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Simulation of Harmonically Excited Simple Linear Pendulum (HESLP) Model using Selected Versions of Fourth Order Runge-Kutta Schemes

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Abstract:
Classical pendulum is a very interesting dynamical system to study; it has been used to introduce the study of dynamic systems and to predict response of more complex systems with similar model. Hence family of fourth order Runge-Kutta (RK4) schemes for solving engineering problems numerically is investigated in this work based on absolute average performance of each version of the fourth order RK4 schemes compared with the analytical result using harmonically excited simple linear pendulum model. The pendulum model has been studied using the popular Runge-Kutta (RK41) formula, as it gives accurate and acceptable result up to a ten thousandth place when compared with the analytical result. The fourth order RK4 methods form a family of formulas with each member having 13 characteristic coefficients based on Butcher's assumptions. This work involves simulation of harmonically excited simple linear pendulum for five different time steps using ten selected versions of the RK4 schemes and the analytical solution as the yard stick for measuring performance over 60 800 numbers of steps. This work reveals the superiority of version five-RK45 for all frequency ratios less than unity and for all time steps over its popular counterpart.

Keywords: Butcher's assumption, Runge-Kutta methods, characteristic coefficient, performance, frequency ratio

I. INTRODUCTION

Classical pendulum is a very interesting dynamical system to study. It has been used to introduce young students to study of dynamic systems as well as being used to study more complicated pendula systems by use of numerical methods [2] in which the pendulum model, though simple, produces an astonishingly complex geometry [5]. In the paper [13], talking about next generation science, he emphasizes the relationship between sciences and engineering practices as one of complementarity. Given the inclusion of engineering in the science standards, the practices complement one another and should be mutually rein-forcing in curricular and instructions. [3] reported the close analogue between classical pendulum and super conducting electronic device called Josephson junction; hence it can be used to predict the temporal response of super conducting devices and by implication the response of a driven classical pendulum can be used to investigate the response of more complex systems with similar model. [14] stated that the pendulum principle has been used for practical application of compound pendulum theory in the determination of the mass distributions of astronauts, farm tractors and sailing boats. [15] reported the use of pendulum slider in health and safety practices. In his work, a slip resistance on floors and stairs were measured by pendulum slider. [6] gave examples of systems that have similar model as driven pendulum such as torsional pendulum, heavy rotating machinery when heavy fly wheel is attached to a shaft and the electrical RLC circuit. [7] also reported the use of visualized results obtained in his work to highlight the hypersensitivity of the pendulum to initial conditions to effectively introduce the physics of chaotic system. In the paper [16], reported the third-order sub harmonic oscillation in weakly non-linear cyclic symmetric structures with multiple degrees using their linear approximation. A simple model consisting of apiece wise linear, single degree of freedom oscillator subject to harmonic excitation was reported for which symmetric double impact motions both harmonic and sub-harmonic were studied. [17] In the paper [18], an excited strong non-linear oscillator of Durffing type was investigated to verify the effect of the initial conditions on the motion. The solution of the non-linear oscillator was assumed in the form which is usual for the linear vibration model. In this work, Harmonically Excited Simple Linear Pendulum (HESLP) model has been chosen to work in the reverse, that is, HESLP has been used to investigate the numerical tool, fourth order Runge-Kutta (RK4) methods. The pendulum model has been studied using the popular Runge-Kutta method, RKv 1, as it gives accurate result to ten thousandth place (10-4) compare with the analytical method. [3] solved the equation of motion of classical simple and double pendulum using the RKv 1 and [8] reported Lyaponuov's spectrum of modified Lorenz system by constant step fourth order Runge-Kutta method, RKv 1, validated reports of Yuxai and Macro(1996). From the on-going, it is obvious that Runge-Kutta scheme is one of the versatile numerical tools for the simulation of physical and engineering systems. [4] and [9] reported selected version of fourth order Runge-Kutta schemes used to simulate oscillators dynamics. According to Lambert (1991), the Runge-Kutta methods represent an important family of implicit and explicit iterative method for approximation of ordinary differential equation in numerical analysis [1]. The fourth order Runge-Kutta formula is one of the most powerful predictor-corrector algorithms of all; one which is so accurate that computer packages designed to find numerical solutions for differential equations will use it by default. [18] Also, one of the advantages of fourth order Runge-Kutta schemes is the ease of changing the step size. [17] It was also reported that high order methods of Runge-Kutta formulas are capable of achieving highly accurate approximation of differential equation solution at lower computational cost than low order methods. [19] The 13 characteristic coefficients of each member of the family of the Runge-Kutta methods is based on J.C Butcher's work which revealed much success in analysis of explicit Runge-Kutta methods[1]. The versatility of popular fourth order Runge-Kutta (RKv 1) formula has been proved by various literatures that discloses it efficiency as a numerical tool. RKv 1, being a member of a family of which there is a 13 characteristic coefficients reported by [4], developed from the same Butcher's assumption need to be tested for performance on accuracy in comparison with each scheme in the family. Hence, the test for accuracy of each scheme is the subject matter in this work. Of interest in recent technological advancement is the manufacturing of drones and this growth requires that modeling tools be accurate as much as possible; all of these inspired the need to test and rank fourth order Runge-Kutta schemes.

