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Anti-Invariant Submanifolds of S-Manifold

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Abstract:
The object of the present paper is to study anti-invariant submanifolds M of S- manifold $\overline{M}$. It is shown that if M is totally umbilical then M is totally geodesic. Also results have been obtained connecting linear span L({{ ξ_α}) ; totally geodesicty and anti-invariance of M. Also we find the necessary and sufficient condition for anti-invariant submanifolds of S-manifold to be T-invariant and anti-invariant and condition for integrability of the distribution D. AMS Subject Classification: 53C15, 53C20, 53C50.

Key Words: Anti-invariant submanifold, S- manifold, T-invariant, Totally geodesic, Totally umbilical, Integrable condition.

1. INTRODUCTION

In 1963, Yano [12] introduced the notion of ϕ-structure on a C^∞ (2n+s)-dimensional manifold $\overline{M}$ as a non-vanishing tensor field ϕ of type (1,1) on $\overline{M}$ which satisfies ϕ3 + ϕ = 0 and has constant rank r=2n. The almost complex (s=0) and almost contact (s=1) are examples of ϕ-structures. In 1970 , S.I.Goldberg and K.Yano [29] defined globally framed ϕ-structures for which the subbundle ker ϕ is parallelizable.Then there exists a global frame { ξ_1. ....., ξ_s } for the subbundle kerφ, (the vector fields ξ_1,......,ξs are called the structure vector 1 with dual 1-forms, η_1 ,....., .., η_s such *that* $g(\phi X,\phi Y) = g(X,Y) - \sum_{\alpha=1}^{s} \eta_\alpha(X)\eta_\alpha(Y)$ for any vector fields X,Y in $\overline{M}$, and then the structure is called a metric ϕ-structure. A wider class of globally framed ϕ-manifolds was introduced by [3] according to the following definition; a metric ϕ-structure is said to be K-structure if the fundamental 2-form Φ given by Φ(X, Y) = g(X, ϕY) for any vector fields X and Y on M is closed and normality condition holds , that is [ϕ, ϕ] + 2 $\sum_{\alpha=1}^{s}$ d$\eta_\alpha \otimes \xi_\alpha$ = 0, where [ϕ, ϕ_] denotes the Nijenhuis tensor of ϕ. A K-manifold is called an S-manifold if d$\eta_\alpha = \Phi$ for all α= 1, ..., s. If (s=1), an S-manifold is a Sasakian manifold. Furthermore, S-manifolds have been studied by several authors (see,for examples, [[1],[2] [6],[7], [8],[29]). The research work on the geometry of invariant submanifols of contact and complex manifolds is carried out by M.Kon [25] in 1973, C.S.Bagewadi [30] in 1982, Yano and Kon [35] in 1984 , and in 2016 ([10] ,[11] ,[15]) and other authors ([9], [25], [26]) etc. Also the study of geometry of anti-invariant submanifolds is carried out by ([13] ,[16] ,[17] ,[18], [19], [27] ,[28], [32] ,[33], [34]) in various contact manifolds. Motivated by the studies of the above authors , we study anti-invariant submanifolds of S-manifolds. The paper is organised as follows: the section 1 consists of preliminaries of S-manifolds and section 2 contains the results as stated in abstract.

2. A (2n+s)-dimensional differentiable manifold $\overline{M}$ is called a metric ϕ-manifold if there exists a (1,1) type tensor field ϕ , s vector fields ξ_1,......,ξs called structure vector fields s-1-forms η_1 ,....., .., η_s and a Riemannian metric g on M such that

$\phi^2 = -I + \sum_{\alpha=1}^{s} \eta_\alpha \otimes \xi_\alpha$, $\eta_\alpha(\xi_\beta) = \delta_{\alpha\beta}$, $\eta_\alpha \circ \varphi = 0$, $\phi(\xi_\alpha) = 0$ (2.1)

$$g(\phi X,\phi Y) = g(X,Y) - \sum_{\alpha=1}^{s} \eta_\alpha(X)\eta_\alpha(Y)$$

For any X,Y∈ TM In addition, we have

$$g(X,\phi Y) = -g(\varphi X,Y), \quad g(X,\xi_\alpha) = \eta_\alpha(X) \qquad (2.2)$$

