ISSN XXXX XXXX © 2019 IJESC

Research Article Volume 9 Issue No.1

Resolution of the Equation of Drying by the Finite Windows Methods: Preliminary Study

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Abstract:
Within the context of the installation of a tool of resolution of the equations of transfer of heat and mass, the finite windows method was proposed. This method was applied for the first time in the study of transfer of heat for a wall plane which is a simple case. This method gave satisfactory results as compared to the old methods which are the numerical methods. This work concerns the application of this method on the equations of drying. The objective being to highlight the criterion of convergence and the stability of this method. On the basis of the equations of Luikov, simplifying assumptions were laid. The equations were discretized by method MWF (finite windows methods). The system was broken up into regular grid whose numbering of the point of grid is in one dimension. One considers the transfer of heat by conduction in a potato and cut in length. The various products are brought up to the same temperature. While varying the initial temperature of drying, the maximum temperature of drying varies too. The results obtained by the finite windows methods are compared with the experimental and numerical results of finite volumes methods. The finite windows methods give better results.

Key words: Drying, equation of transfer, finite windows methods, heat transfer, mass transfer.

I. INTRODUCTION

The simultaneous transfer of mass and heat is generally carried out during drying. During this operation, there is physical and chemical change of food [1]. Consequently, the adverse effects are controlled if the temperature and the mass are predicted. Mathematical models are used to analyze the phenomenon of drying. The later is governed by internal and external phenomenon of coupled transfer of heat and mass. This phenomenon is described by the partial derivative equations which generally admit approximate solutions and are difficult to solve. The resolution of the partial derivative equations in general remains a permanent challenge in science and for the equations of drying in particular. One thus has resorts to the numerical methods. There are several numerical methods: finite difference method, finite element method, and finite volume method [2]-[3]. Finite differences method is a numerical method which consists in replacing the partial derivatives by the divided or combined differences. This method has advantages for its simplicity of calculation. However, this method is limited to simple geometries and converges little. As for the finite element method, it consists of approaching a finite subspace with a problem written in variational form. This method is complex for its implementation with all the equations. The method of finite volume which is a method which integrates on elementary volume. It is generally applied on equations of fluid mechanics. This method treats the case of the complex geometries and it is less expensive. However, this method has little convergent theoretic results [4]. These numerical methods must validate the criteria which are convergence; precision and stability. In literature, many researchers have used numerical methods to solve the equations of drying. Thus [2] solved the equations of drying by using finite difference method. By comparing the experimental and the numerical result, one observes considerable gaps. It arises from the results obtained without stability and with little convergence. [5] were interested in drying of the grains of some cereals such as the barley, corn and soya. Finite difference method was used for the resolution of the equations of drying. The results show that convergence is not better. [6] also applied this method to solve equations of drying applied on a sweet potato. The result obtained is compared to the experimental. The finding which emerge is that convergence is not good. [7] solved the equations of drying of potato by applying the finite volume method. The results obtained show that there is not a good convergence. [8] solved the equations of drying of a wet object by the finite difference method and compared the results with the experimental one. The result obtained shows that there is difference between the experimental and numerical results. [3] solved the equations of drying of a wet object with the finite element method. The results obtained show that the stability is not better. At the end of this work, one notes that the resolution of the partial differential equations remains a problem without having concrete solutions. Thus [9] set up a new technique of resolution which is finite windows methods. It was applied for the first time in the simple case of the wall plane. It gave satisfactory results. Similarly, [10] also applied this method for the resolution of the equations of transfer in exchangers and also obtained good results. In this work, we propose to apply this finite windows method to the resolution of the equations of drying used by [7]. The results of this method will be compared with the experimental results and the numerical results obtained by the other methods.

II. MATERIAL AND METHODS

The Matlab software was used to simulate. This software permits us to obtain the profiles of temperature, mass and to measure the convergence of the method. We also used the software Tecplot and getData. This software also helped us to extract the data of the experimental and numerical curve obtained by [7] and to plot the experimental curve in the same graph as the numerical curve. The product to be dried is a potato subjected to an initial temperature of 303 K

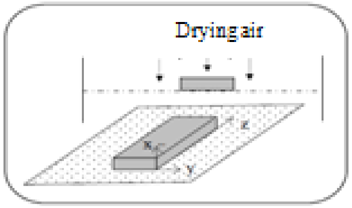

Figure.1. Physical model of drying

Presentation of the finite windows method

It is about the finite windows methods, which is a new technique of approach of the resolution of the partial differential equations.

