



Effects of Chemical Reaction on Unsteady MHD Casson Fluid flow past a moving Infinite Inclined Plate through Porous Medium

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Abstract:

The study of the present paper is the effects of chemical reaction on an unsteady MHD free convection Casson fluid flow past an infinite inclined plate through porous medium in the presence of thermal radiation and heat absorption. When $t^* > 0$, the velocity $u^* = 0$. At that time, the plate the temperature and the concentration are raised to T_w^* and C_w^* . A uniform magnetic field B_0 is applied in y^* -direction. The set of dimensionless governing linear partial differential equations are solved analytically by using the Laplace transform technique. The effects of various non-dimensional parameters on the velocity, the temperature, the concentration, the skin friction, the local Nusselt and the Sherwood numbers have been discussed and analyzed through graphs and tables.

Keywords: Casson fluids, MHD, free convection, thermal radiation, chemical reaction and heat absorption.

1. Introduction: The study of non-Newtonian Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rates of shear, a yield stress below which no flows occurs and a zero viscosity at an infinite rate of shear. If a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, where as if a shear stress greater than yield stress is applied and it starts to move. Few examples of Casson fluids are jelly, tomato sauce, honey, concentrated fruit juice, blood etc. Casson model is sometimes stated to fit rheological data better than general viscoelastic model for many materials. Many authors have their research in Casson fluid for mathematical modeling. S. Ostrach [1] has analyzed the laminar free-convection flow and heat transfer about a flat plate parallel to the direction of the generating body force. Boundary-Layer Behavior on Continuous Solid Surfaces was investigated by B.C.Sakiadas [2]. Free convection flow past an accelerated infinite plate were studied by Pop, I *et al* [3]. Raptis *et al* [4] were found the unsteady free convective flow through a porous medium adjacent to a semi-infinite vertical plate using finite difference scheme. J.L.McGregor [5] has done research on the Application of the Minimal Energy Hypothesis to a Casson Fluid. R.K.Dash *et al* [6] were analyzed the Shear augmented dispersion of a solute in a Casson fluid flowing in a conduit. Effects of mass transfer on MHD Flow of Casson fluid with chemical reaction were studied by S.A.Shehzad *et al* [7]. J.Prakash *et al* [8] were discussed on Dufour effects on unsteady hydro magnetic radiative fluid flow past a vertical plate through porous medium. A.G.Vijayakumar *et al* [9] were found the chemical reaction and radiation effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature and variable mass diffusion. Masthanrao, S *et al* [10] were investigated the chemical reaction and Hall effects on MHD convective flow along an infinite vertical porous plate with variable suction and heat absorption. Diffusion of chemically reactive species in Casson

fluid flow over an unsteady permeable stretching surface was analyzed by Vajravelu, K *et al* [11]. Pramanik, S [12] studied the Casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation. Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating was discussed by Hussanan A *et al* [13]. C. K. Kirubhashankar *et al* [14] have investigated an unsteady MHD flow of a Casson fluid in a parallel plate channel with heat and mass transfer of chemical reaction. Emmanuel Maurice Arthur *et al* [15] have analyzed the Casson fluid flow over a vertical porous surface with chemical reaction in the presence of magnetic field. Newtonian heating effect on steady hydro magnetic Casson fluid flow a plate with heat and mass transfer was analyzed by Das.M *et al* [16]. Gnaneswara Reddy M [17] was studied an unsteady radiative convective boundary layer flow of a Casson fluid with variable thermal conductivity. C.S.K. Raju *et al* [18] have discussed the heat and mass transfer in magneto hydrodynamic Casson fluid over an exponentially permeable stretching surface. Heat and mass transfer in unsteady MHD Casson fluid flow with convective boundary conditions was analyzed by K. Pushpalatha *et al* [19]. Chemical reaction and thermal radiation effects on unsteady MHD free convection flow past an inclined moving plate with TGHS was investigated by S.Rama Mohan *et al* [20]. C. Veerasha *et al* [21] have analyzed the Joule heating and thermal diffusion effect on MHD radiative and convective casson fluid flow past an oscillating semi-infinite vertical porous plate.

In the present study, we analyzed the effect of thermal radiation, chemical reaction and heat absorption and suction on flow of casson fluid past an infinite inclined plate through porous medium. The set linear partial differential equations are solved analytically by using Laplace Transform technique. The influence of various non-dimensional quantities on the velocity, the temperature, the concentration, the skin friction, the local

Nusselt and the Sherwood numbers are thoroughly investigated by graphs and tables.

2. Mathematical formulation:

Consider an unsteady MHD free convection heat and mass transfer flow of a viscous, incompressible, electrically, conducting, radiating and chemically reacting fluid past a semi infinite inclined plate at an angle α to the vertically. A uniform magnetic field of strength B_0 applied in a transverse direction to the fluid flow. Let x^* -axis is taken along the plate and y^* -axis is taken normal to it. Initially, when $t^* \leq 0$, both the fluid and plate are at stationary condition having constant temperature and concentration. When $t^* > 0$, the velocity $u^* = 0$. At the same time, the plate temperature and concentration are raised to T_w^* and C_w^* . For free convection flow, it is also assumed that, the induced magnetic field is negligible as the magnetic Reynolds number of the flow is taken to be very small. The viscous dissipation is negligible in the energy equation. The effects of variation in density (ρ) with temperature and species concentration are considered only in the body force term, in accordance with usual Boussinesq approximation. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. Since the flow of the fluid is assumed to be in the direction of x^* -axis, so the physical quantities are functions of the space coordinate y^* and t^* only. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is as follows.

$$\tau_{ij} = \begin{cases} 2\left(\mu_\alpha + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_\alpha + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi < \pi_c \end{cases}$$

here $\pi = e_{ij} e_{ij}$ and e_{ij} are the $(i, j)^{th}$ component of the deformation rate, π is the product of the component of the deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, π_α is plastic dynamic viscosity of non-Newtonian fluid, and P_y is the yield stress of the fluid.