II. METHODOLOGY

[6] stated that fourth order Runge-Kutta methods are good for numerical approximate solution of first order differential equation (ODE) with known initial condition and can be used to solve second ODE by transforming it into two first ODE with known initial condition to be solved simultaneously for which the steps of solution is shown in steps given by the following equations:

$\frac{dy}{dt} = f(t,y),$

$y_{i+1} = y_i + \emptyset h,$

With $\emptyset$ being an incremental weighting function. The general form of the weighting function $\emptyset$ is given as:

$\emptyset = C_1K_1 + C_2K_2 + C_3K_3 + \cdots + C_nK_n$. n depends on the order of the Runge-Kutta scheme in use.

The functions K_1 to K_4 for the fourth order Runge-Kutta schemes are given by the set of equations below and the corresponding predicting formula is given by:

$$K_1 = f(x_i, y_i),$$
$$K_2 = f(x_i + a_2h, y_i + b_{21}K_1)$$
$$K_3 = f(x_i + a_3h, y_i + b_{31}K_1 + b_{32}K_2)$$
$$K_4 = f(x_i + a_4h, y_i + b_{41}K_1 + b_{42}K_2 + b_{43}K_3)$$
$$y_{i+1} = y_i + h(C_1K_1 + C_2K_2 + C_3K_3 + C_4K_4).$$

[4] gives the Butcher's assumptions and formulas for which members of the fourth order Runge-Kutta family 13 characterizing coefficients are determine. These assumption and formulas are given by equations below

$$\sum_{i=1}^{s} c_i b_{ij} = c_i(1-a_j), j = 2,3,4$$
$$b_{21} = a_2$$
$$c_2 = (1-(2a_3))/(12a_2 * (1-a_2) * (a_3-a_2))$$
$$c_3 = (1-(2a_2))/(12a_3 * (1-a_2) * (a_3-a_2))$$
$$c_4 = ((6*(a_2a_3) - 4(a_2+a_3)+3)/(12*(1-a_2) * (1-a_3))$$
$$c_1 = 1-(c_2+c_3+c_4)$$
$$b_{32} = (a_3 * (a_2-a_3))/(2*a_2((2a_2)-1)$$
$$b_{31} = a_3 - b_{32}$$
$$b_4a = (1-a_2)*(2*(1-a_3)*(1-(2*a_3)) - (a_2-a_3))$$
$$b_4b = (2*a_2*(a_2-a_3))*((6*a_2*a_3) - 4*(a_2+a_3) + 3)$$
$$b_{42} = b_4a/b_4b$$
$$b_4c = (1-(2*a_2))*((1-a_2)*(1-a_3))$$
$$b_4d = (a_3*(a_3-a_2))*((6*a_2*a_3) - 4*(a_2+a_3)+3)$$
$$b_{43} = b_4c/b_4d$$
$$b_{41} = a_4 - (b_{42}+b_{43})$$

Constraining conditions are given by equation below

$$a_2 \neq 0,1; a_3 \neq 0,1; a_2 \neq a_3; c_4 = 0$$
$$\{6a_2a_3 - 4(a_2+a_3)\} \neq 0$$

The success criteria for a member of the family are: the absolute value of all the coefficient be less or equal to 5 for which $a_2 \geq 0.2$ and $a_3 \leq 0.8$. [1] and [10] presented the tedious computational equations of the 13 characteristic coefficients of fourth order Runge-Kutta schemes in their work. RK4 family member are identified by their 13 characteristic coefficients which are shown for Selected RK4 schemes in Table I.

Harmonically excited simple linear pendulum model (1) was developed and was solved analytically to obtain the response of the model to the excitation as shown in (3). The oscillatory response was considered, being the common application.

Using the Lagrange method, a superior method to derive the equation of motion of dynamic system,

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} + \frac{\partial T}{\partial \dot{q}} = Q_{iND} \quad [11]$$

Table. I .the characteristic coefficient of selected 4th order r-k formulas

Selected fourth order RK scheme					
	RK41	RK42	RK43	RK44	RK45
a2	0.50000	0.33333	0.25000	-0.50000	0.50000
a3	0.50000	0.66667	0.50000	0.50000	0.00000
a4	1.00000	1.00000	1.00000	1.00000	1.00000
b21	0.50000	0.33333	0.25000	-0.50000	0.50000
b31	0.00000	-0.33330	0.00000	0.75000	-1.00000
b32	0.50000	1.00000	0.50000	-0.25000	1.00000
b41	0.00000	1.00000	1.00000	-2.00000	-1.00000
b42	0.00000	-1.00000	-2.00000	1.00000	1.50000
b43	1.00000	1.00000	2.00000	-2.00000	0.50000
c1	0.16667	0.12500	0.16667	0.16667	0.08333
c2	0.33333	0.37500	0.00000	0.00000	0.66667
c3	0.33333	0.37500	0.66667	0.66667	0.08333
c4	0.16667	0.12500	0.16667	0.16667	0.16667

Selected fourth order RK scheme					
	RK46	RK47	RK48	RK49	RK410
a2	1.00000	0.33300	0.77680	0.52220	0.74440
a3	0.50000	0.66670	0.53330	0.43330	0.64440
a4	1.00000	1.00000	1.00000	1.00000	1.00000
b21	1.00000	0.33300	0.77680	0.52220	0.74440
b31	0.37500	-0.33350	0.38230	-0.39740	0.55590
b32	0.12500	1.00020	0.15100	0.83070	0.08850
b41	-0.50000	1.00230	-0.07560	-0.18120	0.48870
b42	-0.50000	-1.00370	-0.73550	0.59850	-1.62350
b43	2.00000	1.00150	1.81100	0.58270	2.13480
c1	0.16667	1.06240	-0.91930	1.13530	-3.03400
c2	0.00000	-0.37490	0.13150	-0.50120	1.26490
c3	0.66667	0.18760	1.59170	0.20100	2.47310
c4	0.16667	0.12490	0.19620	0.16480	0.29610

