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Intuitionistic Generalized Semi Regular Cokernal Compact Spaces

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Abstract:
In this paper, we have discussed about intuitionistic generalized semi regular compact sets, intuitionistic generalized semi regular cocompact sets and we have dealt with intuitionistic generalized semi regular cokernal compact spaces.

Keywords: intuitionistic-gsr-C-compact set, intuitionistic-gsr-C-cocompact set, intuitionistic -gsr cokernal compact spaces.

I. INTRODUCTION:

Levine [7] was the first to bring into light the concept of generalized closed sets in general topology. Later its properties such as continuity, connectedness and compactness were studied. All these properties of generalized closed sets were extended to intuitionistic fuzzy set by Atanassov [1]. Coker [2,3] developed all these concepts in intuitionistic topology. He studied the concepts of connectedness, continuity, compactness. Generalized closed sets were further developed as generalized semi regular closed sets in soft topological spaces by Mohana et.al [8]. The same generalized semi regular closed sets in intuitionistic topology were discussed by Mohana.K and Stephy Stephen [11]. Many authors [5, 8, 12] are working out new concepts in intuitionistic topology. This paper of Igsr cokernal compact spaces evolved as a result of the idea proposed by Roja et.al [10] who introduced cokernal compact spaces in intuitionistic topology.

1.PRELIMINARIES:

Definition 1.1: [4] Let X be a non empty set. An intuitionistic set (IS) A is an object having the form A = < X, A_1, A_2>, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 1.2: [4] Let X be a non empty set and let A, B are intuitionistic sets in the form A = < X, A_1, A_2>, B = < X, B_1, B_2> respectively. Then

(a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$
(b) A = B iff $A \subseteq B$ and $B \supseteq A$
(c) $\bar{A}$ = < X, A_2, A_1>
(d) [ ] A = < X, A_1, $(A_1)^c$ >
(e) $A - B = A \cap \bar{B}$.
(f) $\underset{\sim}{\varphi}$ = < X, ϕ, X >, $\underset{\sim}{X}$ = < X, X, ϕ>
(g) $A \cup B$ = < X, $A_1 \cup B_1$, $A_2 \cap B_2$ >.
(h) $A \cap B$ = < X, $A_1 \cap B_1$, $A_2 \cup B_2$ >.

Furthermore, let {A_i: i ∈ J} be an arbitrary family of intuitionistic sets in X, where A_i = < X, $A_i^{(1)}$, $A_i^{(2)}$ >. Then
(i) $\cap A_i$ = < X, $\cap A_i^{(1)}$, $\cup A_i^{(2)}$ >.
(j) $\cup A_i$ = < X, $\cup A_i^{(1)}$, $\cap A_i^{(2)}$ >.

Definition 1.3: [4] An intuitionistic topology (IT in short) on a nonempty set X is a family τ of IS's in X containing $\underset{\sim}{\varphi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. The pair (X,τ) is called an intuitionistic topological space (ITS in short). Any intuitionistic set in τ is known as intuitionistic open set (IOS) in X and the complement of IOS is called intuitionistic closed set (ICS) in X.

Definition 1.4 [11]: Let (X,τ) be an intuitionistic topological space and let A = < X, A_1, A_2 > be an intuitionistic set. Then A is said to be intuitionistic generalized semi regular closed (Igsr-closed) if Iscl(A) ⊆ U whenever A ⊆ U and U is intuitionistic regular open in X.

Definition 1.5: [6] Let X, Y is two non empty sets and f: X→Y be a function. If B = < Y, B_1, B_2 > is an IS in Y, then the preimage of B is denoted by $f^{-1}(B)$, where $f^{-1}(B)$ = < X, $f^{-1}(B_1)$, $f^{-1}(B_2)$ >. If A = < X, A_1, A_2 > is an IS in X, then the image of A under f is denoted by f (A) is the IS in Y defined by f (A) = < Y, f (A_1), f (A_2) >.

Definition 1.6: [6] Let (X, τ) and (Y, δ) be two intuitionistic topological spaces and let a function f: X→Y be defined, then f is said to be continuous iff the preimage of each ICS in Y is intuitionistic closed in X.

