auto regulatory but they are also susceptible to a wide variety of hormones and some hormones here we have mentioned –

A) Insulin- secreted by bête cell of pancreas, It reduce the blood sugar concentration.

B) Glycogen-It secreted by alpha cell of pancreas. Any excessive glucose is stored in the liver in the form of glycogen and this glycogen is converted back into glucose in terms of need for low blood sugar.

C) Thyroxin - Thyroxin is the major hormone secreted by the follicular cells of the thyroid gland. It is important to note that is involved in controlling the rate of metabolic processes in the body and influencing physical development. Diabetic patients have a higher prevalence of thyroid disorders compared with the normal person. The presence of thyroid dysfunction may affect diabetes control. Hyperthyroidism is typically associated with worsening glycaemia control and increased insulin requirements. In patients without any thyroid dysfunction it normally segregates the thyroxin hormone which influence in the metabolism of the body ergo it can either increased or decreased blood sugar levels.

D) Glucocorticoids - Glucocorticoids is the hormone secreted by the adrenal cortex and plays an important role in the metabolism of carbohydrates. The name "glucocorticoid" derives from early observations that these hormones were involved in glucose metabolism. In the fasted state, glucocorticoid stimulates several processes that collectively serve to increase and maintain normal concentrations of glucose in blood. The metabolic effects include the inhibition of glucose uptake in muscle and adipose tissue: A mechanism to conserve glucose and stimulation of gluconeogenesis, particularly in the liver. Gluconeogenesis is a metabolic pathway that results in the generation of glucose from non-carbohydrate carbon. The vast majority of gluconeogenesis takes place in the liver and, to a smaller extent, in the cortex of kidneys. This process occurs during periods of fasting, starvation, or intense exercise and is highly energetic

MATHEMATICAL MODEL

Mathematical models have provided one mean of understanding Diabetes dynamics. There are various models based on glucose and insulin distributions and those models have been used to explain glucose/insulin interaction. All these models are valid under certain conditions and assumption. Although these models may be useful in research setting, they all have limitations in predicting blood glucose in real-time clinical situation because of the inherent requirement of frequently updated information about the models variable like glucose loads and insulin availability [6]. Consider a

mathematical model comprised of glucose level G, glucose uptake activity X and insulin level I. Many parameters have been taken and on the basis of these parameters values a mathematical model is formed. This model includes the basal values also i.e GB and b Ib . The model is defined as:

$$\frac{dG}{dt} = -m_1G + m_2I + m_1Gb + \sin(t)$$

$$\frac{dX}{dt} = -m_2X + m_3I - m_3Ib + m_6Ib$$

$$\frac{dI}{dt} = -m_3I + m_4G + m_4m_5 - m_6I + m_6Ib$$

All the variables and parameters values used in mathematical models are described as:

G (t): The plasma glucose concentration at time t (mg/dl)

X (t): The generalized insulin variable for the remote compartment (min-1)

I (t): The plasma insulin concentration at time t (μ U/ml)

Gb: This is the basal pre injection value of plasma glucose (mg/dl)

Ib: This is the basal pre injection value of plasma insulin(μ U/ml)

m1: Insulin independent rate constant of glucose rate uptake in muscles, liver and adipose tissue (1 min⁻¹).

m2: The rate of decrease in tissue glucose uptake ability (1 min⁻¹).

m3: The insulin independent increase in glucose uptake ability in tissue per unit of insulin concentration Ib (2 min⁻¹ (μ U/ml)).

m4: The rate of the pancreatic β-cells' release of insulin after the glucose injection and with glucose concentration above h [(μ U/ml) min⁻² (mg/dl)-1]

m5: The threshold value of glucose above which the pancreatic β-cells release insulin.

M6: The first order decay rate for insulin in plasma (min-1) pancreatic β-cells release insulin

NUMERICAL SOLUTION

The analysis is done on the normal person as well as on different types of diabetic patient i.e. patient 1, patient 2, patient 3. Basically there are patients who are suffering from Diabetes mellitus but the results from each patient is different and it is explained with the help of graphs and parameters values. Glucose is given to the patients then we studied the plasma glucose concentration, plasma insulin concentration and generalized insulin variable in the body of patients. The graph for these types of patients is obtained by MATLAB.