For a physical pendulum which experiences damping and also driven externally, the kinetic energy, T is given as

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

Where $x = l\ sin\theta$ $\quad \dot{x} = l\dot{\theta}\ cos\theta$

$$y = l\ cos\ \theta \qquad \dot{y} = -l\dot{\theta}\ sin\theta$$

Therefore,

$$T = \frac{1}{2}ml^2\dot{\theta}^2(cos^2\theta + sin^2\theta)$$

Yielding

$$T = \frac{1}{2}ml^2\dot{\theta}^2.$$

The potential energy of the system reference to the support, V is given as

$$V = -mgl\ cos\theta$$

When a body swings through the air it experiences viscous drag resisting its motion. The damping force D is proportional to the velocity of the body as it moves in air. Newton, based on relationships between the interaction of dynamic body in fluid and rate of change of momentum, gave a conclusion that resistance to motion of a given body through a given fluid medium is proportional to the square of the velocity [11], hence

$$D \propto \dot{\theta}^2$$

$$D = \frac{1}{2}C\dot{\theta}^2$$

The external harmonic force acting on the system

$$Q_{iND} = F_0\ \sin\omega t$$

The Lagrange equation in equation 3.1 can thus be expressed as

$$ml^2\ddot{\theta} + C\dot{\theta} + mgl\sin\theta = F_0\ \sin\omega t \quad \text{……..(2)}$$

$$\ddot{\theta} + \frac{C}{ml^2}\dot{\theta} + \frac{g}{l}\sin\theta = \frac{F_0}{ml^2}\sin\omega t \quad \text{………(3)}$$

This can be written in the standard form as

$$\ddot{\theta} + 2\zeta\rho\dot{\theta} + \rho^2\sin\theta = \gamma\sin\omega t \quad \text{……….. (4)}$$

The damping ratio $\zeta = \frac{C}{2*m*\sqrt{(g*l^3)}}$; natural frequency, $\rho = \sqrt{(\frac{g}{l})}$ and forcing component, $\gamma = \frac{F_0}{ml^2}$ [12]

Linearizing (4) gives the model for harmonically excited linear pendulum, as shown in (5)

$$\ddot{\theta} + 2\zeta\rho\dot{\theta} + \rho^2\theta = \gamma\sin\omega t \quad \text{………. . (5)}$$

$$\ddot{\theta} + \varphi\dot{\theta} + \rho^2\theta = \gamma\sin\omega t \quad \text{………… (6)}$$

The analytic solution obtained for a general condition of displacement history is given as

$$\theta(t) = e^{-\frac{\varphi t}{2}}(A\cos\alpha t + B\sin\alpha t) + \frac{\gamma}{\varepsilon^2 + \mu^2}(\varepsilon\sin\omega t - \mu\cos\omega t)$$

For $\varphi^2 < 4\rho^2$

$$\alpha = 4\rho^2 - \varphi^2;\quad \varepsilon = \rho^2 - \omega^2;\quad \mu = \varphi\omega$$

$$A = \frac{a(\varepsilon^2 + \mu^2) + \gamma\mu}{\varepsilon^2 + \mu^2}$$

$$B = \frac{(2b + a\varphi)(\varepsilon^2 + \mu^2) + \gamma(\mu\varphi - 2\omega\varepsilon)}{2\alpha(\varepsilon^2 + \mu^2)}.$$

Comparison was made between the average displacement result given by analytical solution and randomly selected 10 fourth order Runge-Kutta version of the HESLP model using five different time steps of which two of such is reported. Comparison based on different frequency ratio for simulation time of 10 000 seconds is also reported for two of five time steps used. Graphical representation is also given for visual comparison.

III. RESULTS AND DISCUSSION

Simulation of HESLP model was carried out using 1995 version of FORTRAN while results obtained were visualized using Microsoft Excel 2010 version. Table II and III show results obtained for average of absolute percentage error recorded by each of the selected version for total time length of simulation and for different frequency ratio for comparison respectively. From Table II, it is obvious that the popular RK4 (RKv 1) is very accurate but RKv 5 performs better on the average. The average absolute percentage deviation from analytical result appear to be large for all the schemes even for RKv 1 and RKv 5, (0.733% and 0.732% respectively), this is particularly due to the difference in the start-up performance of all of the selected versions of the RK4 family and the analytical results in the transient region as is shown in Figure 3.2a to 3.2j; this transient behavior of the RKv schemes is observed irrespective of the time step or frequency ratio. Table IIA shows the ranking of selected schemes for which RKv 1 and RKv 5 are close competitors in accuracy, therefore, we conclude that the popular fourth order Runge-Kutta (RKv 1) is the most popular because its 13 characteristic coefficient is easy to remember. Schemes RKv 6, 7, 8, 9, 10 show consistency in large variation for all time steps and frequency ratio; we therefore conclude that they will not be good for simulation of linear dynamic system. Figures 3.2(a-j) show transient behavior of HESLP model with zero initial conditions being described by the exact solution which none of 4th order Runge-Kutta schemes recognize irrespective of time step and frequency ratio and the steady state result. For all time steps and frequency ratios, in the stable region the 4th order Runge-Kutta schemes was able to compete favorably well with the exact description of dynamic behavior of HESLP. Table IIIA and figures 3.3E & 3.3F show the numerical value and graphical representation respectively of average of absolute percentage error for different frequency ratio below resonance. Figure 3.3E clearly describes the consistent better performance of RKv 5 over the popular version, RKv 1. Figures 3.3(a-d) show performance based on the average of absolute percentage error for small, medium and large frequency ratios