Definition 1.7: [10] Let (X, τ) be an intutionistic topological space. Then A = < X, A_1, A_2 > ∈ τ is said to be intuitionistic C-compact (IC-compact) set if every $A \subseteq \bigcup_{i\in\tau} A_i^c$ where A_i^c is I-closed set in (X, τ).

The complement of an intuitionisticC-compact set is an intuitionistic C-cocompact set.

Definition 1.8: [10] Let (X, τ) be an intuitionistic topological space and A = < X, A_1, A_2 > be an intuitionistic set in (X, τ). Then the intuitionistic*C*-compact kernel of A and intuitionistic*C*-compact cokernal of A are denoted and defined by

$IK_c^{\circ}(A) = \cup$ { K= < X, K_1, K_2 > : K is an intuitionistic*C*-compact set in (X,τ) and K ⊆ A }

$ICK_c^{-}(A) = \cap$ { K= < X, K_1, K_2 > : K is an intuitionistic*C*-cocompact set in (X,τ) and A ⊆ K }

Remark 1.9: [10] Let (X, τ) be an intuitionistic topological space and A = < X, A_1, A_2 > be an intuitionistic set of X. Then

i) $ICK_c^{-}(A)$ = A ⇔ A is an IC-cocompact set.

ii) $IK_c^{\circ}(A)$ = A ⇔ A is an IC-compact set.

Definition 1.10: [10] an intuitionistic topological space (X, τ) is said to be an intuitionistic cokernal compact space if the intuitionisticC-compact cokernal of every IC-compact set. That is., $ICK_c^{\neg}(A) \subseteq \cup A_i^c$

2. INTUITIONISTIC GENERALIZED SEMI REGULAR COKERNAL COMPACT SPACES

Definition 2.1: Let (X,τ) be an intutionistic topological space. Then A = < X, A_1, A_2 > $\in \tau$ is said to be intuitionistic –gsr *C*-compact (IgsrC-compact) set if every A $\subseteq \cup_{i\in\tau} A_i^c$ where A_i^c is Igsr-closed set in (X,τ). The complement of an intuitionistic-gsr-*C*-compact set is an intuitionistic-gsr-*C*-cocompact (IgsrC-cocompact) set.

Definition 2.2: Let (X, τ) be an intuitionistic topological space and A = < X, A_1, A_2 > be an intuitionistic set in (X, τ). Then the Igsr*C*-compact kernel of A and Igsr*C*-compact cokernal of a are denoted and defined by

$IgsrK_c^{\circ}(A) = \cup$ { K= < X, K_1, K_2 > : K is an Igsr*C*-compact set in (X,τ) and K $\subseteq$ A }

$IgsrCK_c^{\neg}(A) = \cap$ { K= < X, K_1, K_2 > : K is an Igsr*C*-cocompact set in (X,τ) and A $\subseteq$ K }

Remark 2.3: Let (X,τ) be an intuitionistic topological space and A = < X, A_1, A_2 > be an intuitionistic set of X. Then

i) $IgsrCK_c^{\neg}(A)$ = A $\Leftrightarrow$ A is an Igsr*C*-cocompact set.

ii) $IgsrK_c^{\circ}(A)$ = A $\Leftrightarrow$ A is an Igsr*C*-compact set.

Definition 2.4: An intuitionistic topological space (X, τ) is said to be an Igsr cokernal compact space if the Igsr*C*-compact cokernal of every Igsr*C*-compact set. That is., $IgsrCK_c^{\neg}(A) \subseteq \cup A_i^c$.

Example 2.5: Let X = {a,b}, then the intuitionistic set A = < X, ϕ, X >, B = < X, X, ϕ >, C = < X, {a}, ϕ>, D = < X, ϕ, {b} >, E = <X, {a}, {b} >. Then the family τ = {φ, X, A, B, C, D, E } is an intuitionistic topology on X.
Igsr*C*-compact set = {< X, ϕ, X >, < X, X, ϕ >, < X, {a}, ϕ>, < X, ϕ, {b} >}
Igsr*C*-cocompact set = {< X, X, ϕ >, < X, ϕ, X >, < X, ϕ, {a}>, < X, {b}, ϕ >}
Then $IgsrK_c^{\circ}(A)$ = $\cap$ K. Therefore, (X, τ) is Igsr cokernal compact space.