There are some variables and symbols used in the graphs: G(t) The plasma glucose concentration at time t (mg/dl) “*”

X(t) The generalized insulin variable for the remote compartment (min-1) “.” I(t) The plasma insulin concentration at time t (μ U/ml) “o”

DATA FOR NORMAL PERSON

The study of normal person shows that initially when glucose is given glucose concentration become very high and as time passes the level become stable, the values of parameters are given in table 1

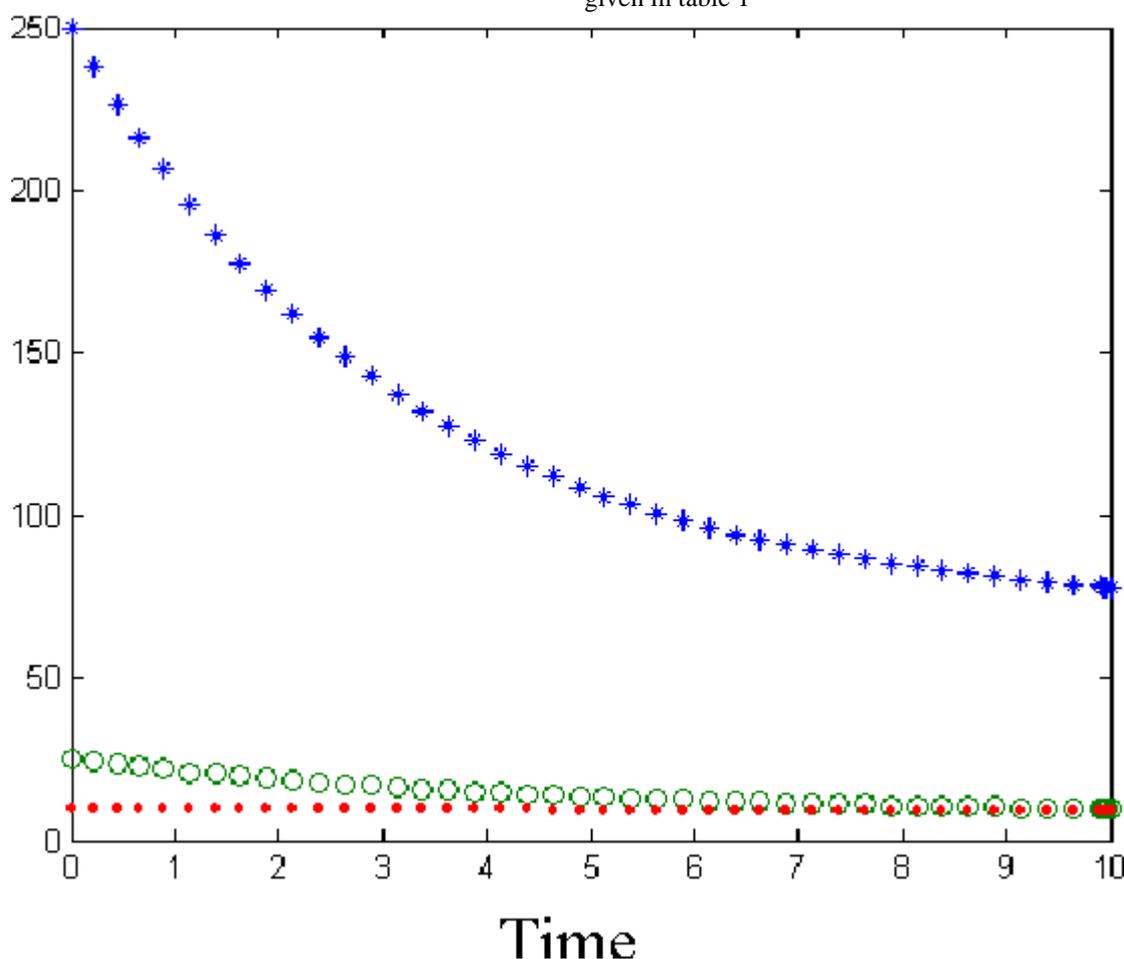


Table 1

Parameter	Values
m1	.0317000
m2	.0123
m3	.00000492
m4	.0039
m5	79.0353
m6	.2659
Gb	80
Ib	7