Table.2. Comparison of simulated absolute average analytical and numerical formula result for different time steps (ts

AVERAGE OF ABSOLUTE PERCENTAGE ERROR RECORDED FOR EACH TIME STEP					
TS	RKv 1	RKv 2	RKv 3	RKv 4	RKv 5
1	1.25863	13.25	1.2896	112.59	1.2574
0.5	0.72751	19.99	0.7293	94.27	0.7263
0.25	0.70665	15.41	0.7091	34.99	0.7069
0.125	0.50057	13.61	0.5033	15.43	0.5008
0.063	0.47346	14.22	0.4764	8.36	0.4736
aver	0.73336	15.298	0.7416	53.13	0.7320

AVERAGE OF ABSOLUTE PERCENTAGE ERROR RECORDED FOR EACH TIME STEP					
TS	RKv 6	RKv 7	RKv 8	RKv 9	RKv 10
1	1.2935	104.96	166.56	150.27	337.67
0.5	0.7348	94.62	171.8	132.64	418.28
0.25	0.7097	35.71	69.58	49.13	184.3
0.125	0.5033	15.83	32.01	21.53	88.04
0.063	0.4764	8.6	17.54	11.58	48.95
ave	0.743	51.94	91.5	73.03	215.4

TABLE.3. Ranking Based On Lowest Average Of Absolute Percentage Error

POSITION BASED ON LOWEST AVERAGE OF ABSOLUTE % ERROR					
time step (TS)	1st	2nd	3rd	4th	5th
1	V5	V1	V3	V6	V2
0.5	V5	V1	V3	V6	V2
0.25	V1	V5	V3	V6	V2
0.125	V1	V5	V3	V6	V2
0.0625	V1	V5	V3,V6	***	V4
Ave on all TS	V5	V1	V3,	V6	V2

POSITION BASED ON LOWEST AVERAGE OF ABSOLUTE % ERROR						
time step (TS)	6th	7th	8th	9th	10th	
1	V7	V4	V9	V8	V11	10
0.5	V4	V7	V9	V11	V8	10
0.25	V7	V4	V9	V11	V8	10
0.125	V4	V7	V9	V11	V8	10
0.0625	V7	V2	V9	V11	V8	10
Ave on all TS	V7	V4	V9	V8	V11	10

*** ranking tie V1= RK4 version 1(RKv 1)

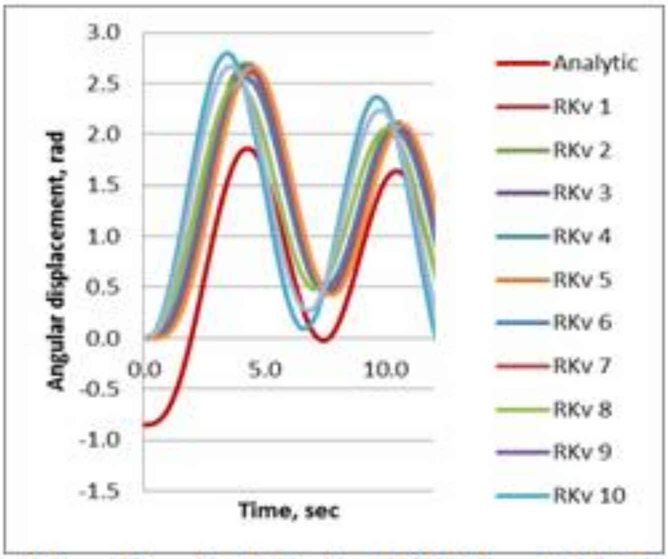

Figure.1. Transient behavior of HESLP model for frequency ratio 5.0 and time step 1.0sec

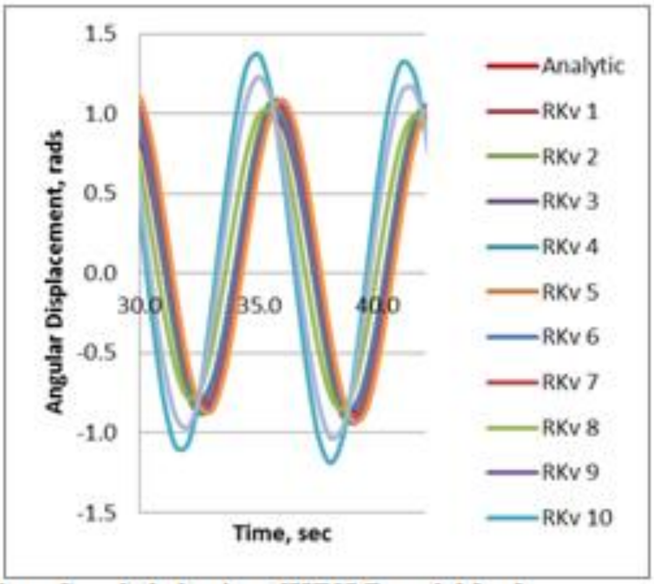

Figure.2. b Steady behavior of HESLP model for frequency ratio 5.0 and time step 1.0sec

