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Analysis of Lumped System in Slab

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Abstract:
Present work deals with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a long slab and cylinder. Polynomial approximation method is used to solve the transient conduction equations for both the slab and tube geometry. A variety of models including boundary heat flux for both slabs and tube and, heat generation in both slab and tube has been analyzed. Furthermore, for both slab and cylindrical geometry, a number of guess temperature profiles have been assumed to obtain a generalized solution. Based on the analysis, a modified Biot number has been proposed that predicts the temperature variation irrespective the geometry of the problem. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time. The result of the present analysis has been compared with earlier numerical and analytical results. A good agreement has been obtained between the present prediction and the available results.

Keywords: Transient Conduction, Lumped model, Polynomial approximation method, Modified Biot number

1. INTRODUCTION

Due to variation in temperature thermal energy transport within a medium by molecular interaction, fluid motion, and electro-magnetic waves. This transport of thermal energy is called Heat transfer. The principle of energy conservation governs this variation in temperature. Which says that energy is "conserved"—that is, it can't be created or destroyed, so that the total amount of energy in any closed system doesn't change. However, energy exists in many forms, and it can change from one form to another. The study of thermodynamics is very important for thermal analysis of a system. System may be defined as "A prescribed, identifiable and fixed collection of matter which is completely enclosed with in a definite region, and whose behavior is being investigated". This investigation is done by defining the boundaries of the system. The system may be a quantity of steam, a mixture of vapour and gas, or an internal combustion engine and its components. The term Surrounding represent the environments which are affected by changes occurring within the system. The Boundary separating a system from its surrounding. When the system undergoes a change from one state to another, there are mass and energy interactions between the system and surroundings. To perform a thermal analysis of a system we need to applying the conservation principles, and examining how the system participates in thermal energy exchange and conversion Heat conduction is increasingly important in various areas, namely in the earth sciences, and in many other evolving areas of thermal analysis. A common example of heat conduction is heating an object in an oven or furnace. The material remains stationary throughout, neglecting thermal expansion, as the heat diffuses inward to increase its temperature. The importance of such conditions leads to analyze the temperature field by employing sophisticated mathematical and advanced numerical tools. The section considers the various solution methodologies used to obtain the temperature field. The objective of conduction analysis is to determine the temperature field in a body and how the temperature within the portion of the body. The temperature field usually depends on boundary conditions, initial condition, material properties and geometry of the body. The solution of conduction problems involves the functional dependence of temperature on space and time coordinate. Obtaining a solution means determining a temperature distribution which is consistent with the conditions on the boundaries and also consistent with any specified constraints internal to the region.

TABLE.1 NOMENCLATURE

B	Biot Number
K	Thermal conductivity
H	Heat transfer coefficient
M	Order of the geometry
T	Time
V	Volume
T	Temperature
S	Shape factor
R	Coordinate
R	Maximum coordinate
G	Internal heat generation
G	Dimensionless internal heat generation
X	Dimensionless coordinate
PAM	Polynomial approximation
P	Modified Biot number
Greek symbol	
A	Thermal diffusivity
τ	Dimensionless time
θ	Dimensionless temperature
$\overline{\theta}$	Dimensionless average temperature
Subscripts	
o	Initial
∞	Infinite

2. THEORTICAL ANALYSIS

The generalized Unsteady state one dimensional temperature distribution of a long slab can be expressed by the following partial differential equation

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r}(r^m \frac{\partial T}{\partial r})$$

Where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively.

Heat transfer coefficient is assumed to be constant, as illustrated in Fig 2.1

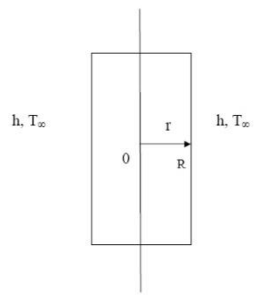

Figure.2. Schematic of slab

Boundary conditions are $\frac{\partial T}{\partial r} = 0$at r = 0 (2)

$k\frac{\partial T}{\partial r} = -h(T - T_\infty)$ at r = R (3)

And initial condition: T=T_0 (4)

In the derivation of Equation (1), it is assumed that thermal conductivity is independent of temperature. If not, temperature dependence must be applied, but the same procedure can be followed.