The principle of the method was described by [9].

It is articulated around the following points:

- Discretization of the space;
- Formulation of the finite windows;
- Calculation of the residues;
- Determination of the matrices of the coefficients;
- Optimization of the residues;
- Calculation of the solution to each mesh.

Drying equations

Several models were set up to describe the mechanism of drying. The choice is based on the model of Luikov. The simplifying assumptions were put forward. Tanks to these assumptions, we find the model of [7].

The equations of the model of Luikov which describe the coupled transfers of heat and mass are as follows:

$$\begin{cases} \frac{\partial T}{\partial t} = \nabla(\alpha \overrightarrow{grad} T) + \xi \frac{\rho_s \Delta Hv}{\rho Cp} \frac{\partial M}{\partial t} \\ \frac{\partial M}{\partial t} = \nabla(D\overrightarrow{grad} M + \delta D \overrightarrow{grad} T) \end{cases} \quad (1)$$

Simplifying assumptions [7]

- The vapor flow inside the sample is not negligible during drying;
- The influence of the heat gradient on the internal migration of water is neglected;
- The temperature and the water content within the product are homogeneous at the beginning of drying;
- The plate support of the sample is perfectly insulating.

Taking into account these assumptions, one leads to the following differential connection:

$$\begin{cases} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\alpha \frac{\partial T}{\partial z}\right) + \frac{\rho_s \Delta Hv}{\rho Cp} \frac{\partial M}{\partial t} \\ \frac{\partial M}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial M}{\partial z}\right) \end{cases} \quad (2)$$

The initial conditions and in extreme cases are given in table 1

Table.1. Initial and boundary conditions

Initial conditions	Boundary conditions
For t=0 et $0 \leq z \leq Lo$; M=Mo and T=T_0 T_0=303 K	for z=0 ; $ht(Ta - T) = \Delta Hv \rho_s D \frac{\partial M}{\partial z} - \lambda \frac{\partial T}{\partial z}$ $hm(M - Meq) = D \frac{\partial M}{\partial z}$ Z=L(t) ;$\frac{\partial T}{\partial z} = 0$ et $\frac{\partial M}{\partial z} = 0$

Let us pose

b = $\Delta Hv \rho_s D$,

System (2) become :

$$\begin{cases} \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\alpha \frac{\partial T}{\partial z}\right) + b \frac{\partial M}{\partial t} \\ \frac{\partial M}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial M}{\partial z}\right) \end{cases} \quad (3)$$

Numerical implementation

The solution of this equation consists of finding the unknown factor on the level of the nodes. The temperature and the mass with each node will be respectively noted T_i and M_i. A continuous approximation of the temperatures and mass will be made using a polynomial approximation. A polynomial function of interpolation N(x, y) will be built to determine the polynomial approximation. That is to say a node of interest k around which one forms a finite window. We introduce two approximate functions which are the approximate function in the partial differential equations to solve. We obtain what is called the residue noted r_k

$$\dot{\tilde{T}}(x,y)=\sum_{i=1}^{i=9} Ni(x,y) * Ti \quad (4)$$

$$\dot{\tilde{M}}(x,y)=\sum_{i=1}^{i=9} Ni(x,y) * Mi \quad (5)$$

By applying the least square criterion to the residue and we obtain the residues:

$$R_k = \iint_{FF} r_k^2 \, dxdy \quad (6)$$

We minimize the residue to determine the unknown factor which is the temperature and we obtain the following result:

$$\frac{\partial \widehat{Rn}}{\partial Tn} = \sum_{k\epsilon\Gamma n} \iint \frac{\partial r_k^2}{\partial Tn} dxdy = 2\sum_{k\epsilon\Gamma n} \iint \frac{\partial r_k}{\partial Tn} r_k \, dxdy = 0 \quad (7)$$

Let

$$\sum_{k\epsilon\Gamma n} \iint \frac{\partial r_k}{\partial Tn} r_k \, dxdy = 0 \quad (8)$$

The same for the mass:

$$\frac{\partial \widehat{Rn}}{\partial Mn} = \sum_{k\epsilon\Gamma n} \iint \frac{\partial r_k^2}{\partial Mn} dxdy = 2\sum_{k\epsilon\Gamma n} \iint \frac{\partial r_k}{\partial Mn} r_k \, dxdy = 0 \quad (9)$$