A 2-form Φ defined by $\Phi(X,Y) = g(X,\phi Y)$ $(\overline{\nabla}_X \phi)Y = \sum_{\alpha=1}^{s}$ [g $(\Phi X,\phi Y)\xi_\alpha + \phi^2 X\, \eta_\alpha(Y)$] (2.3)

$\overline{\nabla}_X \xi_\alpha = -\phi X$ (2.4)

For any X,Y∈T$\overline{M}$, α,β=1,………,.s

Let M be a submanifold of $\overline{M}$ Let $T_x(M)$ and $T^\perp{}_x(M)$ denote the tangent and normal space of M at x ∈ M respectively The Gauss and Weingarten formulas are given by

$$\overline{\nabla}_X Y = \nabla_X Y + \sigma(X,Y) \qquad (2.5)$$

$$\overline{\nabla}_X N = -A_N X + \nabla^\perp{}_X N \qquad (2.6)$$

for any vector fields X,Y tangent to M and any vector field N normal to M , where of $\overline{\nabla}$ and ∇ are the operator of covariant differentiation on and of $\overline{M}$ *and* M , is $\nabla^\perp$ the linear connection induced in the normal space $T^\perp{}_x(M)$ Both A_N an d σ are called the Shape operator and the second fundamental form and they satisfy

$$g(\sigma(X,Y),N) = g(A_N X,Y) \qquad (2.7)$$

If the second fundamental form σ of M is of the form σ(X,Y) = g(X,Y) μ, then M is called totally umbilical. where μ is the mean curvature. If the second fundamental form vanishes identically then M is said to be totally geodesic. If μ = 0, then M is said to be minimal. A submanifold M of a S-manifold $\overline{M}$ is said to be invariant if the structure vector field ξ of $\overline{M}$ is tangent to M and φ($T_x(M) \subset T_x(M)$, where $T_x(M)$ is the tangent space for all x ∈ M and If φ($T_x(M)$ ⊂ $T^\perp{}_x(M)$

where $T^{\perp}{}_x(M)$ is the normal space at x ∈ M then M is said to be anti-invariant in $\overline{M}$. Now we define S-manifold with constant ϕ-holomorphic sectional curvature Let L be the distribution determined by the projection tensor ϕ^2- and let M be the complementary distribution which is determined by ϕ^2+I and spanned by fields ξ_1,......,ξs It is clear that if X ∈L then $\eta_\alpha(X)$ = 0 for any α and if X ∈ TM, then ϕX = 0 , A plane section π on $\overline{M}$. is called an invariant ϕ-section if it is determined by vector X ∈ L(x) , x ∈ $\overline{M}$. such that X, ϕX is an orthonormal pair spanning the section. The sectional curvature of ϕ is called the ϕ-sectional curvature . I f $\overline{M}$ is an S-manifold of constant ϕ-sectional curvature k, then its curvature tensor has the form

$\overline{R}$ (X, Y,Z,W) $=\sum_{\alpha\beta}$ (g (ϕX,ϕ W) $\eta_\alpha(Y)$ $\eta_\beta(Z)$ - g(ϕX,ϕZ) $\eta_\alpha(Y)$ $\eta_\beta(W)$

$\frac{(K+3S)}{4}$[g(ϕX, ϕW)g(ϕY ,ϕZ)-g(ϕX,ϕZ)g(ϕY,ϕW)

+ $\frac{(K-S)}{4}$ g(ϕX,W) g(ϕY, Z)-g(ϕX,ϕZ)g(ϕY,ϕW) − 2g(ϕX, Y) g(ϕZ,W)] (2.8)

for any vector fields X, Y,Z,W on $\overline{M}$ where X, Y,Z,W ∈ T($\overline{M}$)

Such a manifold $\overline{M}$ (k) is called an S-space form . The Euclidean spaceE^{2m+1} and hyperbolic space H^{2m+s} are examples of S-space forms.