Proposition 2.6: Let (X,τ) be any intuitionistic topological space. Let A = < X, A_1, A_2 > be an Igsr*C*-compact set in X. Then the following conditions hold:

i) $\overline{IgsrCK_c^{\neg}(A)} = IgsrK_c^{\circ}(\overline{A})$

ii) $\overline{IgsrK_c^{\circ}(A)} = IgsrCK_c^{\neg}(\overline{A})$

Proof:

i) $IgsrCK_c^{\neg}(A)$ = $\cap$ {K = < X, K_1, K_2 >: K is an Igsr*C*-cocompact set in (X,τ) and A $\subseteq$ K }

Taking complements on both sides,

$\overline{IgsrCK_c^{\neg}(A)} = \cup$ { $\overline{K}$: $\overline{K}$ is an Igsr*C*-compact set in (X,τ) and $\overline{K} \subseteq \bar{A}$}

$= IgsrK_c^{\circ}(\overline{A})$

ii) $IgsrK_c^{\circ}(A) = \cup$ { K = < X, K_1, K_2 > : K is an Igsr*C*-compact set in (X,τ) and K $\subseteq$ A }

taking complements on both sides,

$\overline{IgsrK_c^{\circ}(A)} = \cap$ { $\overline{K}$: $\overline{K}$ is an Igsr*C*-compact set in (X,τ) and $\overline{K} \supseteq \bar{A}$}

$= IgsrCK_c^{\neg}(\overline{A})$

Proposition2.7: Let (X, τ) be an intuitionistic topological space. Then the following statements are equivalent:

i) (X, τ) is an Igsr cokernal compact space.

ii) For each Igsr*C*-cocompact set A, $IgsrK_c^{\circ}(A)$ is an Igsr*C*-cocompact set.

iii) For each Igsr*C*-compact set A, we have $IgsrCK_c^{\neg}(\overline{IgsrCK_c^{\neg}(A)}) = \overline{IgsrCK_c^{\neg}(A)}$

iv) For every pair of Igsr*C*-compact sets A and B with $\overline{B} = IgsrCK_c^{\neg}(A)$, we have $IgsrCK_c^{\neg}(B) = \overline{IgsrCK_c^{\neg}(A)}$

Proof: i) $\Rightarrow$ ii)

Let A be an IgsrC-cocompact set in (X, τ). Then $\overline{A}$ is an IgsrC-cocompact set in (X, τ). By assumption, we have $IgsrCK_c^{\neg}(\overline{A})$ is an IgsrC-compact set in (X, τ).

Now, $IgsrCK_c^{\neg}(\overline{A}) = \overline{IgsrK_c^{\circ}(A)}$. Therefore, $IgsrK_c^{\circ}(A)$ is an IgsrC-cocompact set in (X, τ). Hence proved.

(ii) $\Rightarrow$ (i)

Let A be an IgsrC-compact set in (X, τ). Then $\overline{A}$ is an IgsrC-cocompact set in (X, τ). Given that

$IgsrK_c^{\circ}(\overline{A}) = \overline{IgsrCK_c^{\neg}(A)}$ is an IgsrC-cocompact set. Now,

$IgsrCK_c^{\neg}(\overline{IgsrCK_c^{\neg}(A)})$

$= IgsrCK_c^{\neg}(IgsrK_c^{\circ}(\overline{A})) = \overline{IgsrCK_c^{\neg}(A)}$

(iii) $\Rightarrow$ (iv)

Let A and B be any two IgsrC-compact set in (X, τ) such that

By (iii), $IgsrCK_c^{\neg}(\overline{IgsrCK_c^{\neg}(A)}) = \overline{IgsrCK_c^{\neg}(A)}$

which implies that $IgsrCK_c^{\neg}(B) = \overline{IgsrCK_c^{\neg}(A)}$

Hence (iii) $\Rightarrow$ (iv)

(iv) $\Rightarrow$ (i)

Let A and B be any two IgsrC-compact sets in (X, τ) such that $\overline{B = IgsrCK_c^{\neg}(A)}$

Given that, $IgsrCK_c^{\neg}(B) = \overline{IgsrCK_c^{\neg}(A)}$

i.e., $\overline{IgsrCK_c^{\neg}(A)}$ is an IgsrC-cocompact set in (X, τ).

This implies that $IgsrCK_c^{\neg}(A)$ is an IgsrC-compact set in (X, τ). Thus, (X, τ) is an IgsrC-compact cokernal compact space. Hence the proof.