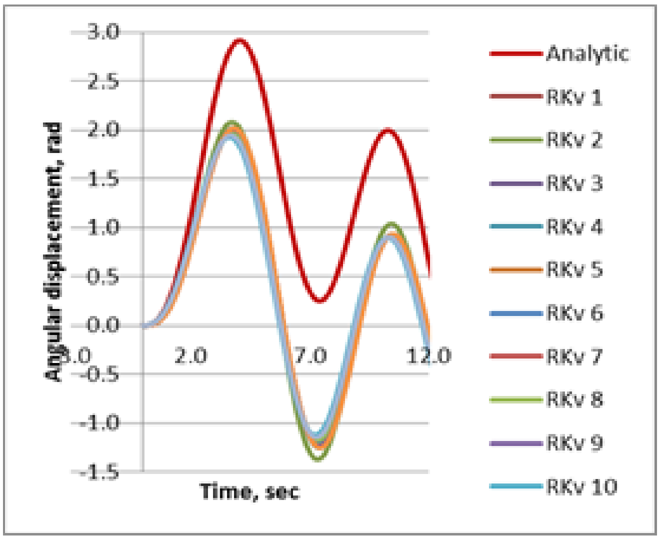

Figure.3. cTransient behavior of HESLP model for frequency ratio 2.0 and time step 0.0625sec

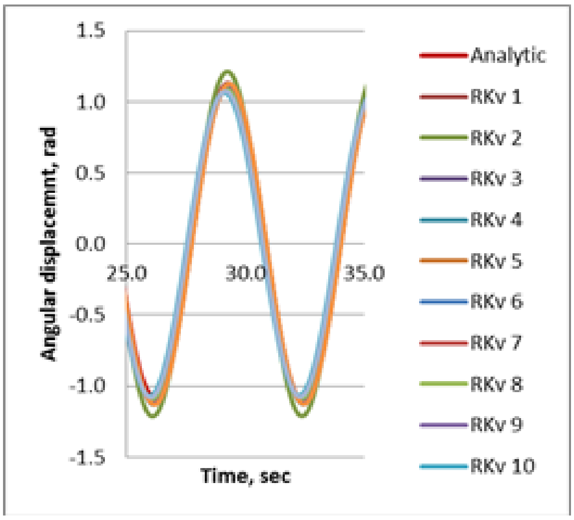

Figure.4.d Steady behavior of HESLP model for frequency ratio 2.0 and time step 0.0625 sec

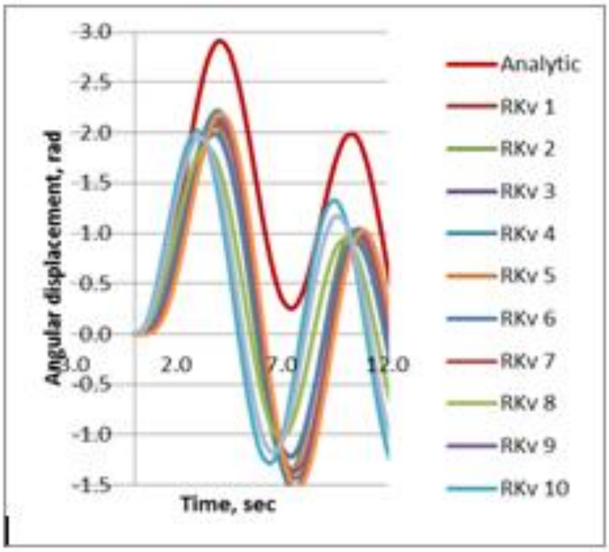

Figure.5.e Transient behavior of HESLP model for frequency ratio 2.0 and time step 1.0sec

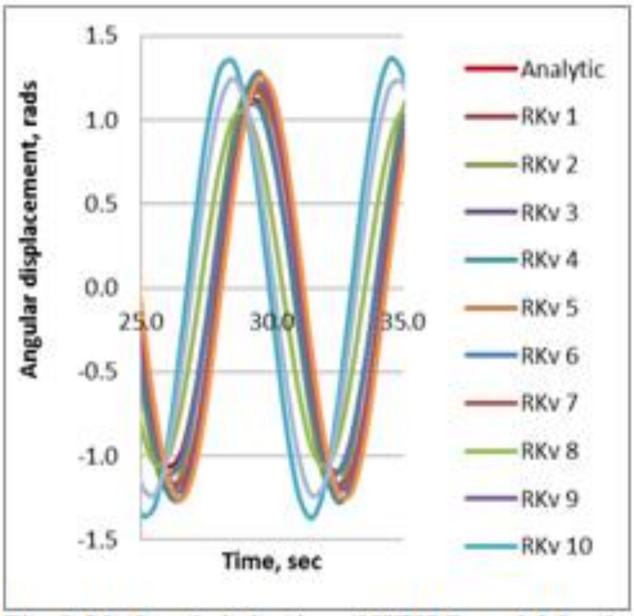

Figure.6.f Steady behavior of HESLP model for frequency ratio 2.0 and time step 1.0sec

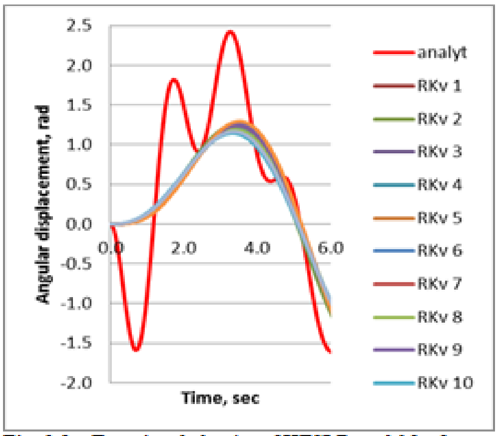

Figure.7.gTransient behavior of HESLP model for frequency ratio 0.8 and time step 0.0625sec