Let us define a tensor field T on $\overline{M}$ by setting

(2.9)

T(X,Y,Z,W)=$\overline{R}$(X, Y,Z,W)+ [g(ϕX,W)g (ϕY,Z)−g (ϕX,Z)g (ϕY,W)− 2g(ϕX, Y)g(ϕZ,W)] (2.9)

The Gauss equation is given by

(2.9) $\overline{R}$ (X, Y,Z,W) = R(X, Y,Z,W) + g(σ(X,Z), σ(Y,W)) − g(σ(X,W), σ(Y,Z))

for any vector fields X,Y,Z,W on $\overline{M}$

A sub manifold M is said to $\overline{R}$-invariant and T-invariant if and only if

$\overline{R}$ (X, Y)Tx(M)) ⊂Tx(M)) and (T(X, Y))Tx(M)) ⊂Tx(M)) respectively.

3. SOME THEOREMS

Theorem 3.1. ;Let M be a submanifold tangent to _-frame {ξ_α}, α=1,....,s of a S-manifold $\overline{M}$

If M is totally umbilical then M is totally geodesic.

Proof. Since ξ_α is tangent to M, we have from Gauss formula

$$\overline{\nabla}_X\xi_\alpha. = \nabla_X\xi_\alpha. + \sigma(X,\xi_\alpha).$$

Using (2.1) (2.5) in the above we have

$-\phi X = \nabla_X\xi_\alpha. + \sigma(X,\xi_\alpha).$ α= 1, ..., s

Equating tangential and normal components

$(\phi X)^T = -\nabla_X\xi_\alpha.$, $(\phi X)^\perp = \sigma(X, \xi_\alpha)$

Putting X = ξ_α. in second equation then by (2.1) we have

$\sigma(\xi_\alpha. \xi_\alpha) = 0$

Let us assume that M is totally umbilical

Then

σ(X, Y) = g(X, Y)μ

for any tangent vectors X,Y to M , where μ denotes the mean curvature vector, Putting

X = Y = ξ_α

$\sigma(\xi_\alpha. \xi_\alpha) = g(\xi_\alpha. , \xi_\alpha.)\mu = 0$

This showes that μ = 0

Hence σ(X, Y) = g(X, Y)μ implies σ(X, Y) = 0

If the second fundamental form σ = 0 then M is totally geodesic.

Remark 3.1 If M is totally geodesic then $(\_\phi X)^\perp = \sigma(X, \xi_\alpha) = 0$, ϕX is tangent to M and hence M is an invariant submanifold of S-manifold. Therefore M will also be S-manifold.

Theorem 3.2. Let M be a submanifold of a S-manifold ¯M tangent to linear span L({ξ_α}) of structure vector field ϕ of . $\overline{M}$ Then L({ξ_α}) is parallel with respect to the induced connection on M if and only if M is anti-invariant submanifold in . $\overline{M}$.

Proof. Suppose each structure vector field __ of L({ξ_α}) is tangent to M. By Gauss formula

(3.1) $\overline{\nabla}_X\xi_\alpha. = -\phi X = \nabla_X\xi_\alpha. + \sigma(X,\xi_\alpha).$

Next suppose L({ ξ_α }) is parallel w.r.t induced connection on M, then each ξ_α is parallel to M. We have $\nabla_X\xi_\alpha = 0$ from equation(2.1)

$-\phi X = \sigma(X, \xi_\alpha)$

i.e $\phi X = \sigma(X, \xi_\alpha)$

Hence ϕX is normal to M

Since _ϕX ∈TxM$^\perp$ thus M is anti-invariant

Conversly; suppose M is anti-invariant Then by definition of anti-invariant submanifolds

$\phi X = \sigma(X, \xi_\alpha)$

Hence from (3.1) , $\nabla_X\xi_\alpha. = 0$

This shows that ξ_α is parallel w.r.to the induced connection M. Hence the theorem.

Theorem 3.3. Let M be a submanifold of S- manifold .$\overline{M}$.. If ξ_α. L({ξ_α. }) is normal to M then M is totally geodesic if and only if M is anti-invariant sub manifold.