Proposition 2.8:
Let (X, τ) be an intuitionistic topological space. Then (X, τ) is an Igsr cokernal compact space iff for each IgsrC-compact set A and IgsrC-cocompact set B such that

$A \subseteq B, IgsrCK_c^{\neg}(A) \subseteq IgsrK_c^{\circ}(B)$

Proof:
Let (X, τ) be an Igsr cokernal compact space. Let A be an IgsrC-compact set and B is an IgsrC-cocompact set in (X, τ) such that $A \subseteq B$.

Then by (ii) of proposition 2.7, $IgsrK_c^{\circ}(B)$ is an IgsrC-cocompact set in (X, τ). Therefore,

$IgsrCK_c^{\neg}(IgsrK_c^{\circ}(B)) = IgsrK_c^{\circ}(B)$. Since A is an IgsrC-compact set and $A \subseteq B, A \subseteq IgsrK_c^{\circ}(B)$.

Now, $IgsrCK_c^{\neg}(A) \subseteq IgsrCK_c^{\neg}(IgsrK_c^{\circ}(B))$
$= IgsrK_c^{\circ}(B) \Rightarrow IgsrCK_c^{\neg}(A) \subseteq IgsrK_c^{\circ}(B)$

Conversely, let B be an IgsrC-cocompact set in (X, τ), then $IgsrK_c^{\circ}(B)$ is an IgsrC-compact set and $IgsrK_c^{\circ}(B) \subseteq B$. By assumption, $IgsrCK_c^{\neg}(IgsrK_c^{\circ}(B)) \subseteq IgsrK_c^{\circ}(B)$.

Also, $IgsrK_c^{\circ}(B) \subseteq IgsrCK_c^{\neg}(IgsrK_c^{\circ}(B))$,

implies $IgsrK_c^{\circ}(B) = IgsrCK_c^{\neg}(IgsrK_c^{\circ}(B))$.

Therefore, $IgsrK_c^{\circ}(B)$ is an Igsr-closed set in (X, τ).

By (ii) of proposition , (X, τ) is an Igsr cokernal compact set.

Definition 2.9: Let (X, τ) and (Y, δ) be any two Igsr cokernal compact spaces. A function f : (X, τ) → (Y, δ) is an Igsr C-compact set in (Y, δ) for each Igsr C-compact set A in (X, τ) .

Proposition 2.10: Let (X, τ) and (Y, δ) be any two Igsr cokernal compact spaces. A function f : (X, τ) → (Y, δ) be an Igsr C-compact open and surjective function. Then $f^{-1}(IgsrCK_c^{\neg}(A)) \subseteq IgsrCK_c^{\neg}(f^{-1}(A))$ for each intuitionistic set A in (Y, δ).

Proof:
Let A be an intuitionistic set in (Y, δ) and $B = f^{-1}(\overline{A})$ Then, $IgsrK_c^{\circ}(f^{-1}(\overline{A})) = IgsrK_c^{\circ}(B)$ is an Igsr C-compact set in (X, τ). Now, $IgsrK_c^{\circ}(B) \subseteq B$. Hence $f(IgsrK_c^{\circ}(B)) \subseteq f(B)$ i.e., $IgsrK_c^{\circ}(f(IgsrK_c^{\circ}(B))) \subseteq IgsrK_c^{\circ}(f(B))$. Since f is an Igsr C-compact open function, $f(IgsrK_c^{\circ}(B))$ is an Igsr C-compact set in (Y, δ). Therefore,

$f(IgsrK_c^{\circ}(B)) \subseteq IgsrK_c^{\circ}(f(B)) = IgsrK_c^{\circ}(\overline{A})$.

Hence, $IgsrK_c^{\circ}(f^{-1}(\overline{A})) \subseteq f^{-1}(IgsrK_c^{\circ}(\overline{A}))$

implies $\overline{IgsrK_c^{\circ}(f^{-1}(\overline{A}))} \supseteq \overline{f^{-1}(IgsrK_c^{\circ}(\overline{A}))}$

implies $IgsrCK_c^{\neg}(f^{-1}(A)) \supseteq f^{-1}(IgsrCK_c^{\neg}(\overline{A})) \Rightarrow$

$f^{-1}(IgsrCK_c^{\neg}(\overline{A})) \subseteq IgsrCK_c^{\neg}(f^{-1}(A))$. Hence the proof.