We obtain:

$$\sum_{k\epsilon\Gamma n} \iint \frac{\partial r_k}{\partial Mn} r_k \, dxdy = 0 \quad (10)$$

Let us note k as the node of interest and r_k as the residues. According to finite window method, each equation forms a residue except the constants. There are 6 equations thus we will have 6 residues

$$r_{k1} = \sum_{i=1}^{9} \tilde{N}iTi - \sum_{i=1}^{9} b.Ni,yMi \quad (11)$$

$$r_{k2} = \sum_{i=1}^{9} \bar{N}iMi \qquad (12)$$

With $\tilde{N}$i=Ni,y - αNi, zz ; $\bar{N}i$ = Ni,y-DNi,zz

$$r_{k1} = \sum_{i=1}^{9} \widetilde{N}\,iTi - \sum_{i=1}^{9} b.Ni,yMi \qquad (13)$$

$$r_{k2} = \sum_{i=1}^{9} \bar{N}iMi \qquad (14)$$

With $\tilde{N}$i=Ni,y - αNi, zz ; $\bar{N}i$ = Ni,y-DNi,zz

III. RESULTS

Figure 2 shows the profiles of temperature compared between the experimental results of [7] and the simulation done in this work. On the basis of the initial temperature of 30 °C and by considering the same thermo physical properties, the result reveals that there is agreement with the experimental one because the two curves converge. It is noticed that the two curves converge starting from 3000 seconds and remain constant at 60°C after 3600 seconds which is the maximal temperature.

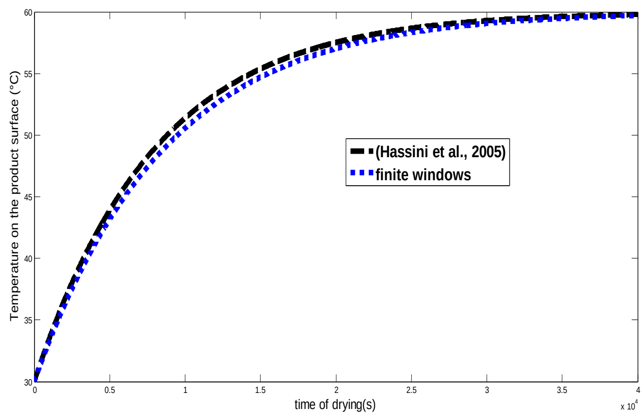

Figure.2. Comparison of the experimental values of hassini *et al,* (2005) and of the values simulated by the finite windows method

Figure 3 makes it possible to make a comparison among the results of the finite windows method, the experimental one and the method of finite volumes resulting from work of [7]. From this figure, one notes that the curve resulting from the finite windows methods converge at 3000 seconds whereas that of the finite volume method converges at 3500 seconds. It is thus concluded that the finite windows methods makes it possible to better approach the experimental one.

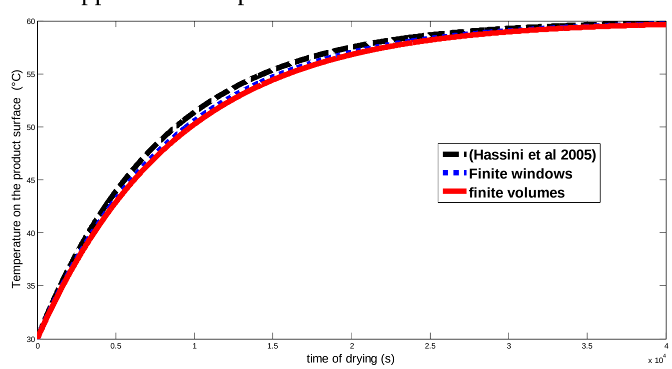

Figure.3. Comparison of the experimental curve of hassini *et al,* (2005) and of the curve simulated by the method of finite volumes and finite windows method

Profile of temperature while varying the size of the product

Figure 4 shows the change of the temperature in the product according to the length. It is noted that the variation in the length of the samples has an influence on their surface temperatures. Indeed, for a weaker length of sample, the temperature of surface believes slightly

