ISSN XXXX XXXX © 2018 IJESC

IJESC

Research Article Volume 8 Issue No.5

Fuzziness in Polynomial Equations

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Abstract:
In this paper we discuss about ranking based clarification in a feature selection problem and consistency of a Fuzzy polynomial equation is explained. By transforming fuzzy polynomial equations to a system of crisp platform, based on three parameters efficiency value, ambiguity and fuzziness. Numerical examples are solved to provide a over view of how ranking Fuzzy numbers method is used to real life applications by solving Fuzzy polynomial equations.

Keywords: Fuzzy polynomial, fuzzy tuplet, triangular fuzzy number, trapezoidal fuzzy number, efficiency value, ambiguity.

1. INTRODUCTION:

Polynomial equation formed with fuzzy coefficients $C_0, C_1, C_2, ..., C_n$ as fuzzy numbers $C_1x + C_2x^2 +, ..., C_nx^n = C_0$ plays an important role in various areas of Mathematics, Engineering and sciences. Using three real indices value, ambiguity, Fuzziness we propose a new method of solving Fuzzy polynomial equation based on ranking method with examples. We review some basic definitions needed in the next section

2.1 Feature ranking Phenomenon:

Let $A = \{a_1, a_2, ..., a_m\}$ be set of 'm' attributes. Let r be a function $r : A_D \to \Re$ having a value of merit of efficiency attribute (Lattice co-efficient) for $a \in A$ from D.A feature ranking is a function F that assigns a value of merit (Relevance) to each attribute $((a - a_l)\alpha + a_l, -(a_u - a)\alpha + a_u)$ and returns a list of attributes $(a_i \in A)$ ordered by its relevance with attributes $i \in \{1, 2, 3, ..., m\}$

$$F(a_1, a_2, ..., a_m\} = (a_1^*, a_2^*, a_3^*, ...a_m^*)$$

Where $r(a_1^*) \geq r(a_2^*) \geq r(a_3^*) \geq, ... \geq r(a_m^*)$

By convention we assume that a high score is indicative of relevant attribute and attributes are sorted in decreasing order of $r(a^*)$.we consider ranking criteria for individual features independently of context.

2.2 Ranking Based clarification in FS Problem

Let S_k^f be a function that returns the sub set of first K attributes provided by a feature ranking method $F(S_k^f : A^m \to A^k)$.

The ranking based accuracy $H(\mathrm{X}(e_i) = y_i)$ will be defined by

$$CA_k(F, \mathrm{H}) = 1 / \mathrm{N} \sum_{i=1}^{n} (H(\mathrm{S}_k^f(\mathrm{x}(e_i) = y_i)$$

S_1^f first best attribute , S_2^f is of first two features and thus up to m.

2.3 Def:

Let C is Fuzzy subset with membership function $C(\mathrm{x}) : \mathrm{R} \to [0,1]$ is called a Fuzzy number if

1.C is normal $\exists$ an element $x_0 \in C$ such that $c(x_0) = 1$

2.C is convex

$C(\alpha x_1 + (1-\alpha)x_2) \geq \mathrm{C}(x_1) \wedge C(x_2) \forall x_1, x_2 \in R, \alpha \in [0,1]$

3. C is upper semi continuous.

4. Support C is bounded where support $C = \{x \in R : c(x) > 0\}$ at varying levels $\alpha \in [0,1]$ must be of finite length.

2.4 Triangular Fuzzy Number

Def:

An α-cut of tuple of three numbers (a_l, a, a_u) denoted by $[\mathrm{L}_{a_\alpha}, \mathrm{R}_{a_\alpha}]$definedas ordered pair $((a - a_l)\alpha + a_l, -(a_u - a)\alpha + a_u)$

For Example

(-3,2,4)= $((2+3)\alpha - 3, -(4-2)\alpha + 4) = (5\alpha - 3, -2\alpha + 4)$ where $\alpha \in (0,1]$

A membership function

$[\mathrm{L}_{a_\alpha}, \mathrm{R}_{a_\alpha}] = (6\alpha - 4, -7\alpha + 9)\alpha \in (0,1]$