Proof. : Suppose L({ξ_α}) is normal to M and so each ξ_α is normal to M ; then Weingarten formula implies

$\overline{\nabla}_X\xi_\alpha. = -A_{\xi_\alpha}X + \nabla^\perp_X\xi_\alpha$ (3.2)

Using (2.4) and (2.2) we have

$g(\phi X, Y) = g(-\overline{\nabla}_X\xi_\alpha. , Y) = g(A_{\xi_\alpha}X, Y) + g(-\nabla^\perp_X\xi_\alpha, Y)$

$= g(A_{\xi_\alpha}X, Y)$ (3.3)

for any X and Y tangent on M. Interchange X and Y in the above then we have

$g(\phi Y, X) = g(A_{\xi_\alpha}Y, X)$ (3.4)

Adding (2.3) and (2.4) and by virtue of (1.2) we have

$g(A_{\xi_\alpha}X, Y) + g(A_{\xi_\alpha}Y, X) = 0$

By (2.7) we have

$g(\sigma(X, Y), \xi_\alpha) = g(A_{\xi_\alpha}X, Y)$

Since A_{ξ_α} is symmetric, $g(A_{\xi_\alpha}X, Y) = 0$.

If M is totally geodesic , then $A_{\xi_\alpha} = 0$, then by (2.2) and (2.4)

$\phi X = \nabla^\perp_X \xi_\alpha \in = T^\perp_X M$

Hence M is anti-invariant

Convresly; suppose M is anti-invariant then $\phi X \in T^\perp_X M$

then from (2.3) we get

$g(A_{\xi_\alpha}X, Y) = 0$

Then by (2.7), ,σ(X, Y) = 0

Hence M is totally geodesic. Thus the theorem is proved.

Theorem 3.4. Let M be an anti-invariant submanifold tangent linear span L($\{\xi_\alpha\}$) of the structure vector fields ξ_α , α= 1, ..., s of S- manifold $\overline{M}$ with constant k . If ANX = 0 for any N ∈ $T_X^\perp$ M then ϕ(Tx(M)) is parallel w r t the normal connection.

Proof. To show that ϕ(Tx(M) is parallel w.r.t. the normal connection $\nabla^\perp$, we have to show that every local section ϕY ∈ϕ(Tx(M), $\nabla_X^\perp$ ϕ(Y) is also a local section in ϕ(Tx(M)). Using Gauss and Weingarten formula

$\nabla_X^\perp$ ϕY = $\overline{\nabla}$ ϕY + AϕYX

$\nabla_X^\perp$ ϕY = $\overline{\nabla}_X$ ϕY + ϕ($\overline{\nabla}_X$ Y) + AϕYX

$= \sum_{\alpha=1}^{s}$ [g $(\phi X, \phi Y)\,\xi_\alpha + \phi^2 X\,\eta_\alpha(Y)$] + AϕYX + ϕ$\nabla_X$ Y + ϕ(σ(X, Y))

by virtue (1.3) and (1.5). Since AN = 0 for any N ∈ $T_X^\perp$ M we have

g($\nabla_X^\perp \phi$ Y,N)

$= \sum_{\alpha=1}^{s}$ [g $(\phi X, \phi Y)$g($\xi_\alpha, N) + g(\phi^2 X, N)\,\eta_\alpha(Y)$] + g(ϕ∇_XY,N)+g(ϕ(σ(X,Y),N)+g($\phi^2 X$,N)+g(AϕYX,N)

=-g(∇_XY,ϕN)-g((σ(X,Y),ϕN)+ g(AϕYX,N)

=-g(∇_XY,ϕN)-g(AϕNX ,N)+ g(AϕYX,N)

Since ϕN is also in $T_X^\perp$ M, R.H.S of the above equation is zero. Thus

g($\nabla_X^\perp \phi$ Y ,N) = 0

Hence the result.

If D denotes the orthogonal subspace of T $\overline{M}$ to L({ ξ_α }) then we can write

T $\overline{M}$= D_⊕L{ ξ_α }.

