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Inventory Model of Perishable Products with Periodic Demand

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Abstract:
In this paper, an inventory model for perishable products with periodic demand and constant deterioration has been proposed. Here model considers both the cases one allowing with shortages and the other without shortages. The deterioration rate of items is taken as constant and the demand rate is taken as periodic function of time. This model has been derived in order to obtain total variable cost.

Keywords: Demand rate, Deterioration, Periodic, Shortages

I. INTRODUCTION

Most of the inventory models in the literature are based on assumption of imperishability of products with infinite useful life. However, in our real life, some products that can be damaged and expired with respect to time or we can say the products that can deteriorate and become unusable after some finite time are called perishable products. Fresh food, blood products, meat, chemicals, composite materials and pharmaceuticals are all examples of perishable products. The usage of perishable products is huge such as food industry and healthcare services. Perishable goods can be broadly classified into two main categories based on deterioration and Obsolescence. Deterioration refers to damage, spoilage, depletion, decay of goods. Obsolescence is loss of value of a product because of arrival of new and better product. Perishable goods are an important part of stocks carried in practice. The study of perishable inventory systems has been the theme of many articles due to its applications in sectors like food, chemicals, pharmaceuticals, photography and blood bank management. Most inventory control models assume that the demand rate to be either constant or time-dependent but independent of the stock status. For perishable products, such as food, fashion products, the demand rate may be influenced by the stock level. However, when dealing with perishable products, deterioration rate can take place during the holding period. So in the study of inventory models of perishable product, deterioration of product plays very important role. Therefore in formulating inventory models of perishable products, the demand rate and the deterioration rate are two important factors. In the last few years several researchers have discussed inventory problem involving time variable demand pattern. Ghare and Schrader (1963) were the first proponents for developing a model for an exponentially decaying inventory. Covert and Philip (1973) extended this model by considering weibull distribution deterioration rate. Stafen and Sivazlian (1975) discussed a finite horizon inventory problem with variable demand. Donaldson (1977) solved the classical inventory problem without shortages with linear demand over finite time horizon. Shah and Jaiswal (1977) developed an order level inventory model with constant rate of deterioration. Dave and Patel (1981) considered an inventory model without shortages for perishable products with time dependent demand. Sachan (1984) then extended this model by allowing shortages. Dave (1989) developed models with shortages taking time dependent demand. But it was computationally complicated. Data and Pal (1992) developed same model which was very simple and easy to calculate the values of the decision variables. Hariga (1996) developed optimum EOQ models for perishable products with time varying demand. Teng (1999) and Yang (2001) developed deterministic lot size inventory models with shortages and deterioration for fluctuating demand. Goyal and Giri (2001) wrote the survey on recent trends in modeling of deteriorating inventory. Pande, Gautam and katyar (2015) developed an inventory model with periodic demand and constant deterioration by allowing shortages. The rest of the paper is organized as follows. In section II, we describe the assumptions and notations used throughout this paper. In section III, we develop the mathematical model without shortage to obtain the total cost per year. We then develop the mathematical model with shortages to obtain the total cost per year in section IV. Finally, we draw the conclusion and the future research in section V.

II. ASSUMPTIONS AND NOTATIONS

For modelling the inventory system, we will use the following notations and assumptions.

ASSUMPTIONS

1) The demand is periodic function of time and is given by $asint$, whose value is sensitive to the selling price.
2) The deterioration rate θ is constant, where $0 < \theta \ll 1$. It is assumed that deterioration starts when the products are stored at the warehouse.
3) The lead time is zero.
4) The replenishment time is infinite.
5) Inventory holding cost is charged only on the amount of undecayed stock.
6) Shortages are allowed in second case only.
7) There is no repair or replacement of the deteriorated items.

NOTATIONS

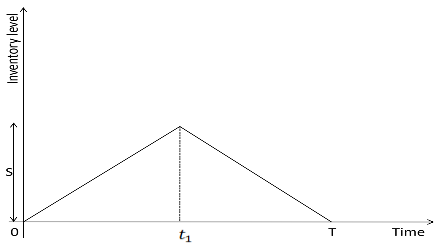
1) $asint$ = the demand rate

2) θ = the deterioration rate of on hand inventory, where $0 \le \theta < 1$
3) $q(t)$ = the inventory level at time $t, 0 \le t \le T$
4) C_r = the replenishment cost
5) C_0 = the ordering cost per order
6) C_d = the deterioration cost
7) C_s = shortage cost
8) C_b = backorder cost per unit
9) C_h = the holding cost
10) C = the purchasing cost per unit
11) h = unit holding cost per unit time
12) S = the maximum inventory level
13) R = the backorder level
14) Q = total inventory level, $Q = R + S$
15) T = the replenishment cycle time
16) P = the production rate
17) T_1 = time during which production runs and decreases due to demand and deterioration
18) T_2 = time during which inventory level becomes zero due to demand and deterioration
19) T_3 = time during which shortages are allowed
20) T_4 = production time with backorder
21) $Z(T)$ = the total relevant cost per year

where the total relevant cost consists of replenishment cost, deterioration cost, holding cost and shortage cost.

