Design and Development of Harmonic Drive for Shredder Machine to Achieve Higher Gear Ratio up to 750:1

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Abstract:
Harmonic drive system are precise and specific transmission gear system in which we have got high transmission ratio with maximum torque. This system are generally applicable in robotics space application and highly automated machinery just because of very precise and accurate work. In this project we are going design and developed very high ratio with maximum torque harmonic drive for one of the shredder machine system in which where xIPE [cross slinking polymer] type material produced by shredder machine. the xIPE material is very low density material and it’s tear strength is also very low so when we have wind this material in roll form just because of it tear strength and low density very accurate work required at high ratio with maximum torque for smooth operation of all winding so that’s why we have to developed such kind of for smooth operation mechanism.

Keywords: Designing of harmonic drive, Design Parameter calculation of harmonic drive, 3d model of harmonic drive, Analysis.

I. INTRODUCTION
The basic concept of harmonic drive had given by C.W. Musser in 1957. Now present era so many number of modification and design are available there are different kind of companies are developed harmonic drive among them HDs and AG are the famous manufacture of harmonic drive designing. There are different kind of harmonic drive with shape size and operation wise are available in market. The based on strain wave gearing principle this is work as strain wave generating gear just because of its wave generator part. The main component of harmonic drive is circular spline, flex spline, and wave generator in which Bearing and wave plug is part of wave generator. circular spline is fixed internal gearing mechanism when flex spline is flexible cup type or simple external gear teeth mechanism when wave generator are in elliptical shape are inserted in flex spline then the flex spline change shape elliptical as like wave generator when flex spline deflect in elliptical shape that time only few teeth are comes to in contact with circular spline teeth in opposite direction now from input shaft side power is given and other end the output shaft are available which is connect with our required machine or component are fixed and then the harmonic drive has worked. Generally harmonic drive are used for better accuracy and high speed reduction in robotics, space application, medical equipment, etc. Just because of its better accuracy and high speed reduction ratio harmonic drive is better option than other drive. The harmonic drive is also available in compact size so it’s utilize very little bit space, above defined advantage harmonic drive now a day are spread worldwide.

II. DOUBLE STAGE HARMONIC DRIVE DESIGNING:--

Principle:--
The harmonic drive design calculation is based on harmonic drive standard by C.W. Musser found in 1957. In this article we have design to different kind of harmonic drive in which we have taken this two harmonic drive as first stage and Second stage in first stage harmonic drive we have take 25:1 ratio harmonic drive and in second stage harmonic drive we have taken 30:1 harmonic drive for achieve higher gear reduction ratio up to 750:1. This kind of such work is only possible in harmonic drive in very compact model so we have taken harmonic drive for such big ratio.

Design Parameter:--
The design parameter for developed harmonic drive is based on shredder machine mechanism just because of this kind of drive is only used for smooth and accurate work in shredder machine when shredder machine are produce xIPE material by waste. So we have take 750:1 ratio for smooth operation of wind roll minimum RPM required to winding the xIPE material in roll form is 2 so we have take 750:1 ratio and the torque generate on system is 1150 Nm. So we have based on this two parameter we have designing harmonic drive for shredder machine for xIPE roll material winding process.

A) Harmonic Drive Design Base parameter
The harmonic drive designing base parameter are generally calculated based on shredder machine because of the double stage harmonic drive are used in shredder machine so RPM and torque calculation are getting from shredder machine based calculation. for shredder machine required RPM minimum 2 RPM and Torque are generate on system is 1150 Nm so base on this mention parameter we have selected in this paper is double stage harmonic drive in which one is 25:1 and second is 30:1 for final output ratio 750:1 at required 2 RPM at 1500 RPM motor capacity. step by step we have seen based parameter shredder machine system.
<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Motor available RPM</td>
<td>1500 RPM</td>
</tr>
<tr>
<td>2</td>
<td>System Required Torque</td>
<td>1150 Nm</td>
</tr>
<tr>
<td>3</td>
<td>Required Machine ratio</td>
<td>750 : 1</td>
</tr>
<tr>
<td>4</td>
<td>Material Manufacture type</td>
<td>XLPE in Roll form</td>
</tr>
<tr>
<td>5</td>
<td>Thickness of XLPE Material Sheet</td>
<td>10 mm</td>
</tr>
<tr>
<td>6</td>
<td>Diameter of XLPE Material Sheet</td>
<td>Ø800 or Ø1500</td>
</tr>
<tr>
<td>7</td>
<td>Mass of XLPE Material</td>
<td>30 Kg/m³</td>
</tr>
<tr>
<td>8</td>
<td>Velocity of Output Material sheet</td>
<td>2000 mm/min</td>
</tr>
<tr>
<td>9</td>
<td>Weight of Ø800 roll size</td>
<td>21.21 Kg</td>
</tr>
<tr>
<td>10</td>
<td>Weight of Ø1500 roll size</td>
<td>78.11 Kg</td>
</tr>
</tbody>
</table>

B) Required RPM and Torque calculation for shredder machine. 