Definition 2.11: Let (X, τ) and (Y, δ) be any two Igsr cokernal compact spaces. A function f: (X, τ) → (Y, δ) is called an Igsr C-compact continuous function if $f^{-1}(A)$ is Igsr C-compact set in (X, τ) for every Igsr C-compact set A in (Y, δ).

Remark 2.12: Let (X, τ) and (Y, δ) be any two Igsr cokernal compact spaces. Let f: (X, τ) → (Y, δ) be any function. Then the following statements are equivalent:

(i) f: (X, τ) → (Y, δ) is an Igsr C-compact continuous function.

(ii) $IgsrCK_c^{\neg}(f^{-1}(A)) \subseteq f^{-1}(IgsrCK_c^{\neg}(A))$ for each Igsr C-compact set A in (Y, δ).

Proof:
(i)⇒(ii)
Given f: (X, τ) → (Y, δ) is an Igsr C-compact continuous function. Let A = < X, A_1, A_2 > be an IgsrC-compact set in (Y, δ). Let $IgsrCK_c^{\neg}(A)$ is an Igsr C-compact set in (Y, δ) and hence $f^{-1}(IgsrCK_c^{\neg}(A))$ is an Igsr C-compact set in (X, τ).
Therefore,
$IgsrCK_c^{\neg}(f^{-1}(IgsrCK_c^{\neg}(A))) = f^{-1}(IgsrCK_c^{\neg}(A))$
Since, $A \subseteq IgsrCK_c^{\neg}(A)$, $f^{-1}(A) = f^{-1}(IgsrCK_c^{\neg}(A))$.
Therefore,
$IgsrCK_c^{\neg}(f^{-1}(A)) \subseteq IgsrCK_c^{\neg}(f^{-1}(IgsrCK_c^{\neg}(A)))$

$= f^{-1}(IgsrCK_c^{\neg}(A))$

i.e., $IgsrCK_c^{\neg}(f^{-1}(A)) \subseteq f^{-1}(IgsrCK_c^{\neg}(A))$

(ii)⇒(i)
Given that $IgsrCK_c^{\neg}(f^{-1}(A)) \subseteq f^{-1}(IgsrCK_c^{\neg}(A))$, for each Igsr C-compact set in (Y, δ). Let A be an Igsr C-cocompact set in (Y, δ). It is enough to show that $f^{-1}(V)$ is an Igsr C-compact set in (X, τ). Since V $= IgsrCK_c^{\neg}(A)$, $f^{-1}(A) = f^{-1}(IgsrCK_c^{\neg}(A))$ but it is given that $IgsrCK_c^{\neg}(f^{-1}(A)) \subseteq f^{-1}(IgsrCK_c^{\neg}(A))$.

Hence $IgsrCK_c^{\neg}(f^{-1}(A)) \subseteq f^{-1}(A) \subseteq f^{-1}(IgsrCK_c^{\neg}(A))$.

Thus $f^{-1}(A) = IgsrCK_c^{\neg}(f^{-1}(A))$.

i.e., $f^{-1}(A)$ is an Igsr C-cocompact set in (X, τ). This proves that f is an Igsr C-compact continuous function.

Proposition 2.13: Let (X, τ) and (Y, δ) be any two Igsr cokernal compact spaces. Let f: (X, τ) → (Y, δ) be a bijective function. Then f is an Igsr C-compact continuous function if for every intuitionistic set A in (X, τ), $f(IgsrCK_c^{\neg}(A) \subseteq IgsrCK_c^{\neg}(f(A))$.

Proof:
Let us assume that f is an Igsr C-compact continuous function and A be an intuitionistic set in (X, τ). Hence, $f^{-1}(IgsrCK_c^{\neg}(f(A)))$ is an Igsr C-cocompact set in (X, τ).
By remark 2.12,
$IgsrCK_c^{\neg}(f^{-1}(f(A))) \subseteq f^{-1}(IgsrCK_c^{\neg}(f(A)))$
Since f is an injective function,
$IgsrCK_c^{\neg}(A) \subseteq f^{-1}(IgsrCK_c^{\neg}(f(A)))$
Taking f on both sides,
$f(IgsrCK_c^{\neg}(A)) \subseteq f(f^{-1}(IgsrCK_c^{\neg}(f(A))))$
Since f is a surjective function,
$f(IgsrCK_c^{\neg}(A)) \subseteq IgsrCK_c^{\neg}(f(A))$.