Range of x in R

$x = 6\alpha - 4$

if $\alpha = \dfrac{x+4}{6}\ fromL_{C_\alpha}$

$-7\alpha + 9$ if

$\alpha = \dfrac{9-x}{7}\ fromR_{C_\alpha}$

Further C(X)=0 if $x < -4, x > 9$

$= \dfrac{x+4}{6}$ if $-4 \leq x \leq 2$

$= \dfrac{9-x}{7}$ if $2 < x \leq 9$

In general (a_l, a, a_u) tuple can be defined as C(x)=0 If

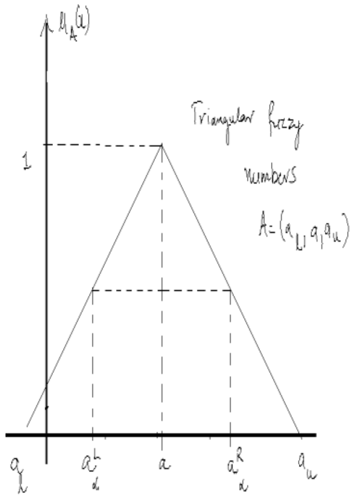
$x < a_l,$

$x > a_u$

$= \frac{x-a_l}{a-a_l} a_l \le x \le a$

$= \frac{a_u - x}{a_u - a} a < x \le a_u$

Which is a triangular Fuzzy number.



2.5 Trapezoidal Fuzzy Number TrFN

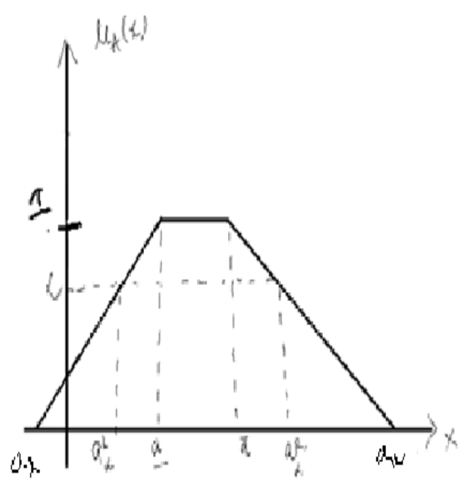
A trapezoidal Fuzzy number denoted by Quadruplet $A = (a_l, a, \bar{a}, a_u)$ and has shape of Trapezoidal.

α -cut of TrFN

$A_\alpha = ((a - a_l)\alpha + a_\alpha, -(a_u - \bar{a})\alpha + a_u)$ where

$\alpha \in [0,1]$ or $(\mathrm{L}_{C_\alpha}, R_{C_\alpha})$

Denoted by



2.6 Support that C is a Fuzzy number $r \in (0,1)$ and r cut $(\mathrm{L}_{C_r}, R_{C_r})$.

A function S: X→[0,1] is said to be increasing S(0)=0,S(1)=1. Let simplest most natural reducing function is uniformly S(r) =r With respect to uniform reducing function,

Efficiency value $V(\mathrm{c}) = \int_0^1 r[\mathrm{L}_{C_r} + \mathrm{R}_{C_r}]\mathrm{d}_r$

Ambiguity $\mathrm{A(c)} = \int_0^1 r[\mathrm{R}_{C_r} - L_{C_r}]\mathrm{d}_r$

Fuzziness

$\mathrm{F(c)} = \int_0^{1/2} (\mathrm{R}_{C_r} - L_{C_r})\mathrm{d}_r + \int_0^{1/2} (\mathrm{L}_{C_r} - R_{C_r})\mathrm{d}_r$

May be V(c) is efficiency value representing Fuzzy number; A(C) is global spread of C(x).
F(C) is global difference
Also say $C_1 = C_2$
if

$V(C_1) = V(C_2)$

$\mathrm{A}(C_1) = A(C_2)$ in ranking method, we compare C_1, C_2

$\mathrm{F}(C_1) = F(C_2)$

Otherwise use iterative algorithm

Let $C = (a, b, \alpha, \beta)$ TrFN

$\mathrm{V}(C) = \frac{a+b}{2} + \frac{\beta - \alpha}{6}$

$A(C) = \frac{b-a}{2} + \frac{\beta + \alpha}{6}$

$\mathrm{F}(C) = \frac{\beta + \alpha}{4}$

Let $C = (\mathrm{m}, \alpha, \beta)$ Triangular Fuzzy Number

$V(C) = m + \frac{\beta - \alpha}{6}$

$A(C) = \frac{\beta + \alpha}{6}$

$\mathrm{F}(C) = \frac{\beta + \alpha}{4}$

3. FUZZY POLYNOMIAL EQUATION

Fuzzy polynomial expression constituted with $C_1, C_2, C_3, \ldots C_n$ Fuzzy numbers may be triangular or trapezoidal number is denoted by

$C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n$

Hence a polynomial Equation can be written as

$C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n = C_0$

$\forall \ x \in R$

Obviously

$V(C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n) = V(C_0)$

$\mathrm{A}(C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n) = A(C_0)$

$\mathrm{F}(C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n) = F(C_0)$

$\Rightarrow$

$V(C_1)x + V(C_2)x^2 + V(C_3)x^3 + \ldots + V(C_n)x^n = V(\mathrm{C}_0)$

$\mathrm{A}(C_1)x + A(C_2)x^2 + A(C_3)x^3 + \ldots + A(C_n)x^n = A(\mathrm{C}_0)$ ------> (1)