Data for patient1

The study of Glucose, insulin, and plasma concentration is done on the Diabetic patient for 10 hours. It show that initially the glucose level is very high but after giving the insulin to the patient there is still no major fall in glucose level. After 10 hours from 250 mg/dl it falls to only about 225 mg/dl. But when we see the graph (Fig.2) for generalized insulin variable as well as for plasma insulin concentration even after some time it is same and no change in its concentration level. The values of parameters for patient 1 is given in Table:2

TABLE:2

Parameter	value
m1	0
m2	0.017
m3	0.0000053
m4	0.0042
m5	80.25
m6	0.264
Gb	80
Ib	7

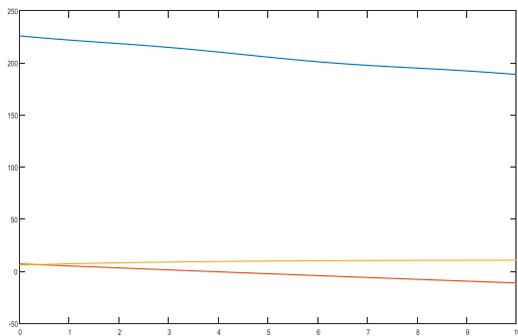


Fig. 2

Data for patient2

The study of Glucose, insulin, and plasma concentration is done on the Diabetic patients of another type. The study of this case is shown by the graph Fig.3 shows the study for 10 hours. It show that initially the glucose level is very high but after giving the insulin to the patient 2 there is still no major fall in glucose level. In this graph plasma insulin concentration remains the same but when we see the graph for generalized insulin variable there is the minor fall after some time. The values of parameters given in table3

Parameters	values
m1	0
m2	0.072
m3	.000216
m4	.0038
m5	77.5783
m6	.2465
Gb	80
Ib	7

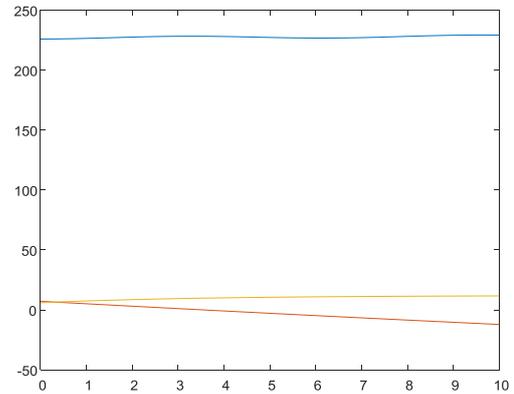


Fig: 3

Data for patient3

The study of Glucose, insulin, and plasma concentration is done on the Diabetic patient's type 3. The study of this case is shown by the graph Fig.4 shows the study for 10 hours. Again the level of glucose is same as in the case of other patients. In this graph plasma insulin concentration & generalized insulin variable have a minor fall after some time. The values of parameters for patient 3 are given in Table: 4.

TABLE 4

Parameters	Values
m1	0
m2	.0142
m3	.00000994
m4	.0046
m5	82.9370
m6	.2814
Gb	80
Ib	7

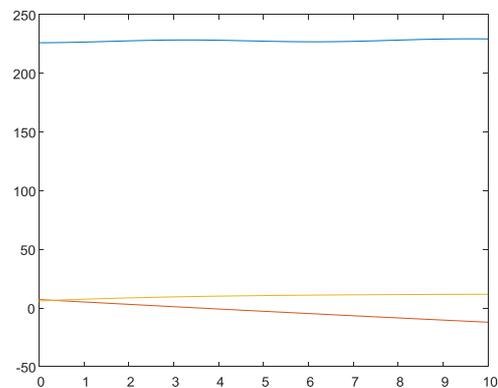


Fig: 4

Conclusion:

This model shows the difference of glucose-insulin regulatory system, between a normal person and diabetic person. The glucose concentration of diabetic patient does not come down after a certain time which shows the evidence that the person suffer from diabetes. This model makes the resulting model directly useful for clinical purposes through a careful assessment of the relevant parameters.

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