Proposition 2.14: Let (X, τ) and (Y, δ) be any two Igsr cokernal compact spaces. Let f: (X, τ) → (Y, δ) be any function. Then the following statements are equivalent:

(i) f :(X, τ) → (Y, δ) is an Igsr C-cocompact continuous function.

(ii) $IgsrCK_c^{-}(f(A)) \subseteq f(IgsrCK_c^{-}(A))$, for each Igsr C-compact set A=<X, A_1, A_2 > in (X, τ).

Proof:

(i)⇒(ii)

Let A = < X, A_1, A_2 > be an Igsr C-compact set in (X, τ). Clearly $IgsrCK_c^{-}(A)$ is an Igsr C-cocompact set in (X, τ). Since f is an Igsr C-cocompact function, $f(IgsrCK_c^{-}(A))$ is an Igsr C-cocompact set in (Y, δ).

Thus $IgsrCK_c^{-}(f(A)) \subseteq IgsrCK_c^{-}(f(IgsrCK_c^{-}(A)))$

$= f(IgsrCK_c^{-}(A))$

Hence proved.

(ii)⇒(i)

Let A be any Igsr C-cocompact set in (X, τ).

Then $A = IgsrCK_c^{-}(A)$. By (ii),

$IgsrCK_c^{-}(f(A)) \subseteq f(IgsrCK_c^{-}(A))$

$\Rightarrow f(A) \subseteq IgsrCK_c^{-}(f(A))$

Thus $f(A) = IgsrCK_c^{-}(f(A))$

and hence f(A) is an Igsr C-cocompact set in (Y, δ). Therefore, f is an Igsr C-cocompact function. Hence (ii)⇒(i).

Definition 2.15: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. A function f: (X, τ) → (Y, δ) is called an Igsr C-compact irresolute function if $f^{-1}(A)$ is Igsr C-compact set in (X, τ) for each Igsr C-compact set A in (Y, δ).

Proposition 2.16: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. A function f: (X, τ) → (Y, δ) is an Igsr C-compact irresolute function if and only if $f(IgsrCK_c^{-}(A)) \subseteq IgsrCK_c^{-}(f(A))$, for every Igsr C-compact set in (X, τ).

Proof:

Suppose that f is an Igsr C-compact irresolute function and let A be an Igsr C-compact set in (X, τ). Then, $IgsrCK_c^{-}(f(A))$ is an Igsr C-cocompact set in (Y, δ).

By assumption, $f^{-1}(IgsrCK_c^{-}(f(A)))$ is an Igsr C-cocompact set (X, τ). Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(IgsrCK_c^{-}(f(A)))$

$A \subseteq f^{-1}(IgsrCK_c^{-}(A))$

now $IgsrCK_c^{-}(A) \subseteq IgsrCK_c^{-}(f^{-1}(IgsrCK_c^{-}(f(A))))$

$IgsrCK_c^{-}(A) \subseteq f^{-1}(IgsrCK_c^{-}(f(A)))$

i.e., $f(IgsrCK_c^{-}(A)) \subseteq IgsrCK_c^{-}(f(A))$.

Conversely , suppose that A is an Igsr C-cocompact set in (Y, δ).Then $IgsrCK_c^{-}(A) = A$. Now, by assumption,

$f(IgsrCK_c^{-}(f^{-1}(A))) \subseteq IgsrCK_c^{-}(f(f^{-1}(A)))$

$= IgsrCK_c^{-}(A) = A.$

This implies that, $IgsrCK_c^{-}(f^{-1}(A)) \subseteq f^{-1}(A)$

but, $IgsrCK_c^{-}(f^{-1}(A)) \supseteq f^{-1}(A)$.

Hence $IgsrCK_c^{-}(f^{-1}(A)) = f^{-1}(A)$ i.e., $f^{-1}(A)$ is an Igsr C-cocompact set in (X, τ). Hence f is an Igsr C-compact irresolute function.