$\mathrm{F}(C_1)x + F(C_2)x^2 + F(C_3)x^3 + \ldots + F(C_n)x^n = F(\mathrm{C}_0)$

Corresponding to any Triangular Fuzzy number $C = (\mathrm{m}, \alpha, \beta)$

for each $C_i = (\mathrm{m}_i, \alpha_i, \beta_i)$

$V(C_i) = \mathrm{m}_i + \frac{\beta_i - \alpha_i}{6}$ $\quad i = 0,1,2,...,n$ *are calculated*

$V(C_i) = \frac{\beta_i + \alpha_i}{6}$

$\mathrm{F}(C_i) = \frac{\beta_i + \alpha_i}{4}$

For TrFN

$C_i = (a_i, \mathrm{b}_i, \alpha_i, \beta_i)$

$V(C_i) = \frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}$

$\mathrm{A}(C_i) = \frac{b_i - a_i}{2} + \frac{\beta_i + \alpha_i}{6}$

$\mathrm{F}(C_i) = \frac{\beta_i + \alpha_i}{4}$

Triangular set of Fuzzy Polynomial Equations

$$(m_1 + \frac{\beta_1 - \alpha_1}{6})x + (m_2 + \frac{\beta_2 - \alpha_2}{6})x^2 + ... + (m_n + \frac{\beta_n - \alpha_n}{6})x^n = (m_0 + \frac{\beta_0 - \alpha_0}{6})$$
$$(\frac{\beta_1 + \alpha_1}{6})x + (\frac{\beta_2 + \alpha_2}{6})x^2 + ... + (\frac{\beta_n + \alpha_n}{6})x^n = (\frac{\beta_0 + \alpha_0}{6})$$
$$(\frac{\beta_1 + \alpha_1}{4})x + (\frac{\beta_2 + \alpha_2}{4})x^2 + ... + (\frac{\beta_n + \alpha_n}{4})x^n = (\frac{\beta_0 + \alpha_0}{4})$$

→(2)

This can be solved easily.
A trapezoidal Fuzzy polynomial set of equations imply that

$$(\frac{a_1 + b_1}{2} + \frac{\beta_1 - \alpha_1}{6})x + ... + (\frac{a_n + b_n}{2} + \frac{\beta_n - \alpha_n}{6})x^n = (\frac{a_0 + b_0}{2} + \frac{\beta_0 - \alpha_0}{6})$$
$$(\frac{b_1 - a_1}{2} + \frac{\beta_1 + \alpha_1}{6})x + ... + (\frac{b_n - a_n}{2} + \frac{\beta_n - \alpha_n}{6})x^n = (\frac{b_0 - a_0}{2} + \frac{\beta_0 + \alpha_0}{6})$$
$$(\frac{\beta_1 + \alpha_1}{4})x + ... + \frac{\beta_n + \alpha_n}{4})x^n = \frac{\beta_0 + \alpha_0}{4}$$

→(3)

4. NUMERICAL EXAMPLES:

4.1Solving Fuzzy polynomial equations

$$(0,2,2)x + (0,2,2)x^2 = (0,4,4)$$

With Fuzzy parameter Coefficients

$$V(\mathrm{c}_1) = 0, V(\mathrm{c}_2) = 0, V(\mathrm{c}_0) = 0,$$
$$\mathrm{A}(\mathrm{c}_1) = 4/6, \mathrm{A}(\mathrm{c}_2) = 4/6, \mathrm{A}(\mathrm{c}_0) = 8/6,$$
$$\mathrm{F}(\mathrm{c}_1) = 1, \mathrm{F}(\mathrm{c}_2) = 1, \mathrm{F}(\mathrm{c}_0) = 2$$

We have 0=0, $x + x^2 = 2, x + x^2 = 2$

Exact solution $x = 1, x = -2 \in \Re$

4.2 Consider polynomial equation

$$(2,0,1,4)x + (1,2,3,1)x^2 + (1,1,2,3)x^3 = (1,2,1,15)$$

Here

$$V(\mathrm{c}_1) = 9/6, V(\mathrm{c}_2) = 7/6, V(\mathrm{c}_3) = 7/6, V(\mathrm{c}_0) = 23/6,$$

Hence the implied equation

$$7x^3 + 7x^2 + 9x = 23$$

Has got exact solution x=1 is verified.
Also, follows that other two equations of the system.

5.CONCLUSION:

Solving Fuzzy polynomial equations is explained in above two examples. Some fuzzy polynomial equations are existing with no real roots and hence the system is in-consistent. In case of system not having exact solution there is a choice of iteration method for approximating the solution.

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