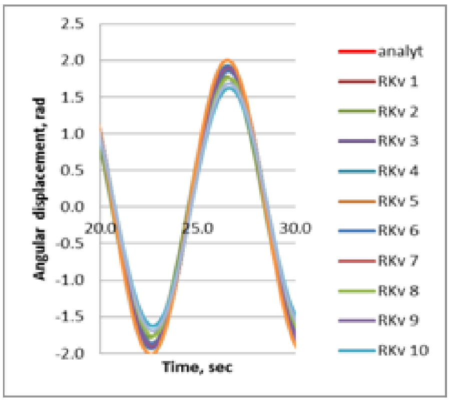

Figure.8. h Steady behavior of HESLP model for frequency ratio 0.8 and time step 0.0625sec

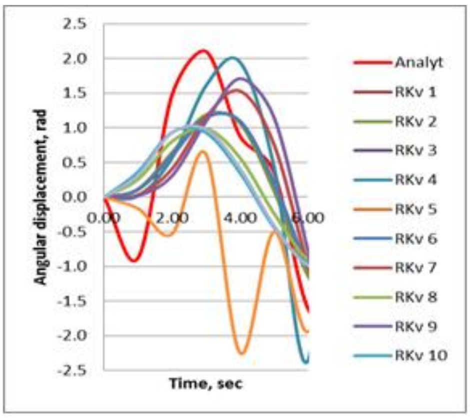

Figure.9.i Transient behavior of HESLP model for frequency ratio 0.8 and time step 1.0sec

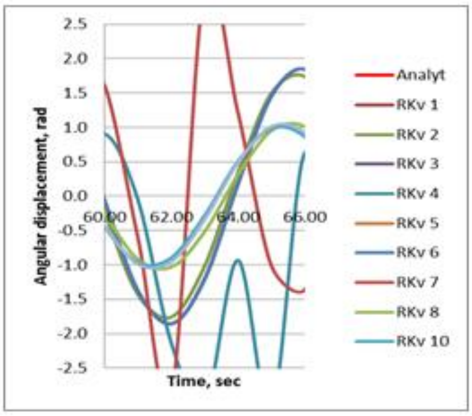

Figure.10. j steady behavior of HESLP model for frequency ratio 0.8 and time step 1.0sec

Table.4. cumulative average of absolute percentage error (caape) for different frequency ratio below resonance.

Ao	RKv 1	RKv 2	RKv 3	RKv 4	RKv 5
0.2	0.00047	5.10883	0.00141	22.2654	0.00047
0.3	0.00023	4.77515	0.00129	14.2495	0.00022
0.4	0.00031	8.82464	0.00243	19.3171	0.00029
0.5	0.00051	11.5139	0.00336	19.0263	0.00049
0.6	0.00081	12.9293	0.00404	16.1977	0.00079
0.7	0.00151	18.1054	0.00603	16.6239	0.00149
0.8	0.00285	22.2293	0.0083	14.0236	0.00284
0.9	0.00345	24.0794	0.00919	9.33818	0.00346
aver	0.00127	13.4457	0.00451	16.3802	0.00126

A

Ao	RKv 6	RKv 7	RKv 8	RKv 9	RKv 10
0.2	0.0022	0.65895	7.02239	0.93773	4.45159
0.3	0.00156	0.87313	5.42457	1.22431	6.03598
0.4	0.00266	1.60045	7.52316	2.24437	10.0511
0.5	0.00353	1.99241	7.31804	2.80219	13.0544
0.6	0.00416	1.95631	5.90207	2.76301	15.9825
0.7	0.00614	1.76262	4.68304	2.50545	23.9614
0.8	0.0084	3.36614	7.10264	4.56405	33.1672
0.9	0.00927	5.80143	11.7611	7.97933	41.5432
aver	0.00474	2.25143	7.09213	3.12756	18.5309

B

Table.5. Cumulative average of absolute percentage error for different frequency ratio at and above

Ao	RKv 1	RKv 2	RKv 3	RKv 4	RKv 5
1.0	0.00769	33.47969	0.01539	5.25355	0.00774
1.1	0.00725	31.51229	0.01418	8.54028	0.00733
1.2	0.00622	29.0896	0.01253	12.25834	0.00634
1.3	0.00971	27.63485	0.01559	15.22268	0.00986
1.4	0.01416	26.22981	0.01967	17.47587	0.01434
1.5	0.02428	32.67917	0.03105	25.32081	0.02456
1.6	0.04156	29.93413	0.04772	26.25392	0.04187
1.7	0.03848	17.06245	0.04201	16.05607	0.03867
1.8	0.05716	17.55289	0.06078	18.05597	0.05738
1.9	0.06372	23.47824	0.06849	26.88369	0.06406
2.0	0.08869	72.89917	0.10315	95.73942	0.08993
2.1	0.10235	13.52746	0.10512	16.44707	0.10256
2.2	0.42234	18.2162	0.42595	24.59743	0.42266
2.3	0.62254	20.28981	0.6265	29.33982	0.62292
2.4	0.12497	11.05482	0.12724	14.92153	0.12517
2.5	0.06715	15.90641	0.07034	24.61959	0.06748
2.6	0.07678	14.60441	0.07971	23.20425	0.0771
2.7	0.07678	14.28842	0.07964	23.54799	0.0771
2.8	0.06828	11.90279	0.07068	19.45475	0.06855
2.9	0.08016	9.8656	0.08217	15.662	0.08038
3.0	0.05638	14.04915	0.05916	25.86082	0.05674
3.1	0.06623	9.84362	0.06822	16.71461	0.06646
3.2	0.08151	11.99065	0.08389	22.50929	0.08183
3.3	0.03166	8.35825	0.03337	13.78502	0.03186