We prove the following Theorem.

Theorem 3.5. ; Let M be a submanifold of an S-manifold $\overline{M}$ then M is anti-invariant if and only if D is integrable

Proof. ; Let X, Y ∈ D then X, Y ∈ T $\overline{M}$

g([X, Y], ξ_α) = g($\overline{\nabla}_X$ Y − , $\overline{\nabla}_Y$ X, ξ_α)

=g($\overline{\nabla}_X$Y, ξ_α) − g($\overline{\nabla}_Y$ X, ξ_α) (3.5)

=Xg(Y, ξ_α)-g(Y, $\overline{\nabla}_X\xi_\alpha$)-Yg(X, ξ_α)+g(X, $\overline{\nabla}_Y\xi_\alpha$)

Using (1.4) we have (2.6)

g([X, Y], , ξ_α) = 2g(ϕX, Y) (3.6)

Thus [X, Y] ∈D if and only if ϕX is normal to Y

i.e [X, Y] ∈ D if and only if ϕX ∈ $T_X^\perp$ M

i.e [X, Y] ∈ D if and only if M is anti-invariant

i.e D is integrable if and only if M is anti-invariant

Hence the theorem.

We have the following known result.

Proposition 2.6. ,[36] Let M^{n+s} be a submanifold tangent to the structure vector fields of an S-manifold $\overline{M}^{2n+s}$ (k) (k ≠ s). Then ($\overline{R}$ (X, Y,Z)W)$^\perp$ = 0 for any X, Y,Z,W ∈ T(M), if and only if M^{n+s} is invariant or anti-invariant.

On the basis of the above we can prove the following theorem.

Theorem 3.6. Let M be a submanifold tangent to linear span L(|, ξ_α|) of structure vector fields , ξ_α , ξ_α = 1, ..., s of a S- manifold with constant k (k ≠ 3) then M is T- invariant if only if M is invariant or anti-invariant.

Proof. Easily follows from the Proposition 2.6

4. REFERENCES

[1] Cabrerizo.J.L, Fernandez.L.M and Fernandz. M. ,A Classification of Certain submanifold of an Smanifold, A˙ nn polinici Mathematici 54(21),117-123 (1991)

[2] Cabrerizo.J.L, Fernandez.L.M., and Fernandz.M. ,A Classification Totally f-umbilical submanifold of an S-manifold, ˙ Soochow J. Math.18(21),211-121(1992)

[3] Blair,DE, Geometry of manifolds with structural group U(n)×O(S) , J.Differ.geom.Vol.4(1970),155-167

[4] J.VanzuraK. , Almost r-contact Structures , Ann.Scuola Norm.sup.pisa.Sci.Fis.mat.26(1972),97-115

[5] Blair,DE, Contact Manifolds in Riemannian geometry , Lecture Notes in Mathematics.Vol.509. Springerverlag, Berlin and NewYork. (1976)

[6] K.R.Vidyavathi and C.S Bagewadi. A study on Ricci Soliton in S-manifolds, IOSR-JM J. VOL 13.Jan- Feb (2017).

[7] Mukut Mani Tripathi. Submanifolds of framed metric manifolds and S-manifolds, Note di.Matematica 20, N.2, (2001), 135-164

[8] K.R.Vidyavathi and C.S Bagewadi. Irrotational _-Curvature tensor In S-manifolds, IJMMS VOL 13.Jan- Jun (2017);155-166.

[9] Anitha B.S and C.S.Bagewadi. Invariant submanifolds of Sasakian manifolds Admitting Semisymmetric Nonmetric Connection, International Joumal of Mathematics and Mathematical science (2012) 18 pages

[10] C.S Bagewadi and Anitha B.S . Invariant submanifolds of TransSasakian manifolds, Ukrainian mathematical Jounal vol 67 no, 10 (2016);

[11] M.S.Siddesha and C.S Bagewadi. Submanifold of a (k, μ)-Contact manifold, CUBO A mathematical Journal J. VOL 18. N 01 (59-68).(2016)

[12] K.Yano. , On structure f-satiesfying f3 + f = 0, Techinical report N o 12 . Univ.of Washington (1961)