III. MODEL FORMULATION
(WITHOUT SHORTAGES)

Let $q(t)$ is the inventory level at any time$t$. We consider S is the highest inventory.



The inventory level increases with the production rate P and decreases due to the market demand and deterioration during $(0, t_1)$ and during(t_1, T), the inventory depletes to zero due to demand and deterioration.

So, the model formulated by the differential equations,

$$\frac{dq}{dt} = P - asint - \theta q, \;\; 0 \le t \le t_1 \qquad (1)$$

With the boundary condition $q(0) = 0$ (2)

and $\frac{dq}{dt} = -asint - \theta q, t_1 \le t \le T$ (3)

With the boundary condition $q(T) = 0$ (4)

Solving (1) with boundary condition (2), we get

$$q(t) = \frac{P}{\theta}\left(1 - e^{-\theta t}\right) - \frac{a}{1+\theta^2}\left(\theta sint - cost + e{-}\theta t\right) \qquad (5)$$

Solving (3) with boundary condition (4), we get

$$q(t) = -\frac{a}{1+\theta^2}(\theta sint - cost) + \frac{ae^{\theta(T-t)}}{1+\theta^2}(\theta sinT - cosT) \qquad (6)$$

Total replenishment cost is given by,

$$C_r = \frac{C_0}{T} \qquad (7)$$

Total holding cost is given by,

$$C_h = \frac{h}{T}\int_0^T q(t)\,dt$$

$$= \frac{h}{T}\left[\int_0^{t_1} q(t)\,dt + \int_{t_1}^{T} q(t)\,dt\right]$$

$$= \frac{h}{T}\left[\int_0^{t_1}\left(\frac{P}{\theta}\left(1 - e^{-\theta t}\right) - \frac{a}{1+\theta^2}\left(\theta sint - cost + e{-}\theta t\right) dt + t1T{-}a1+\theta 2\theta sint{-}cost+ ae\theta T{-}t1+\theta 2\theta sinT{-}cosT\,dt\right.\right.$$

$$= \frac{h}{T}\left[\frac{P}{\theta^2}\left(\theta t_1 + e^{\theta t_1} - 1\right) + \frac{a}{1+\theta^2}(\theta cost_1 + \quad sint_1) + a\theta e\theta t11+\theta 2{-}1+a\theta cosT{-} e\theta(T{-}t1)1+\theta 2\theta sinT{-}cosT{-} \quad a\,1+\theta 2\theta cost1+sint1\right.$$

$$= \frac{h}{T}\left[\frac{P}{\theta^2}\left(\theta t_1 + e^{\theta t_1} - 1\right) + \frac{a}{\theta}\left(\frac{e^{\theta t_1}}{1+\theta^2} - 1\right) + \quad \frac{a}{\theta}\left(cosT - e\theta(T{-}t1)1+\theta 2\theta sinT{-}cosT\right.\right.$$

Since $e^{\theta t_1} = 1 + \theta t_1$

$\Rightarrow \theta t_1 + e^{\theta t_1} - 1 = 2\theta t_1$ (Neglecting second and higher order terms ofθ)

$$\therefore C_h = \frac{h}{T}\left[\frac{2Pt_1}{\theta} + \frac{a}{\theta}\left(cosT - 1 + \frac{e^{\theta t_1}}{1+\theta^2} - \frac{e^{\theta(T-t_1)}}{1+\theta^2}(\theta sinT - cosT\right.\right. \qquad (8)$$

Total deterioration cost is given by,

$$C_d = \frac{C}{T}\left(S - \int_0^T asint\,dt\right)$$

$$= \frac{C}{T}(S + acosT - a)$$

$$= \frac{C}{T}(S - a(1 - cosT)) \qquad (9)$$

The total inventory cost per unit time is given by,

$$Z(T) = C_r + C_h + C_d$$

$$= \frac{C_0}{T} + \frac{h}{T}\left[\frac{2Pt_1}{\theta} + \frac{a}{\theta}\left(cosT - 1 + \frac{e^{\theta t_1}}{1+\theta^2} - \quad \frac{e^{\theta(T-t_1)}}{1+\theta^2}(\theta sinT - cosT + \quad CTS{-}\,a(1{-}cosT)\right.\right. \qquad (10)$$