3.4	0.01201	11.89174	0.01432	23.17563	0.01234
3.5	0.00503	9.21618	0.00687	16.5678	0.00527
3.6	0.03183	8.6812	0.03356	15.61966	0.03205
3.7	0.13322	13.15492	0.13534	23.19261	0.13354
3.8	0.10714	8.27536	0.1087	13.56442	0.10734
3.9	0.24586	7.90519	0.24739	13.71859	0.24605
ave	0.092371	19.15248	0.096291	21.45212	0.092649

o	RKv 6	RKv 7	RKv 8	RKv 9	RKv 10
1.0	0.01549	10.91326	21.27575	15.11977	66.24839
1.1	0.01426	13.43456	25.84964	18.66227	73.93171
1.2	0.0126	15.2297	29.34682	21.15012	80.71376
1.3	0.01566	17.09646	33.20413	23.70072	89.96478
1.4	0.01974	18.69209	36.6933	25.85907	99.07864
1.5	0.03113	26.66727	52.83205	36.8204	143.1517
1.6	0.0478	27.26543	54.54495	37.57364	148.3865
1.7	0.04206	16.40741	33.12291	22.56927	90.44777
1.8	0.06082	18.50241	37.54798	25.41228	103.1123
1.9	0.06855	27.75355	56.62285	38.09119	156.4754
2.0	0.10338	99.74324	204.3047	136.83	568.0964
2.1	0.10515	16.99342	34.88557	23.25745	97.14846
2.2	0.426	25.6061	52.2962	34.92536	145.8235
2.3	0.62655	30.68763	62.66331	41.79337	175.1201
2.4	0.12726	15.60295	32.26318	21.30766	90.75511
2.5	0.07038	25.94537	53.84497	35.46929	152.2242
2.6	0.07975	24.53408	50.98942	33.52319	144.496
2.7	0.07968	24.98528	51.99898	34.12946	147.7034
2.8	0.07072	20.68895	43.06205	28.24977	122.5879
2.9	0.08219	16.6871	34.7486	22.77014	99.04463
3.0	0.0592	27.69429	57.80375	37.814	165.1978
3.1	0.06824	17.91598	37.35921	24.44428	106.8927
3.2	0.08392	24.21317	50.72913	33.04246	145.3435
3.3	0.03338	14.83804	31.03143	20.24693	89.04718
3.4	0.01435	25.04704	52.42528	34.18524	150.7051
3.5	0.00689	17.91331	37.51725	24.44627	107.9583
3.6	0.03358	16.89763	35.29601	23.04016	101.6591
3.7	0.13537	25.1937	52.49451	34.29672	151.3344
3.8	0.10871	14.73319	30.71415	20.0595	88.53091
3.9	0.2474	14.85351	30.61964	20.18124	88.29008
ave	0.09634	23.0912	47.26959	31.63237	132.9823

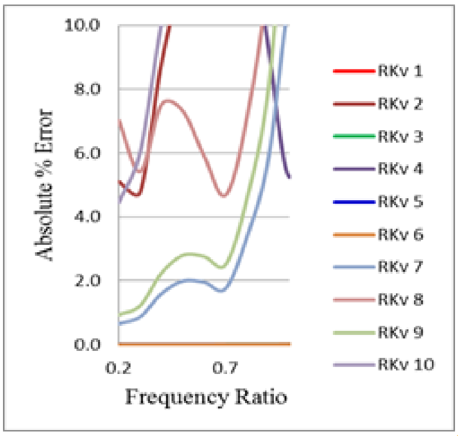

Figure.11.a Graph showing performance of selected 4th order RK versions for small frequency ratio, Ao based on CAAPE

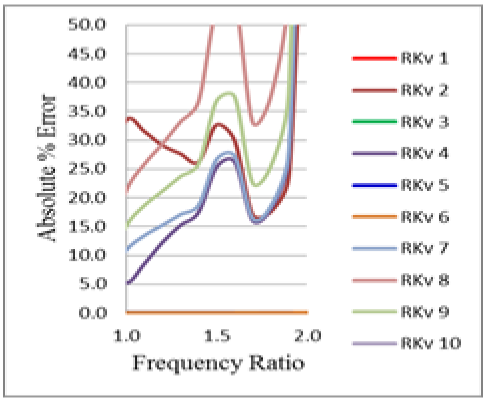

Figure.12.b Graph showing performance of selected 4th order RK versions for medium frequency ratio, Ao based on CAAPE

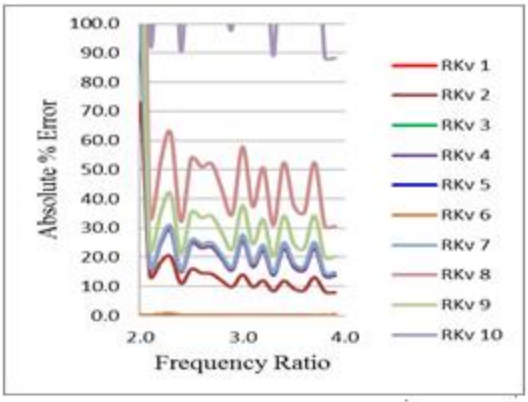

Figure.13.c Graph showing performance of selected 4th order RK versions for large frequency ratio, Ao based on CAAPE