3. PROPERTIES OF IGSR R-COMPACT SPACES

Definition 3.1: Let (X, τ) be an Igsr cokernal compact space and let A = < X, A_1, A_2 > be any intuitionistic set in (X, τ). Then A is said to be an Igsr RC-compact if $A = IgsrK_c^{\circ}(IgsrCK_c^{-}(A))$

Definition 3.2: Let (X, τ) be an Igsr cokernal compact space and let A = < X, A_1,A_2 > be any intuitionistic set in (X, τ). Then A is said to be an Igsr RC-cocompact if $A = IgsrCK_c^{-}(IgsrK_c^{\circ}(A))$.

Remark 3.3: Every Igsr RC-compact is an Igsr C-compact.

Proposition 3.4: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. A function f: (X, τ) → (Y, δ) is an Igsr C-compact continuous function of (X, τ) into an Igsr cokernal compact space (Y, δ) and if V= < X, V_1,V_2 > is an Igsr RC-compact in (Y, δ), then $f^{-1}(V)$ is an Igsr RC-compact in (X, τ).

Proof:

Since V is an Igsr RC-compact in (Y, δ), it follows that V is an Igsr C-compact in (Y,). Since f is Igsr continuous, $f^{-1}(V)$ is an Igsr C-compact in (X, τ). That is, $IgsrK_c^{\circ}(f^{-1}(V)) = f^{-1}(V)$ ----------1

Since (Y, δ) is an Igsr cokernal compact space and since V is an Igsr RC-compact in (Y,), $V = IgsrK_c^{\circ}(IgsrCK_c^{-}(V))$

$= IgsrCK_c^{-}(IgsrK_c^{\circ}(IgsrCK_c^{-}(V)))$

That is, $V = IgsrCK_c^{-}(V)$--------2

$IgsrCK_c^{-}(f^{-1}(V)) \subseteq f^{-1}(IgsrCK_c^{-}(V))$ because f is an Igsr C-compact continuous function. Therefore,

$IgsrK_c^{\circ}(IgsrCK_c^{-}(f^{-1}(V)))$

$\subseteq IgsrK_c^{\circ}(f^{-1}(IgsrCK_c^{-}(V)))$

From 2, it follows that

$IgsrK_c^{\circ}(f^{-1}(IgsrCK_c^{-}(V))) = IgsrK_c^{\circ}(f^{-1}(V))$ -----3

From 1 and 3, it follows that

$IgsrK_c^{\circ}(IgsrCK_c^{-}(f^{-1}(V))) \subseteq f^{-1}(V)$ -----4

Since $f^{-1}(V) \subseteq IgsrK_c^{-}(f^{-1}(V))$, then

$IgsrK_c^{\circ}(f^{-1}(V)) \subseteq IgsrK_c^{\circ}(IgsrCK_c^{-}(f^{-1}(V)))$

From 1, it follows that $f^{-1}(V) \subseteq IgsrK_c^{\circ}(IgsrCK_c^{-}(f^{-1}(V)))$ --------5

Therefore, from 4 and 5, it follows that

$f^{-1}(V) = IgsrK_c^{\circ}(IgsrCK_c^{-}(f^{-1}(V)))$. Hence $f^{-1}(V)$ is an Igsr RC-compact in (X, τ).

Definition 3.5: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. If f: (X, τ) → (Y, δ) be a function, then f is said to be Igsr C-compact function if the image of each Igsr C-compact set in (X,τ) is an Igsr C-compact set in (Y, δ).

Definition 3.6: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. If f: (X, τ) → (Y, δ) be a function, then f is said to be Igsr C-cocompact function if the image of each Igsr C-cocompact set in (X, τ) is an Igsr C-cocompact set in (Y, δ).

Proposition 3.7: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. If f: (X, τ) → (Y, δ) is an Igsr continuous bijective function of an Igsr cokernal compact space (X, τ) into a space (Y, δ). If V= < X, V_1,V_2 > is an Igsr RC-compact set in (X, τ) , then f(V) is an Igsr RC-comapct set in (Y, δ).

Proof:
Since V is an Igsr RC-compact set in (X, τ) and since (X, τ) is an Igsr cokernal compact space,

$V = IgsrK_c^{\circ}(IgsrK_c^{-}(V)) = IgsrK_c^{-}(V)$.