Under the above assumptions the flow is governed by the following equations:

Momentum Equation:

$$\frac{\partial u^*}{\partial t^*} = g\beta_r(T^* - T_\infty^*) \cos \alpha + g\beta_c(C^* - C_\infty^*) \cos \alpha + v \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho} u^* - \frac{v}{K} u^* \quad (1)$$

Energy Equation:

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y^*} - \frac{Q}{\rho C_p} (T^* - T_\infty^*) \quad (2)$$

Diffusion Equation:

$$\frac{\partial C^*}{\partial t^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - K_r(C^* - C_\infty^*) \quad (3)$$

The initial and boundary conditions are:

$$t^* \leq 0, u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \text{ for all } y^*$$

$$t^* > 0, u^* = 0, T^* = T_\infty^* + A(T_w^* - T_\infty^*) e^{n^* t^*},$$

$$C^* = C_\infty^* + A(C_w^* - C_\infty^*) e^{n^* t^*} \text{ at } y^* = 0$$

$$u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \text{ as } y^* \rightarrow \infty \quad (4)$$

Where $u^*, \beta_T, \beta_c, B_0, v, \kappa, \rho, T^*, C^*, C_p, C_s, q_r, Q, \sigma, D_m, t, \alpha, \beta, Sc, K, Pr$ and K_r are respectively the fluid velocity in the x^* -direction, coefficient of thermal expansion, coefficient of expansion with concentration, external magnetic field, kinematic viscosity, thermal conductivity, fluid density, temperature of the fluid, Species concentration, Specific heat at constant pressure, Concentration susceptibility, radiative heat flux, heat absorption, electric conductivity, Coefficient of mass diffusivity, time, inclined angle, Casson parameter, Schmidt number, porosity parameter, Prandtl number and chemical reaction parameter.

The radiative heat flux q_r , under Rosseland approximation of the form

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^*}{\partial y^*} \quad (5)$$

Here σ^* denotes the Stefan-Boltzmann constant and k^* denotes the mean absorption coefficient.

It is assumed that the temperature differences within the flow are sufficiently small and that T^{*4} may be expressed as a linear function of the temperature. This is obtained by expanding T^{*4} in a Taylor series about T_∞^* and neglecting the higher order terms, thus we get

$$T^{*4} = 4T_\infty^{*3} T^* - 3T_\infty^{*4} \quad (6)$$

From Equations (5) and (6), Equation (2) reduces to

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{16\sigma^* T_\infty^*}{3\rho C_p k^*} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q}{\rho C_p} (T^* - T_\infty^*) \quad (7)$$

On introducing the following non-dimensional quantities

$$u = \frac{u^*}{u_0}, y = \frac{y_0}{v} y^*, t = \frac{u_0^2}{v} t^*, K = \frac{k^* u_0^2}{v},$$

$$M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \theta = \frac{(T^* - T_\infty^*)}{(T_w^* - T_\infty^*)}, \phi = \frac{(C^* - C_\infty^*)}{(C_w^* - C_\infty^*)},$$

$$Gr = \frac{v g \beta_T (T_w^* - T_\infty^*)}{u_0^3}, Gm = \frac{v g \beta_c (C_w^* - C_\infty^*)}{u_0^3},$$

$$Pr = \frac{\rho v C_p}{\kappa}, k = \frac{k^* u_0^2}{v}, Q = \frac{Q^* v}{\rho C_p u_0^2}, R = \frac{16\sigma^* T_\infty^*}{3k^* k},$$

$$Sc = \frac{v}{D}, n = \frac{v}{u_0^2} n^* \quad (8)$$

In view of (8) the Equations (1), (3) and (7), reduce to the following non-dimensional forms

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right) u + G_r \theta \cos \alpha + G_m \phi \cos \alpha \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{1+R}{Pr}\right) \frac{\partial^2 \theta}{\partial y^2} - Q\theta \quad (10)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Kr\phi \quad (11)$$

The corresponding boundary conditions reduce to

$$\begin{aligned} t \leq 0, u = 0, \theta = 0, \phi = 0 \text{ for all } y \\ t > 0, u = 0, \theta = 1 + Ae^{nt}, \phi = 1 + Ae^{nt} \text{ at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \quad (12)$$