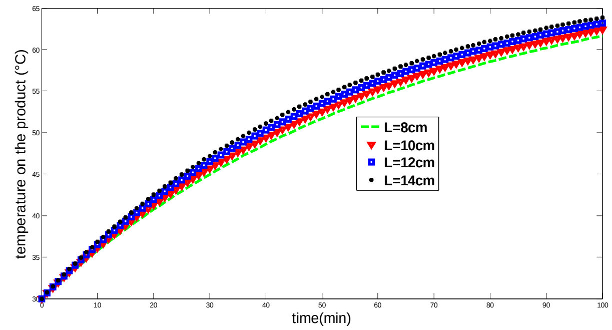

Figure.4. Profile of temperature in function of the length in the product at initial temperature of 30°c

- Comparison of the experimental curve and the curve simulated by finite windows methods of the evolution of the water content in the product

Figure 5 shows the profile of evolution of the content in the product. The numerical result obtained is compared with the experimental result obtained by [7]. It comes out from this analysis that there is better agreement between the numerical result and the experimental result.

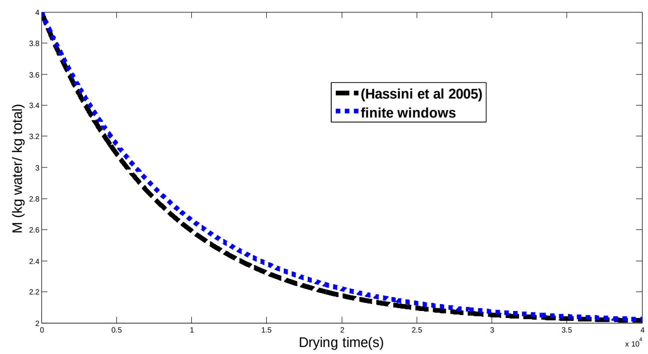

Figure.5.Comparison Of the Experimental Curve and the Curve of Simulation

- Comparison of the experimental, numerical curve by the method the finite differences and finite windows

The curve of simulation resulting from the finite windows methods is also compared with the experimental and numerical curve by the method of finite volumes of [7] (figure 6).The finding which emerges is that the curve resulting from the finite windows methods is very close to the curve experimental. At 2.5×10^4 seconds the result of the finite windows methods converges whereas that of the method of finite volumes converges to 3.5×10^4 seconds.

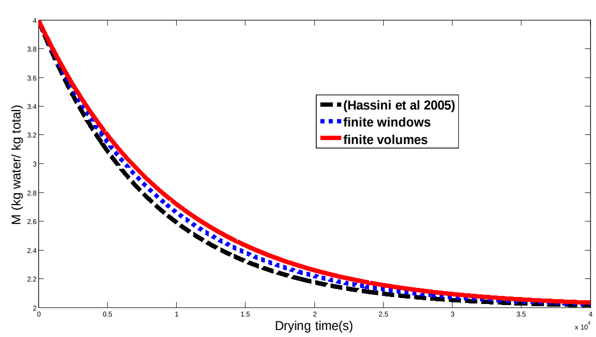

Figure.6.Comparison Of the Experimental Curve, the Curve of Simulation by the Method of Finite Volumes and Finite Windows

IV. CONCLUSION

It was a question in this work of solving the equations of drying with the new method of resolution which is the finite windows method. We based ourselves on the equations of Luikov and by putting forth the simplifying assumptions, we obtain the equations used by [7]. The system obtained is a system coupled in mass and temperature. Also we took account of the boundary conditions and the known physical parameters. By applying the finite windows methods one could obtain the mass and the temperature which are the parameters of the unknown factors. An initial temperature is subjected to the product. Four products of different lengths were chosen and it is noticed that the time of drying is not equal. The profile of temperature and mass are in agreement with that of [7]. We arrived at some results which are the profiles of convergences.

The study reveals to us that by changing the steps of discretization, the solution does not change. At the end of this study, we must be satisfied for attaining our objectives by the results which we presented. It would be judicious to apply this method in a more complex problem which takes into account several parameters of unknown factors which are speed, temperature, pressure and the mass.

V. NOTATION

C_p	specific heat, J/kg.K
D	Mass diffusion coefficient, (m/s^2)
L	Length (m)
Hr	Relative humidity
Hv	Latent heat of vaporization
h_m	Mass coefficient of transfer by convection
h_t	Thermal coefficient of transfer by convection
M	Moisture content $kg_{water}/ kg_{M.S}$
T	Temperature (°C)
t	Time
λ_e	Thermal conductivity (w/m.K)
μ	Dynamic viscosity (Pa.s)
v	Kinematic viscosity (m^2/s)
ρ	Density (kg/m^3)

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