That is, $V = IgsrK_c^{-}(V)$. Since f is an Igsr C-compact continuous bijective function,

$f(V) = f(IgsrK_c^{-}(V)) \subseteq IgsrK_c^{-}(f(V))$

Since f is an intuitionistic-gsr continuous function,

$f(V) = IgsrK_c^{\circ}(f(V)) \subseteq IgsrK_c^{\circ}(IgsrK_c^{-}(f(V)))$ i.e.,

$f(V) \subseteq IgsrK_c^{\circ}(IgsrK_c^{-}(f(V)))$ ------6

Now $IgsrK_c^{\circ}(IgsrK_c^{-}(f(V))) \subseteq IgsrK_c^{-}(f(V))$

Since f is an Igsr C-compact bijective function, f is an Igsr C-cocompact function. Hence,

$IgsrK_c^{-}(f(V)) \subseteq f(IgsrK_c^{-}(V)) = f(V)$

then, $IgsrK_c^{\circ}(IgsrK_c^{-}(f(V))) \subseteq f(V)$ ------7

From 6 and 7, it follows that $IgsrK_c^{\circ}(IgsrK_c^{-}(f(V))) = f(V)$ i.e., f(V) is Igsr RC-compact set in (Y, δ).

Definition 3.8: Let (X, τ) be an intuitionistic topological space. If a family {G_i = < X, $G_i^{(1)}$, $G_i^{(2)}$ >; i ∈ J } of Igsr RC-compact (X, τ) satisfies the condition ∪{G_i ; i ∈ J} =X, then it is called Igsr RC-compact cover of X.

Definition 3.9: An intuitionistic topological space (X, τ) is said to be Igsr RC-compact space if and only if every Igsr RC-compact cover of (X, τ) has a finite subfamily, the Igsr C-compact cokernals of whose members cover the space (X, τ).

Proposition 3.10: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. Let f: (X, τ) → (Y, δ) be an Igsr C-compact function of an Igsr RC-compact space (X, τ) onto an Igsr cokernal compact space (Y, δ), then (Y, δ) is an Igsr R-compact space.

Proof: Let V_j= < X, $V_i^{(1)}$, $V_i^{(2)}$ > be an Igsr RC-compact cover of (Y, δ). Since f is an IgsrC-compact continuous function and (Y, δ) is an Igsr cokernal compact space. From Proposition 3.4, {$f^{-1}(V_j)$: j ∈ J} is an Igsr RC-compact cover of (X, τ). Since (X, τ) is an Igsr R-compact space, there exists a finite subfamily such that $X = \bigcup_{i=1}^{n} IgsrK_c^{-}(f^{-1}(V_{ji}))$.

Thus, $Y = f(\bigcup_{i=1}^{n} IgsrK_c^{-}(f^{-1}(V_{ji}))) \subseteq \bigcup_{i=1}^{n} IgsrK_c^{-}(V_{ji})$

Hence $Y = \bigcup_{i=1}^{n} IgsrK_c^{-}(V_{ji})$.

Proposition 3.11: Let (X, τ) and (Y, δ) be any two intuitionistic topological spaces. If f: (X, τ) → (Y, δ)is an Igsr C-compact continuous bijective function of an Igsr cokernal compact space (X, τ) onto an Igsr R-compact space (Y, δ), then (X, τ) is an Igsr R-compact space.

Proof:
Let V_α = < X, $V_\alpha^{(1)}$, $V_\alpha^{(2)}$ > be an Igsr RC-compact cover of (X, τ) . From Proposition 3.7, is an Igsr RC-compact cover of (Y, δ). Since (Y, δ) is an Igsr R-compact space, there exists $f^{-1}(V_{\alpha 1}),.....f^{-1}(V_{\alpha n})$ such that $Y = \bigcup_{i=1}^{n} IgsrK_c^{-}(f^{-1}(V_{\alpha i}))$.

Then $Y = f^{-1}(\bigcup_{i=1}^{n} IgsrK_c^{-}(f^{-1}(V_{\alpha i})))$. Since f is an Igsr C-cocompact function. Thus

$$X = \bigcup_{i=1}^{n} f^{-1}(f(IgsrK_c^{-}(V_{\alpha i}))) .$$

$$= \bigcup_{i=1}^{n} (IgsrK_c^{-}(V_{\alpha i}))$$

Therefore, (X, τ) is an Igsr R-compact space.

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