Dimensionless parameters defined as

θ = $\frac{T-T_\infty}{T_0-T_\infty}$, B = $\frac{hR}{k}$, τ = $\frac{\alpha t}{R^2}$, x = $\frac{r}{R}$ (5)

For simplicity, Eq. (1) and boundary conditions can be rewritten in dimensionless form

$\frac{\partial \theta}{\partial \tau} = \frac{1}{x^m}\frac{\partial}{\partial x}(x^m \frac{\partial \theta}{\partial x})$ (6)

$\frac{\partial \theta}{\partial x} = 0$ At x=0 (7)

$\frac{\partial \theta}{\partial x} = -B\theta$ At x=1 (8)

θ= 1 at τ= 0 (9)

For a long slab with the same Biot number in both sides, temperature distribution is the same for each half, and so just one half can be considered

2.1. PROFILE1

The guess temperature profile is assumed as

$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$ (10)

Differentiating the above equation with respect to x we get

$\frac{\partial\theta}{\partial x} = a_1 + 2a_2x$ (11)

Applying first boundary condition we have

$a_1 + 2a_2x = 0$ (12)

Thus$a_1 = 0$

Applying second boundary condition we have

$a_1 + 2a_2 = -B\theta$ (13)

$a_2 = -\frac{B\theta}{2}$ (14)

We can also write the second boundary condition as

$\frac{\partial\theta}{\partial x} = -B(a_0 + a_1 + a_2)$(15)

Using the above expression, we have

$a_0 = \theta\left(1 + \frac{B}{2}\right)$ (16)

Average temperature for long slab can be written s

$\bar{\theta} = \int_0^1 \theta dx$ (17)

Substituting the value of θ and integrating we have

$\bar{\theta} = \theta + \frac{B\theta}{3}$ (18)

Integrating non-dimensional governing equation, we have

$\int_0^1 x^m \frac{\partial\theta}{\partial\tau}dx = \int_0^1 \frac{\partial}{\partial x}(x^m \frac{\partial\theta}{\partial\tau})dx$ (19)

Simplifying the above equation, we may write

$\int_0^1 \frac{\partial\theta}{\partial\tau}dx = -B\theta$ (20)

Considering the average temperature, we may write

$\frac{\partial\bar{\theta}}{\partial\tau} = -B\theta$ (21)

Substituting the value of θ at equation (4) we have

$\frac{\partial\bar{\theta}}{\partial\tau} = -\frac{3B\bar{\theta}}{B+3}$ (22)

Integrating the equation (5) we may write as

$\int_0^1 \frac{\partial\bar{\theta}}{\bar{\theta}} = -\int_0^1 \frac{3B}{B+3}\partial\tau$ (23)

Thus by simplifying the above equation we may write

$\bar{\theta} = exp\left(-\frac{3B}{B+3}\tau\right)$ (24)

Or $\bar{\theta} = exp(-P\tau)$

Where P=$\frac{3B}{B+3}$

Several profiles have been considered for the analysis. The corresponding modified Biot number, P, has been deduced for the analysis and is shown in Table.

3. RESULT AND DISCUSSION

We have considered a variety of temperature profiles to see their effect on the solution. Based on the analysis a modified Biot number has been proposed, which is independent of geometry of the problem.

Fig (3.1-3.2) shows the variety of temperature with time for different values of modified Biot number, P. It is seen that, for higher values of P represent higher values of Biot number. Therefore the heat removed from the solid to surrounding is higher at higher Biot number.

This leads to sudden change in temperature for higher value of P. This trend is observed in the present prediction and is shown in fig (3.1).