[13] H.B.Pandey and Anilkumar ,Anti-invariant Submanifolds of almost para contact manifolds, ˙Indian J.Pure appl.Math. 16(6); 586-590 (1985)

[14] K.Yano. , On structure defined by a tensor field of type (1,1) satiesfying f3+f = 0, T˙ ensor 14(1963),99-109

[15] D.Nirmala, C.S Bagewadi and M.S.Siddesha. Semi-invariant Submanifold of a (k, μ)-Contact manifold, Journal-Bulletin of cal. math. Soc Journal VOL 109. No 02 93-100.(2016)

[16] MOHAMMED HASAN SHAHID. Some Results on Anti-invariant Submanifolds of a Trans- Sasakianmanifold., Bull.Malays.Math.Sci.(2) 27 (2004),117-127

[17] A.Brasil, G.A.Lobos, M.Marlano , C-Totally Real submanifolds with Parallel mean Curvature in _ - Sasakian space Forms, Matematica Contemporanca, Vol.34, (2008), sociedade Brasileia de Matematicano.

[18] Yano.K, M.Kon, Anti- -Invariant Submanifolds of a Sasakian Space forms, II., J.Korean Math Soc.13(1976).1-14

[19] Yildiz, A.Murathan , C.Arslan, K.Ezentas,, R, C-totally real pseudo parallel submanifold of Sasakian space forms, Monatsh.Math 151, (2007), 247-256.

[20] Santos,W,,, Submanifolds with parallel mean curvature vector in spheres, Tohoku Math.J.46(1994), 403- 415.

[21] Alegre .P, Carriazo,, A Structures on generalized Sasakian-space forms-, Differential Geom.App). 26 (2008). 656 666.

[22] Blair,DE, Contact Manifolds in Riemannian geometry , Lecture Notes in Mathematics.Vol.509. Springerverlag, Berlin and NewYork. (1976)

[23] D.E. Blair, T. Koufogiorgos and B. J. Papantoniou Contact metric manifolds satisfing a nullity condition, Israel J. Math., 91 (1995), 189214.

[24] B.Y. Chen, Geometry of submanifolds and its applications, Science University of Tokyo, Tokyo (1981).

[25] M. Kon, Invariant submanifolds of normal contact metric manifolds, Kodai Math. Sem. Rep., 27 (1973), 330-336.

[26] C. Ozgur, C. Murathan, On invariant submanifolds of Lorenzian para-Sasakian manifolds, The Arabian J. Sci. Engg., 34, 24(2009), 177-185.

[27] Gerald D.Ludden,Masafumi Okumura,and Kentaro Yano, Anti- invariant submanifolds of almost contact metric manifolds, Math.Ann. Springer-Verlag 225.253-261 (1977),

[28] Sibel Sular, Cihan Ozgur and Cengizhan Murathan Pseudoparallel anti-invariant submanifolds of kenmotsu manifolds, Hacettepe, Journal of mathematics and statistics. vol.39(4)(2010),535-543

[29] S.I.GOldberg and K.Yano , Globally framed f-manifold, III J.Math 15 (1971), 456-474.

[30] C.S.Bagewadi, On totally real submanifolds of a Kahlerian manifold addmitting semisymmetric metric, Indian J.Pure Appl.Math.13(5) (1982),528-536

[31] J.A. Schouten, Ricci Calculas(second Edition), Springer-Verlag., (1954), 322.

[32] K.Yano and M.Kon, Anti-invariant submanifolds,pure and Applied Mathematics, N0.21 Marcel Dekker Inc Newyork,(1976).

[33] K.Yano and M.Kon, Anti-invariant submanifolds of a Sasakian space form Tohokumath J 9(1977),9-23. 12 C.S. Bagewadi and Venkatesha.S

[34] M.Hasan Shahid, Anti-invariant submanifolds of a Kenmotsu manifold, Kuwait J.Sci. and Eng.23(2), (1996).

[35] K. Yano, M. Kon, Structures on manifolds, World scintific publishing, (1984)