Maximum RPM Calculation for Winding Roll

\[ V = \frac{\pi \times D \times N}{60} \]

\[ = \frac{(2000 \times 60)}{(200 \times 3.142)} \]

\[ = 190.9612 \text{ RPM} \]

\[ = 190.9612 \times 100 \]

\[ = 2 \text{ RPM is Maximum.} \]

Minimum RPM calculation for Winding Roll

\[ V = r \times \omega \]

\[ = \omega = 2000/400 \]

\[ = \omega = 5 \text{ rad/sec} \]

\[ \omega = \frac{\pi \times D \times N}{60} \]

\[ = N = (60 \times 5) / (\pi \times 800) \]

\[ = N = 0.119 \text{ RPM Minimum} \]

Maximum Torque calculation for Shredder Machine Winding Roll

\[ T = F \times I \]

\[ = T = \text{Torque of system} \]

\[ = F = \text{Force on system (Kg)} \]

\[ = r = \text{Radius of roll (mm)} \]

\[ T = F \times F.O.S \times r \] (Factor of safety = 2 in weight)

\[ = T = 156.22 \times 2 \times 750 \text{ (r = 750 for Ø1500 dia. Roll)} \]

\[ = T = 117161.771 \text{ kg-mm} \]

\[ = T = 117.17 \text{ kg-m} \] (1 kg-m = 9.81 N-m)

\[ = T = 1150 \text{ N-m} \] (at Maximum size Roll dia.Ø1500)

\[ = T = 166.423 \text{ N-m} \] (at Minimum size Roll dia. Ø800)

C) Double Stage Harmonic Drive Ratio calculation. 

Diametral Pitch (Dp) = 0.393700787 teeth/mm (10 teeth/in)

Tooth difference = 2 selected

Ratio calculation of Harmonic drive first stage

\[ I = F.S. / (C.S. - F.S.) \]

\[ I = 50 / (52 - 50) \]

\[ I = 50 / 2 \]

\[ I = 25 \]

First stage reduction 25:1.

Ratio calculation of Harmonic drive second stage

\[ I = F.S. / (C.S. - F.S.) \]

\[ I = 60 / (62 - 60) \]

\[ I = 60 / 2 \]

\[ I = 30 \]

Second stage reduction 30:1.

Total reduction ratio (i) of harmonic drive.

\[ 25 \times 30 = 750 \]

\[ \text{Ratio} = 750:1 \]

Ratio = input speed/output speed

Output speed = 1440 rpm of motor / 25 harmonic drive ratio

Output speed = 57.6 RPM.

Ratio = input speed/output speed

Output speed = 57.6 rpm of first stage reduction / 30 harmonic drive ratio

Output speed = 1.92 RPM.

C.S. – Circular Spline

F.S. – Flex Spline

W.G. – wave generator

D) Designing of Circular Spline First Stage. 

Data given for 25:1 Ratio First Stage Harmonic Drive

Number of teeth of circular spline (N) - 52

Diametral pitch (Dp) – 0.393700787 teeth/mm., (10teeth/inch)

D = 10 \div 25.4

D = 0.393700787

Pressure Angle = 30° degree

Torque (Ts) = 1150 Nm

(1) Pitch circle diameter (p)

\[ p = \frac{\text{Number of teeth of circular spline}}{\text{Diametral pitch}} \]

\[ p = \frac{52}{0.393700787} \]

\[ p = 132.08 \text{ mm.} \] (as per standard bearing size pitch value taking as Ø130)

(2) Circular Pitch (c)

\[ c = \pi \div \text{Diametral pitch} \]

\[ c = \frac{\pi}{0.393700787} \]

\[ c = 7.979645 \text{ mm/teeth} \] (0.314 in/teeth)

\[ \approx (\text{as per standard bearing size circular pitch value taking as 7 mm}) \]

(3) Addendum (A)

\[ A = \frac{0.5}{\text{diametral pitch}} \]

\[ A = 1.27 \text{ mm.} \]

(4) Base circle diameter (Bd)

\[ Bd = \frac{\text{pitchcircle diameter } \times \cos 30°}{\cos 30°} \]

\[ Bd = 132.08 \times \cos 30° \]

\[ Bd = 114.3762 \text{ mm.} \]

\[ \approx (\text{as per standard bearing size Bd value taking as 113 mm}) \]

(5) Circular Tooth Thickness (Tt)

\[ Tt = \frac{2 \times \text{diametral pitch}}{3.142} \]

\[ Tt = \frac{2 \times 0.393700787}{3.142} \]

\[ Tt = 3.99 \text{ mm.} \]

\[ \approx (\text{as per standard bearing size Tt value taking as 3 mm}) \]

(6) Major internal diameter (Mdi)

\[ Mdi = \frac{\text{Number of teeth} + 1.8}{\text{diametral pitch}} \]
Mdi = \frac{52 + 1.8}{0.393700787} = 136.6520 \text{ mm.}

\approx \text{(as per standard bearing size Mdi value taking as 135 mm)}

(7) Minor external diameter (Mde)

\[ Mde = \frac{\text{Number of teeth} - 1.8}{0.393700787} \]

\[ Mde = \frac{52 - 1.8}{0.393700787} \]

Mde = 127.508 mm.