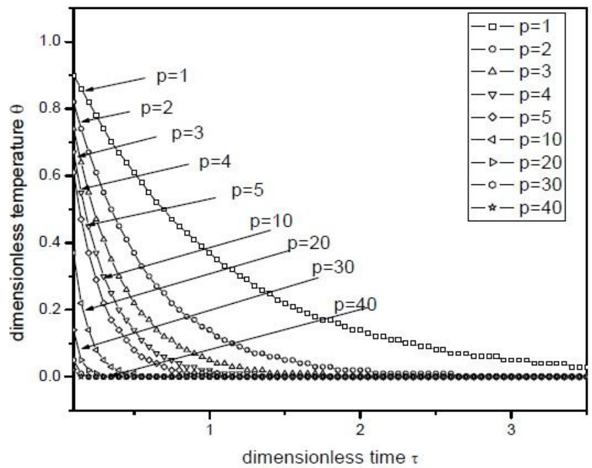

Figure.1. Variation of average temperature with dimensionless time, for P=1 to 40 for a slab.

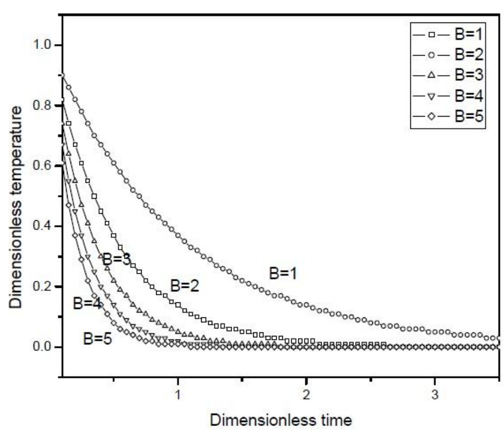

Figure.2. Variation of average temperature with dimensionless time, for B=1 to 5 for a slab

Fig (3.3) shows the comparison of present analysis with the other available results. These include classified lumped system analysis and exact solution by E.J. Correa and R.M. Cotta [4] of a slab. It is observed that the present prediction shows a better result compared to CLSA. The present prediction agrees well with the exact solution of E.J. Correa and R.M. Cotta [4] at higher time. However, at shorter time, the present analysis under predicts the temperature in solid compared to the exact solution. This may be due to the consideration of lumped model for the analysis.

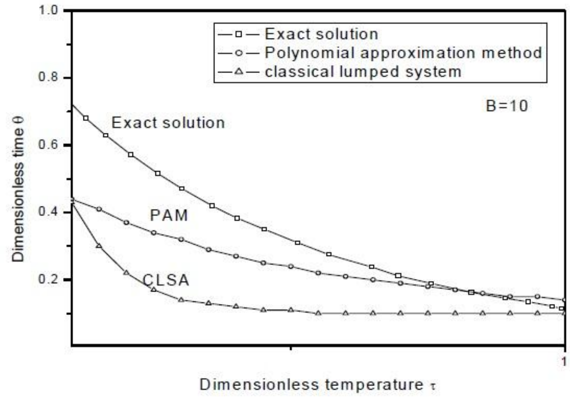

Figure.3. Comparison of solutions of PAM, CLSA and Exact solution for a slab having internal heat generation

4. CONCLUSION

An improved lumped parameter model is applied to the transient heat conduction in a long slab and long cylinder. Polynomial approximation method is used to predict the transient distribution temperature of the slab and tube geometry. Four different cases namely, boundary heat flux for both slab and tube and, heat generation in both slab and tube has been analyzed. Additionally, different temperature profiles have been used to obtain solutions for a slab. A unique number, known as modified Biot number is, obtained from the analysis. It is seen that the modified Biot number, which is a function of Biot number, plays important role in the transfer of heat in the solid.

→ Based on the analysis a unique parameter known as modified Boit number obtained from the analysis and is shown in Table. With higher value of heat source parameter, the temperature inside the tube does not vary with time. However, at lower values of heat source parameters, the temperature decreases with increase of time. With lower value of Biot numbers, the temperature inside the tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

5. SCOPE FOR FUTURE WORK

→ Polynomial approximation method can be used to obtain solution of more complex problem involving variable properties and variable heat transfer coefficients, radiation at the surface of the slab.

→ Other approximation method, such as Heat Balance Integral method, Biots variation method can be used to obtain the solution for various complex heat transfer problems.

→ Efforts can be made to analyze two dimensional unsteady problems by employing various approximate methods.

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