3. Solution of the Problem:

The system of equations (9), (10) and (11) with subject to the boundary conditions in (12), are solved analytically by using Laplace Transform technique and the expressions for

$$\begin{aligned} u(y,t) = & b \left[\frac{1}{c} + \frac{A}{c-n} \right] B_1 + d \left[\frac{1}{e} + \frac{A}{e-n} \right] B_2 - \left[\frac{b}{c} + \frac{d}{e} \right] B_3 \\ & - A \left[\frac{b}{c-n} + \frac{d}{e-n} \right] B_4 - b \left[\frac{1}{c} + \frac{A}{c-n} \right] B_5 - \frac{b}{c} B_6 - \frac{bA}{c-n} B_7 \\ & - d \left[\frac{1}{e} + \frac{A}{e-n} \right] B_8 - \frac{1}{e} B_9 - \frac{A}{e-n} B_{10} \end{aligned} \quad (13)$$

$$\theta(y,t) = B_6 + AB_7 \quad (14)$$

$$\phi(y,t) = B_9 + AB_{10} \quad (15)$$

Here

$$B_1 = \frac{e^{ct}}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_3(c+A_4)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} - \sqrt{(c+A_4)t}\right) \\ & + \exp\left(y\sqrt{A_3(c+A_4)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} + \sqrt{(c+A_4)t}\right) \end{aligned} \right]$$

$$B_2 = \frac{e^{et}}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_3(e+A_4)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} - \sqrt{(e+A_4)t}\right) \\ & + \exp\left(y\sqrt{A_3(e+A_4)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} + \sqrt{(e+A_4)t}\right) \end{aligned} \right]$$

$$B_3 = \frac{1}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_3A_4}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} - \sqrt{A_4t}\right) \\ & + \exp\left(y\sqrt{A_3A_4}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} + \sqrt{A_4t}\right) \end{aligned} \right]$$

$$B_4 = \frac{e^{nt}}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_3(n+A_4)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} - \sqrt{(n+A_4)t}\right) \\ & + \exp\left(y\sqrt{A_3(n+A_4)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_3}}{2\sqrt{t}} + \sqrt{(n+A_4)t}\right) \end{aligned} \right]$$

$$B_5 = \frac{e^{ct}}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_1(c+Q)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_1}}{2\sqrt{t}} - \sqrt{(c+Q)t}\right) \\ & + \exp\left(y\sqrt{A_1(c+Q)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_1}}{2\sqrt{t}} + \sqrt{(c+Q)t}\right) \end{aligned} \right]$$

$$B_6 = \frac{1}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_1Q}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_1}}{2\sqrt{t}} - \sqrt{Qt}\right) \\ & + \exp\left(y\sqrt{A_1Q}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_1}}{2\sqrt{t}} + \sqrt{Qt}\right) \end{aligned} \right]$$

$$B_7 = \frac{e^{nt}}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{A_1(n+Q)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_1}}{2\sqrt{t}} - \sqrt{(n+Q)t}\right) \\ & + \exp\left(y\sqrt{A_1(n+Q)}\right) \operatorname{erfc}\left(\frac{y\sqrt{A_1}}{2\sqrt{t}} + \sqrt{(n+Q)t}\right) \end{aligned} \right]$$

$$B_8 = \frac{e^{et}}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{Sc(e+kr)}\right) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(e+Kr)t}\right) \\ & + \exp\left(y\sqrt{Sc(e+kr)}\right) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(e+Kr)t}\right) \end{aligned} \right]$$

$$B_9 = \frac{1}{2} \left[\begin{aligned} & \exp\left(-y\sqrt{Sc kr}\right) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Krt}\right) \\ & + \exp\left(y\sqrt{Sc kr}\right) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Krt}\right) \end{aligned} \right]$$

$$B_{10} = \frac{e^{nt}}{2} \left[\begin{array}{l} \exp(-y\sqrt{Sc(n+kr)}) \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(n+Kr)t} \right) \\ + \exp(y\sqrt{Sc(n+kr)}) \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(n+Kr)t} \right) \end{array} \right]$$

$$D_2 = e^{et} \left[\sqrt{A_3(e+A_4)} \operatorname{erf}(\sqrt{(e+A_4)t}) + \sqrt{\frac{A_3}{\pi t}} e^{-(e+A_4)t} \right]$$

$$D_3 = \sqrt{A_3 A_4} \operatorname{erf}(\sqrt{A_4 t}) + \sqrt{\frac{A_3}{\pi t}} e^{-A_4 t}$$

Where

$$A_1 = \frac{Pr}{1+R}, A_2 = 1 + \frac{1}{\beta}, A_3 = \frac{1}{A_2}, A_4 = M + \frac{1}{K},$$

$$b = \frac{A_3 Gr \cos \alpha}{A_1 - A_3}, c = \frac{A_3 A_4 - A_1 Q}{A_1 - A_4},$$

$$d = \frac{Gm A_3 \cos \alpha}{Sc - A_3}, e = \frac{A_3 A_4 - ScKr}{Sc - A_3}$$

4. Skin friction:

The rate of change of velocity is given by

$$\tau = - \left(1 + \frac{1}{\beta} \right) \left[\frac{\partial u}{\partial y} \right]_{y=0} \quad (16)$$

From equations (13) and (16), we get Skin friction as follows

$$\tau = \left[1 + \frac{1}{\beta} \right] \left[b \left[\frac{1}{c} + \frac{A}{c-n} \right] D_1 + d \left[\frac{1}{e} + \frac{A}{e-n} \right] D_2 - \left[\frac{b}{c} + \frac{d}{e} \right] D_3 \right. \\ \left. - A \left[\frac{b}{c-n} + \frac{d}{e-n} \right] D_4 - b \left[\frac{1}{c} + \frac{A}{c-n} \right] D_5 - \frac{b}{c} D_6 - \frac{bA}{c-n} D_7 \right. \\ \left. - d \left[\frac{1}{e} + \frac{A}{e-n} \right] D_8 - \frac{1}{e} D_9 - \frac{A}{e-n} D_{10} \right]$$