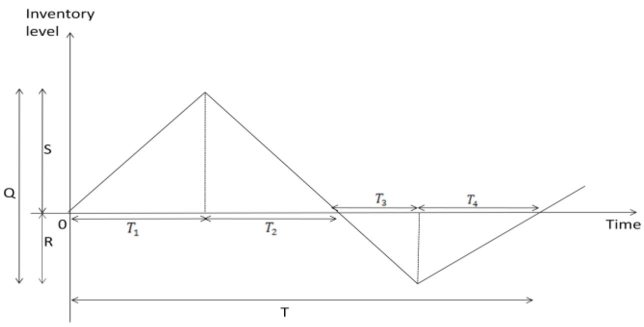
To get the optimum value of T, one need to apply the criteria of optimization as $\frac{dZ}{dT} = 0$,which can be solved numerically to get optimum value of T that is T^*. We can find the total optimum cost of the system $Z^* = Z(T^*)$ by substituting the value of T^* in equation (10).

IV. MODEL FORMULATION

(WITH SHORTAGES)

A model considered in this section allows shortages to be backordered. This situation is shown in below figure. In this model the inventory level decreases below the 0 level. This means a portion of the demand is backlogged.

The maximum inventory level is S and occurs when the order arrives. The maximum backorder level is R, where $R = Q - S$. A backorder is represented by a negative inventory level in the figure. Figure given below depicts the inventory level of inventory model with allowable shortages and deteriorating items.



It is necessary to mention that, during the first period T_1, production runs and also decreases due to deterioration and demand as deterioration starts as soon as production being produced. During time period T_2, there is no production and inventory level depletes to zero due to demand and deterioration. In T_3, there is not any product to fulfill market demands which are going to be backlogged to the next period. Moreover, in time period T_4, the manufacturer produces items to satisfy the arriving as well as the previous period's demand. Now, we firstly obtain the inventory level function at any time t, for formulating the inventory system.

The inventory level in T_1 $(0 \le t \le T_1)$:-
The inventory increases with production rate P and decreases because of demand $asint$ and deterioration θ. Hence, the inventory level in this period can be described by,

$$\frac{dq}{dt} = P - asint - \theta q,\ 0 \le t \le T_1 \qquad (11)$$

With the boundary condition $q(0) = 0$
Solving (11), we get

$$q(t) = \frac{P}{\theta}\left(1 - e^{-\theta t}\right) - \frac{a}{1+\theta^2}\left(\theta sint - cost + e^{-\theta t}\right) \qquad (12)$$

The inventory level in T_2 $(T_1 \le t \le T_1 + T_2)$:-
Here inventory level depletes to zero due to demand $asint$ and deterioration θ. Thus, the inventory level is governed by the differential equation,

$$\frac{dq}{dt} = -asint - \theta q, (T_1 \le t \le T_1 + T_2) \qquad (13)$$

With the boundary condition $q(T_1 + T_2) = 0$
So, the solution of (13) is

$$q(t) = \frac{a}{1+\theta^2}\left[e^{\theta(T_1+T_2-t)}\left(\theta sin(T_1 + T_2) - cos(T_1 + T_2)\right) - \right.$$
(θsint− cost) (14)

The inventory level in T_3 $(T_1 + T_2 \le t \le T_1 + T_2 + T_3)$:-
Since there is no on hand inventory and demand would be backlogged in this period.
So, $q(t)$ satisfies the following differential equation,

$$\frac{dq}{dt} = -asint,\ T_1 + T_2 \le t \le T_1 + T_2 + T_3 \qquad (15)$$

With the boundary condition $q(T_1 + T_2) = 0$
Thus, the inventory level in T_3 is given by,

$$q(t) = a\left(cost - cos(T_1 + T_2)\right) \qquad (16)$$

The inventory level in $T_4 (T_1 + T_2 + T_3 \le t \le T)$:-
The produced items during this period would be depletes due to the demand as well as backorders.
Therefore, the differential equation representing the inventory level of the system is

$$\frac{dq}{dt} = P - asint,\ T_1 + T_2 + T_3 \le t \le T \qquad (17)$$

With the boundary condition $q(T) = 0$
So, the solution of (17) is given by,

$$q(t) = P(t - T) + a(cost - cosT) \qquad (18)$$

Inventory model consists of the replenishment cost, stock holding cost, shortage cost and deterioration cost. These elements are formulated below.