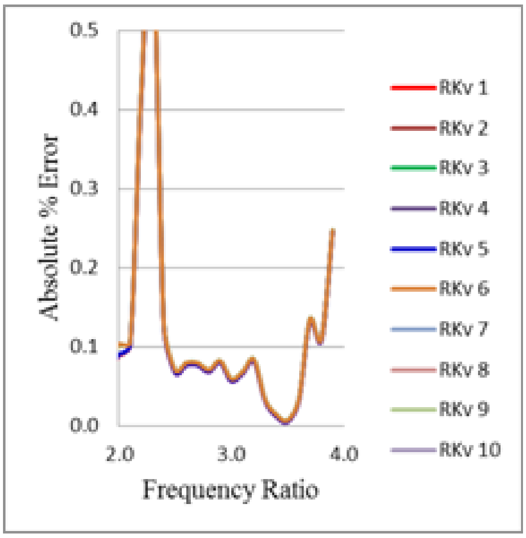

Figure.14.d Graph showing performance of selected 4th order RK versions for small frequency ratio, Ao based on CAAPE

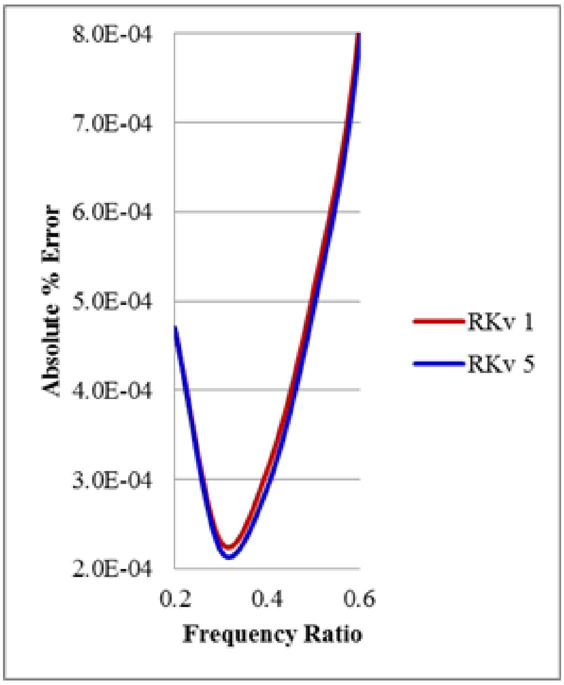

Figure.15.e graph showing superiority of rkv 5 over popular rkv 1 based on CAAPE fig. 14. e

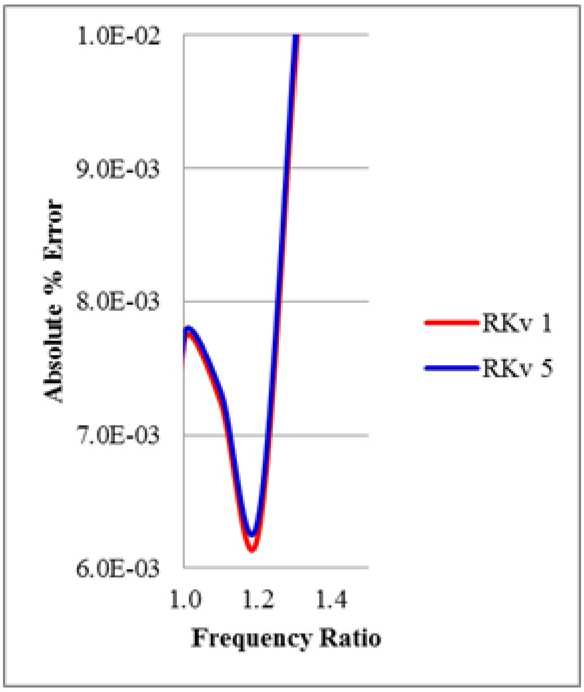

Figure.16.f graph showing superiority ofpopular rkv 1 over rkv 5 at and above resonance based on caape.

At resonance and above resonance, the simulation result of a dynamic system using popular RKv 1shows superiority in performance over others as seen in table IIIB and figure 3.3F. Comparison of figure 3.3E and figure 3.3F, it is obvious that absolute deviation of the numerical simulation of a dynamic system is minimal when the system operates below resonance An obvious behavior of all the RK versions for all time steps and irrespective of what frequency ratio is used in defining the dynamic system, they all did not recognize the transient behavior as the analytical solution does.

IV. CONCLUSIONS

One degree of freedom harmonically excited simple pendulum with prescribed initial conditions has been used to study performance of versions of fourth order Runge-Kutta method using the exact solution as the standard of measurement. During transient period the performance of studied versions are poor for all time steps as their prescribed corresponding result were not accurate and acceptable relative to exact result. However during steady state the results by each of the studied versions compare qualitatively well with the exact results, but quantitatively differed due essentially to choice combinations of the 13-characteristic coefficients of each version. Furthermore the study has shown that version five outperform other versions at all frequency ratios below resonance including the popular version which is mostly used for teaching and simulation purposes. It is therefore recommended that availability of infinite versions be stressed during the teaching of numerical Runge-Kutta schemes and that the popular versions gained its popularity essentially from the ease of remembering its 13-characteristic coefficients only.

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