5. Nusselt number:

The rate of change of heat transfer is given by

$$Nu = - \left[\frac{\partial \theta}{\partial y} \right]_{y=0} \quad (17)$$

From equations (14) and (17), we get Nusselt number as follows

$$\theta(y, t) = D_6 + AD_7$$

6. Sherwood Number:

The rate of change of mass transfer is given by

$$Sh = - \left[\frac{\partial \phi}{\partial y} \right]_{y=0} \quad (18)$$

From equations (15) and (18), we get Sherwood number as follows

$$\phi(y, t) = D_9 + AD_{10}$$

Here

$$D_1 = e^{ct} \left[\sqrt{A_3(c+A_4)} \operatorname{erf}(\sqrt{(c+A_4)t}) + \sqrt{\frac{A_3}{\pi t}} e^{-(c+A_4)t} \right]$$

$$D_4 = e^{nt} \left[\sqrt{A_3(n+A_4)} \operatorname{erf}(\sqrt{(n+A_4)t}) + \sqrt{\frac{A_3}{\pi t}} e^{-(n+A_4)t} \right]$$

$$D_5 = e^{ct} \left[\sqrt{A_1(c+Q)} \operatorname{erf}(\sqrt{(c+Q)t}) + \sqrt{\frac{A_1}{\pi t}} e^{-(c+Q)t} \right]$$

$$D_6 = \sqrt{A_1 Q} \operatorname{erf}(\sqrt{Qt}) + \sqrt{\frac{A_1}{\pi t}} e^{-Qt}$$

$$D_7 = e^{nt} \left[\sqrt{A_1(n+Q)} \operatorname{erf}(\sqrt{(n+Q)t}) + \sqrt{\frac{A_1}{\pi t}} e^{-(n+Q)t} \right]$$

$$D_8 = e^{et} \left[\sqrt{Sc(e+Kr)} \operatorname{erf}(\sqrt{(e+Kr)t}) + \sqrt{\frac{Sc}{\pi t}} e^{-(e+Kr)t} \right]$$

$$D_9 = \sqrt{ScKr} \operatorname{erf}(\sqrt{Krt}) + \sqrt{\frac{Sc}{\pi t}} e^{-Krt}$$

$$D_{10} = e^{nt} \left[\sqrt{Sc(n+Kr)} \operatorname{erf}(\sqrt{(n+Kr)t}) + \sqrt{\frac{Sc}{\pi t}} e^{-(n+Kr)t} \right]$$

$$\text{Where } A_1 = \frac{Pr}{1+R}, A_2 = 1 + \frac{1}{\beta}, A_3 = \frac{1}{A_2}, A_4 = M + \frac{1}{K},$$

$$b = \frac{A_3 Gr \cos \alpha}{A_1 - A_3}, c = \frac{A_3 A_4 - A_1 Q}{A_1 - A_4},$$

$$d = \frac{Gm A_3 \cos \alpha}{Sc - A_3}, e = \frac{A_3 A_4 - ScKr}{Sc - A_3}$$

erfc = Complementary Error function

erf = Error function

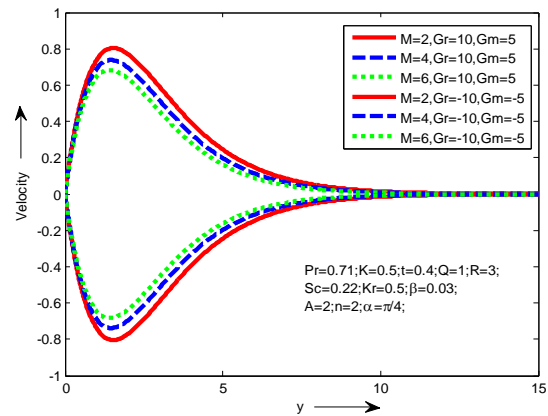
7. Result and Discussion:

The systems of linear non-dimensional equations, with the boundary conditions are solved analytically by using the Laplace Transform technique. The obtained results show the effect of the various non-dimensional governing parameters, such as Casson parameter (β), Magnetic parameter (M), Porosity parameter (K), Prandtl number (Pr), thermal Grashof number (Gr), mass Grashof number (Gm), inclined angle (α) thermal Radiation parameter (R), heat absorption parameter (Q), Schmidt number (Sc), chemical reaction parameter (Kr) and time (t) on the flow of velocity, temperature and concentration profiles. Also the Skin friction, the Nusselt number and the Sherwood number are presented in the tabular form. From **figures 1-18** for the case of cooling ($Gr > 0, Gm > 0$) and heating ($Gr < 0, Gm < 0$) of the plate. The heating and cooling takes place by setting up free convection currents due to temperature and concentration gradient.