Total replenishment cost is given by,

$$C_r = \frac{C_0}{T} \qquad (19)$$

Total inventory holding cost per unit time is given by,

$$C_h = \frac{h}{T}\left[\int_{T_1} q(t)\,dt + \int_{T_2} q(t)\,dt\right]$$
$$= \frac{h}{T}\left[\int_0^{T_1} q(t)\,dt + \int_{T_1}^{T_1+T_2} q(t)\,dt\right]$$
$$= \frac{h}{T}\left[\int_0^{T_1}\left(\frac{P}{\theta}\left(1 - e^{-\theta t}\right) - \frac{a}{1+\theta^2}\left(\theta sint - \quad cost + \right.\right.\right.$$
e−θtdt+ T1T1+T2a1+θ2eθ(T1+T2−t)θsinT1+T2−cosT1+T2−(θsint−cost) dt

$$= \frac{h}{T\theta^2}\left[P\left(\theta T_1 + e^{\theta T_1} - 1\right) + \frac{a\theta}{1+\theta^2}\left(e^{\theta T_1} + e^{\theta T_2}\left(\theta sin(T_1 + \right.\right.\right.$$
T2−cosT1+T2+aθcosT1+T2−1 (20)

Total shortage cost is calculated based on the areas under T_3 and T_4, and its value per unit time is given by,

$$C_s = \frac{-C_b}{T}\left[\int_{T_3} q(t)\,dt + \int_{T_4} q(t)\,dt\right]$$
$$= \frac{-C_b}{T}\left[\int_{T_1+T_2}^{T_1+T_2+T_3} q(t)dt + \int_{T_1+T_2+T_3}^{T} q(t)\,dt\right]$$
$$= \frac{-C_b}{T}\left[\begin{array}{l}\int_{T_1+T_2}^{T_1+T_2+T_3}\left(a\left(cost - cos(T_1 + T_2)\right)\right)dt + \\ \int_{T_1+T_2+T_3}^{T}\left(P(t - T) + a(cost - cosT)\right)dt\end{array}\right]$$
$$= \frac{-C_b}{T}\left[P\left(T(T_1 + T_2 + T_3) - T_1T_3 - T_3T_4 - T_4T_1\right) - \right.$$
asinT1+T2+sinT1+T2+T3+ T3cosT1+T2+ T4cosT

$$= \frac{C_b}{T}\left[P\left(T_1T_3 + T_3T_4 + T_4T_1 - T(T_1 + T_2 + T_3)\right) + \right.$$
asinT1+T2+sinT1+T2+T3+ T3cosT1+T2+ T4cosT
(21)

Total deterioration cost is given by,

$$C_d = \frac{C}{T}\left[S - \int_0^{T_1+T_2} asint\,dt\right]$$
$$= \frac{C}{T}\left[S + a\cos(T_1 + T_2) - a\right]$$
$$= \frac{C}{T}\left[S + a(\cos(T_1 + T_2) - 1)\right] \qquad (22)$$

Therefore the total inventory cost per unit time is given by,

$$Z(T) = C_r + C_h + C_s + C_d$$
$$= \frac{C_0}{T} + \frac{h}{T\theta^2}\left[P\left(\theta T_1 + e^{\theta T_1} - 1\right) + \frac{a\theta}{1+\theta^2}\left(e^{\theta T_1} + \quad e^{\theta T_2}\left(\theta sin(T_1 + \right.\right.\right.$$
T2−cosT1+T2+ aθcosT1+T2−1+
$$\frac{C_b}{T}\left[P\left(T_1T_3 + T_3T_4 + T_4T_1 - T(T_1 + T_2 + T_3)\right) + \quad a(sin(T_1 + \right.$$
T2+sinT1+T2+T3+ T3cosT1+T2+T4cosT+CTS+
a(cosT1+T2−1) (23)

To get the optimum value of T, one need to apply the criteria of optimization as $\frac{dZ}{dT} = 0$, which can be solved numerically to get optimum value of T that is T^*. We can find the total optimum cost of the system $Z^* = Z(T^*)$ by substituting the value of T^* in equation (23).

V. CONCLUSION

The inventory model has three principal tasks. (1) Constructing the mathematical model, (2) finding the values of the model parameters and (3) finding the optimum solution. This paper deals with the inventory model of perishable products with periodic demand and constant deterioration rate. This model attempts to obtain the average total cost per unit time of the inventory system. This paper has presented only the total optimum cost function of the inventory system for the both situation, with shortage and without shortage. This model can further be extended for other functional relations of demand rate and deterioration rate. We could generalize the model to allow trade credits and time discount, quantity discount, inflation rates, and others.

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