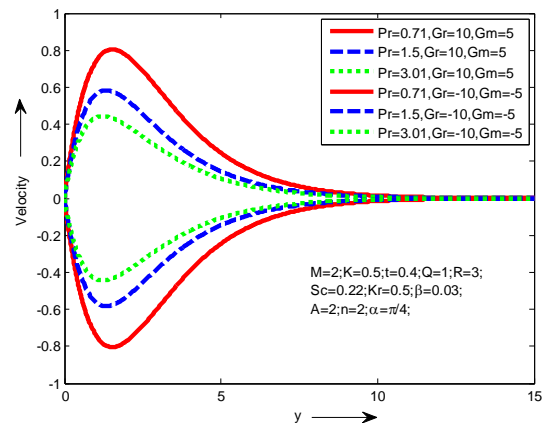
The influence of Magnetic parameter (M) and Prandtl number (Pr) on the fluid velocity profiles are shown **figures (1) and (2)** in the cases cooling and heating of the plate. It is observed that the velocity decreases as M or Pr increases in case of cooling of the plate and opposite phenomenon is observed in case of heating of the plate. **Figure (3)** shows the effects of porosity parameter (K) on the velocity profiles in case of cooling and heating of the plate. It is found that the velocity increases as K increases in case of cooling of the plate. But a reverse effect is identified in case of heating of plate. From **figure (4) and (5)** depicts the effects of thermal Grashof number (Gr), mass Grashof number (Gm) and heat absorption parameter (Q) on the velocity profiles in cases cooling and heating of the plate. From this we observed that the velocity increases as Gr or Gm or Q increases in case of cooling of the plate and a reverse effect is noticed in case of heating of the plate. The effect of thermal radiation (R) on the velocity profiles is shown **figure (6)** in cases cooling and heating of the plate. It is seen that the velocity increase as R increases in case of cooling of the plate and opposite phenomenon is observed in case of heating of the plate. From **figures (7)-(8)** exhibits that the effects of Schmidt number (Sc) and chemical reaction (Kr) on the velocity profiles in cases cooling and heating of the plate. It is found that the velocity increases as Sc or Kr increases in case of cooling of the plate. But reverse effect is identified in case of heating of plate. The effects of casson parameter (β) on the velocity profiles is shown **figure (9)** in case cooling and heating of the plate. It is observed that the velocity increases initially and slowly downwards as β increases in case of cooling of the plate and it is noticed opposite phenomenon in case of heating of the plate. From **figure (10)** reveals that the behavior of α on the velocity profiles in case of cooling and heating of the plate. It is clear that the velocity increases as the inclined angle (α) increases both cooling and heating of the plate. When $\alpha = \frac{\pi}{2}$ we get the velocity flow in vertically. The influence of time (t) on the velocity profiles are shown **figure (11)** in case cooling and heating of the plate. From this it is seen that the velocity increases as time (t) increases in case of cooling and reverse phenomenon is observed in case of heating of the plate.

The effect of different flow parameters on the fluid temperature are given in **Figures 12-15**. The Prandtl number (Pr) is defined as the ratio of kinematic viscosity to thermal diffusivity. The effects of the Prandtl number (Pr) and the heat source parameter (Q) on the temperature profiles are shown **figures (12)-(13)**. It is observed that the temperature decreases as Pr or Q increases on the fluid flow. The effect of thermal radiation (R) on the temperature profiles in **figures (14)**. From this it is seen that the temperature increases as R increases on the fluid flow. The influence of time (t) on the temperature profiles in **figure (15)**. It is noticed that the temperature increases as t increases on the fluid flow.

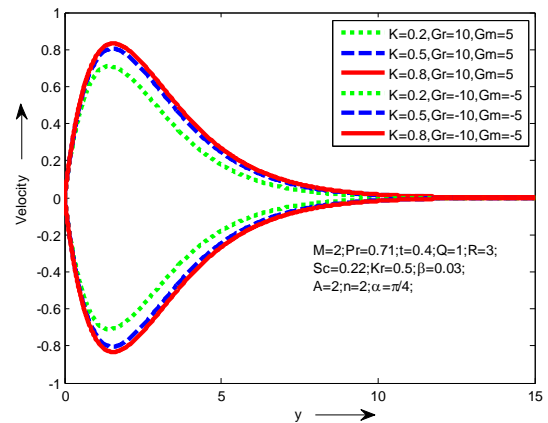
The effects of Schmidt number (Sc) and chemical reaction (Kr) on the concentration profiles in **figures (16) and (17)**. It is observed that the concentration profiles decreases as Sc or Kr increases on the fluid flow. The influence of time (t) on the concentration profiles in **figure (18)**. From this it is seen that the concentration increases as t increases on the fluid flow.



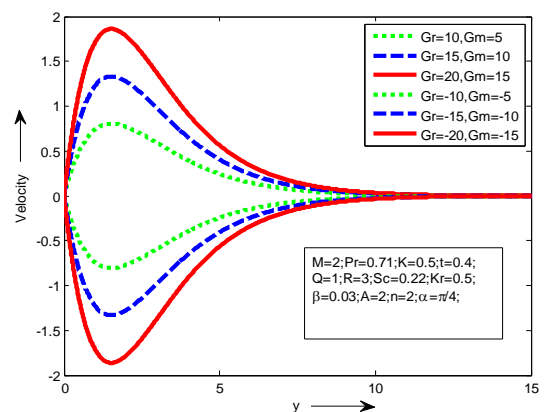
1. The influence of M on the velocity profiles.



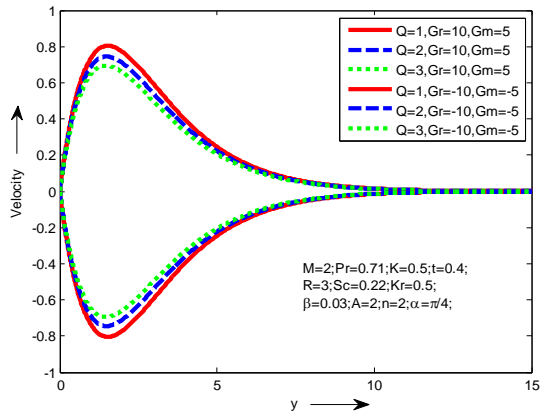
2. The influence of Pr on the velocity profiles.



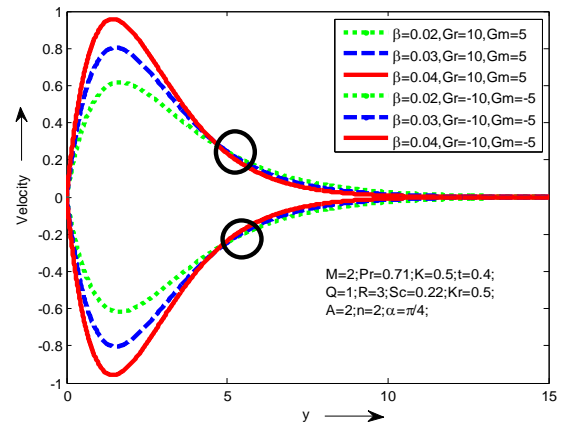
3. The influence of K on the velocity profiles.



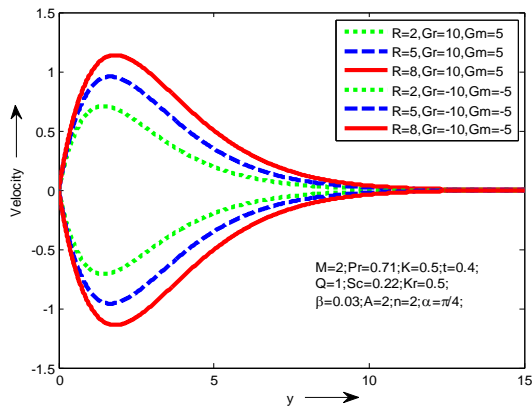
4. The influence of Gr and Gm on the velocity profiles.



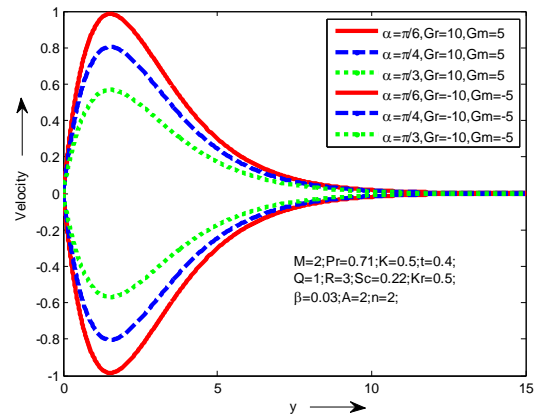
5. The influence of Q on the velocity profiles.



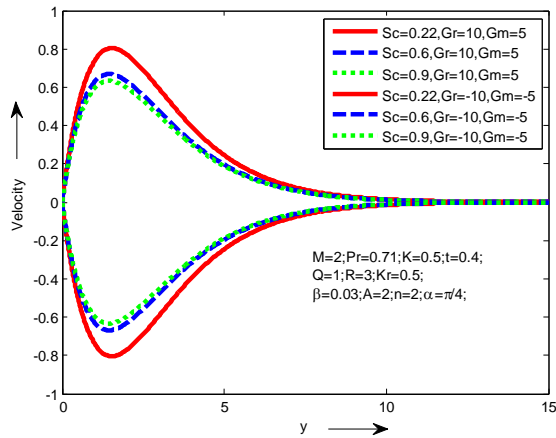
9. The influence of β on the velocity profiles.



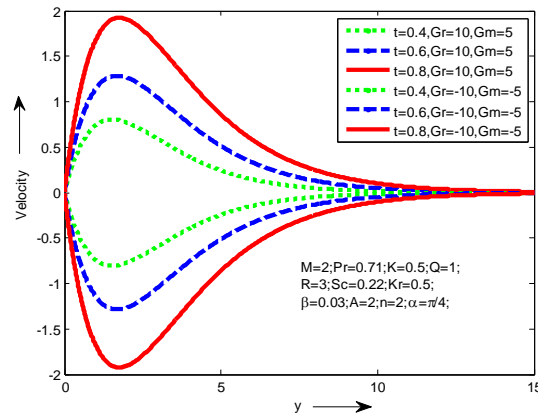
6. The influence of R on the velocity profiles.



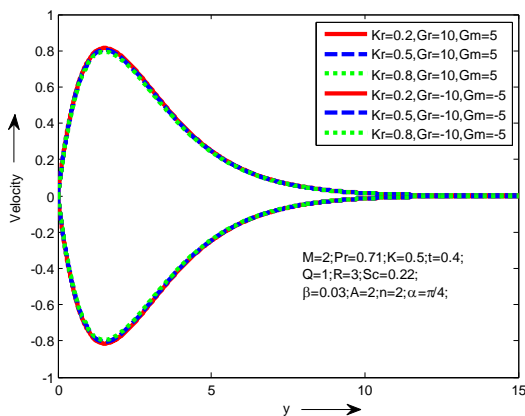
10. The influence of α on the velocity profiles.



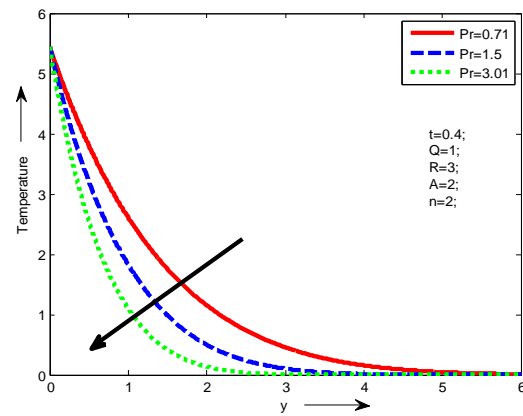
7. The influence of Sc on the velocity profiles.



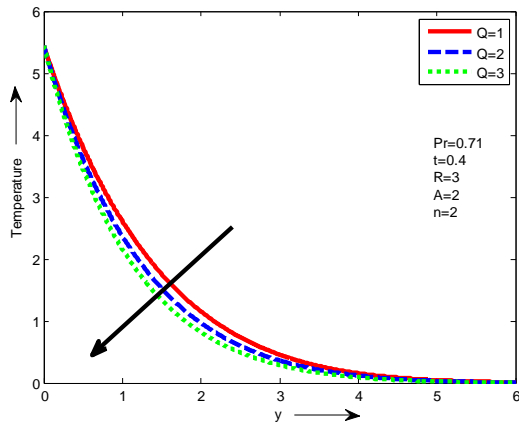
11. The influence of t on the velocity profiles.



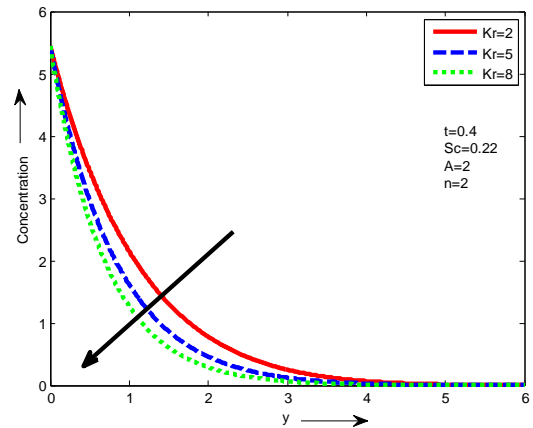
8. The influence of Kr on the velocity profiles.



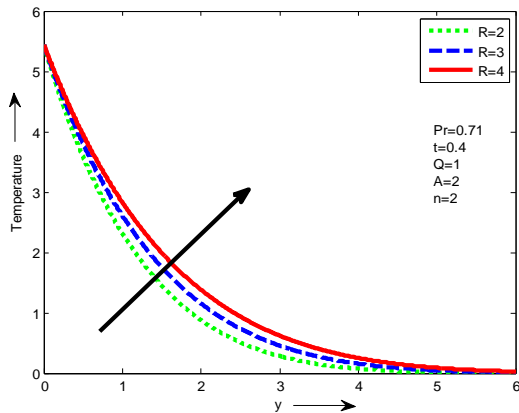
12. The influence of Pr on the temperature profiles.



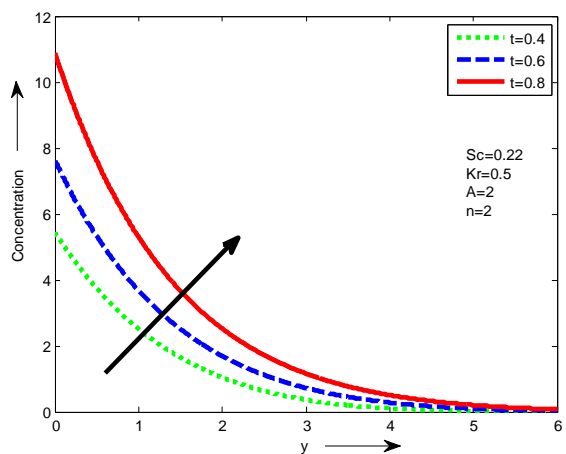
13. The influence of Q on the temperature profiles.



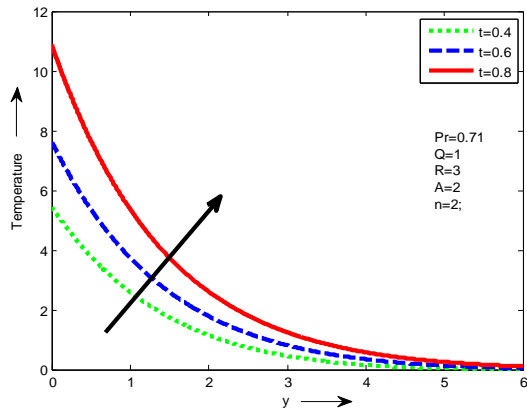
17. The influence of Kr on the concentration profiles.



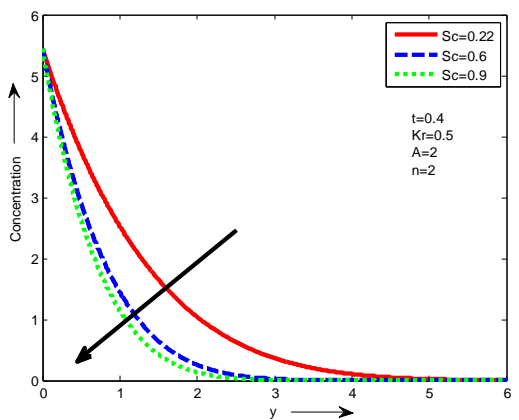
14. The influence of R on the temperature profiles.



18. The influence of t on the concentration profiles.



15. The influence of t on the temperature profiles



16. The influence of Sc on the concentration profiles.

Table-1: Nusselt number

Pr	R	Q	t	Nu
0.71	3	1	0.4	0.8527
1.5	3	1	0.4	1.2394
3.01	3	1	0.4	1.7556
0.71	4	1	0.4	0.7627
0.71	5	1	0.4	0.6962
0.71	3	2	0.4	1.0232
0.71	3	3	0.4	1.1773
0.71	3	1	0.4	1.0206
0.71	3	1	0.4	1.1885
0.71	3	1	0.6	0.9648
0.71	3	1	0.8	1.1802

Table-2: Sherwood number

Sc	Kr	t	Sh
2	0.5	0.4	0.8461
3	0.5	0.4	1.3973
4	0.5	0.4	1.7113
2	0.3	0.4	0.8031
2	0.4	0.4	0.8247
2	0.5	0.4	1.0195
2	0.5	0.4	1.1928
2	0.5	0.6	0.9414
2	0.5	0.8	1.1478

Effect of Casson parameter (β), Schmidt number (Sc), magnetic parameter (M), thermal Grashof number (Gr), mass Grashof number (Gm), porosity parameter (K), Radiation parameter (R), chemical reaction parameter (Kr), heat absorption parameter (Q), Prandtl number (Pr) inclined angle

(α) and time (t) on Skin friction, Nusselt number and Sherwood number is presented in tables.

From table-1, the effects of the Nusselt number increases with the values Pr , Q or t increases and reduces only when R increases. **From table-2**, the influence of the Sherwood number increases with the values of Sc , Kr or t increases.

Table-3: Skin-friction; (for cooling $Gr>0, Gm>0$, for heating $Gr<0, Gm<0$)

M	Pr	K	t	Q	R	Sc	Kr	β	α	τ ($Gr=10, Gm=10$)	τ ($Gr=-10, Gm=-10$)
2	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.3857	0.3857
3	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.3778	0.3778
4	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.3707	0.3707
2	1.5	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.4175	0.4175
2	3.01	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.2744	0.2744
2	0.71	0.3	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.3754	0.3754
2	0.71	0.4	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.3817	0.3817
2	0.71	0.5	0.6	1	3	2	0.5	0.03	$\pi/4$	-0.4834	0.4834
2	0.71	0.5	0.8	1	3	2	0.5	0.03	$\pi/4$	-0.5943	0.5943
2	0.71	0.5	0.4	2	3	2	0.5	0.03	$\pi/4$	-0.3661	0.3661
2	0.71	0.5	0.4	3	3	2	0.5	0.03	$\pi/4$	-0.3497	0.3497
2	0.71	0.5	0.4	1	4	2	0.5	0.03	$\pi/4$	-0.4058	0.4058
2	0.71	0.5	0.4	1	5	2	0.5	0.03	$\pi/4$	-0.4226	0.4226
2	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.4370	0.4370
2	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/4$	-0.4884	0.4884
2	0.71	0.5	0.4	1	3	3	0.5	0.03	$\pi/4$	-0.3674	0.3674
2	0.71	0.5	0.4	1	3	4	0.5	0.03	$\pi/4$	-0.3560	0.3560
2	0.71	0.5	0.4	1	3	2	0.3	0.03	$\pi/4$	-0.3881	0.3881
2	0.71	0.5	0.4	1	3	2	0.4	0.03	$\pi/4$	-0.3869	0.3869
2	0.71	0.5	0.4	1	3	2	0.5	0.04	$\pi/4$	-0.4899	0.4899
2	0.71	0.5	0.4	1	3	2	0.5	0.05	$\pi/4$	-0.5857	0.5857
2	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/6$	-0.4738	0.4738
2	0.71	0.5	0.4	1	3	2	0.5	0.03	$\pi/3$	-0.2736	0.2736

The bold values in table indicate the variation of the parameters and variables present in the corresponding Column's

From table-3: the effects of the skin friction in case cooling and heating. From this it is clear that the Skin friction increases with the values of M , Pr , Q , Sc , Kr or α increase and it decreases with the values of R , K , t or β increases in case of cooling. But it is opposite phenomenon in case of heating

8. Conclusion: From the above work the following conclusions are made:

- ❖ The velocity increases with K , Gr , Gm , R or t increases in case of cooling and reverse phenomenon is observed in case of heating. The velocity decreases with M , Pr , Q , β , Sc or Kr increases in case of cooling and it is noticed opposite behavior in case of heating. But both the cases the velocity increases with an inclined angle (α) increases.
- ❖ The temperature increases as R or t raises and it is reduces as Pr or Q rises.
- ❖ The concentration decreases as Kr or Sc increases and it is increases as A and t increases.
- ❖ The Nusselt number increase with the values of Pr , Q , A and t increases and it decay only as R rises.
- ❖ The Sherwood number increases as of Sc , Kr , A or t increases.
- ❖ The Skin friction increases with the values of M , Pr , Q , Sc , Kr or α increases and it is decreases as R , K , t or β

increases. But the reverse effect is noticed in case of heating.

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