\approx \text{(as per standard bearing size Mde value taking as 125 mm)}

(8) Internal Dedendum (Id)

\[ Id = \frac{0.900}{\text{diametral pitch}} \]

Id = 0.900

Id = \frac{0.393700787}{2.286} mm.

(9) External Dedendum (Ed)

\[ Ed = \frac{0.900}{\text{diametral pitch}} \]

Ed = 0.900

Ed = \frac{0.393700787}{2.286} mm.

(10) Effective spline Length (Le)

\[ Le = \left(127.508^3 \times \left(1 - \left(\frac{0.4}{127.508^4}\right)\right) \right)/(10 \times 25.4)^2 \]

Le = 32.131 mm.

\approx \text{(as per standard bearing size Le value taking as 30 mm)}

(11) Shear stress (\tau)

\[ \tau = \frac{16 \times Ts}{\pi \times Dp^2 \times Le} \]

\[ \tau = \frac{3.141593 \times (10 \times 25.4)^2 \times 32.131}{16 \times 1150 \times 10^3} \]

\[ \tau = 3.695 \text{ N/mm}^2 \]

if we put Le = 30 mm new value in above equation then shear stress will be

\[ \tau = 3.027 \text{ N/mm}^2 \]

(12) Static Tooth load or beam strength or endurance strength calculation based on Lewis formula.

\[ Ws = \sigma e x b x \pi x m x y \]

\[ Ws = 362.25 \text{ MPa} \]

\[ b = 32.131 \]

\[ m = 2.5 \]

\[ y = 0.6 \]

\[ \pi = 3.141593 \]

\[ Ws = 362.25 \times 32.131 \times 2.5 \times 3.141593 \times 0.6 \]

\[ Ws = 54849.64 \text{ MPa} \]

\[ \text{Now as per Buckingham suggest formula for on pulsating load for Dynamic load Calculation} \]

\[ \text{Pulsating loads} = Ws \geq 1.35 \text{ Wd} \]

Table 1. Mechanical Properties of the steel for 40Cr.

<table>
<thead>
<tr>
<th>Tensile strength (N/mm²)</th>
<th>Yield strength (N/mm²)</th>
<th>Elongation After Break (when actual HRC-25)</th>
<th>Rate of Reduction in area</th>
<th>Impact Absorbing energy (J)</th>
<th>Brinell hardness (HB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥ 810</td>
<td>≥ 785</td>
<td>9</td>
<td>45</td>
<td>47</td>
<td>207</td>
</tr>
</tbody>
</table>

Figure 1. 3D Model of circular spline first stage

E) Designing of Flex Spline First Stage. [17][18][19][20][24][25]

Data given for 25:1 Ratio First Stage Harmonic Drive

Number of teeth of flex spline (N) - 50

Diametral pitch (Dp) = 0.393700787 teeth/mm, (10 teeth/inch)

Pressure Angle = 30° degree

Torque (Ts) = 1150 Nm

Root Diameter = \((p) - (2 \times hf)\) = 127-(2x2.286) = 122.43 mm

Inside Diameter = 121.23 mm.

Bed Thickness = \((122.43 - 119.5) \div 2 = 0.6 \text{ mm.}\)

Mean Bed Diameter = 121.23 + 0.6 = 121.83 mm.

Mean Bed Radius = 121.83 \div 2 = 60.92 mm.

(13) Pitch Circle Diameter (p)

\[ p = \frac{\text{Number of teeth of flex spline}}{\text{Diametral pitch}} \]

\[ p = \frac{50}{0.393700787} \]

\[ p = 127.00 \text{ mm.} \]

(14) Circular Pitch (c)

\[ c = \frac{\pi}{\text{Diametral Pitch}} \]

\[ c = \frac{0.393700787}{\pi} \]

Circular pitch = 7.9797 mm/teeth

(15) Addendum circle (hf)

\[ hf = \frac{0.5}{\text{Diametral Pitch}} \]

hf = 1.27 mm.
(16) Base circle diameter (Bd)
\[ Bd = 127.00 \times \cos 30^\circ \]
\[ Bd = 109.99 \text{ mm} \]

(17) Flex spline Tooth Thickness (Tt)
\[ Tt = \frac{2 \times \text{diametral pitch}}{\pi} \times 3.142 \]
\[ Tt = 3.99 \text{ mm} \]

(18) Major internal diameter (Mdi)
\[ Mdi = \frac{\text{Number of teeth} + 1.8}{\text{pitch circle diameter}} \times 50 + 1.8 \]
\[ Mdi = 131.6 \text{ mm} \]

(19) Minor external diameter (Mde)
\[ Mde = \frac{\text{Number of teeth} - 1.8}{\text{pitch circle diameter}} \times 50 - 1.8 \]
\[ Mde = 122.43 \text{ mm} \]

(20) Internal Dedendum (Id)
\[ Id = \frac{0.9}{\text{diametral pitch}} \]
\[ Id = 2.286 \text{ mm} \]

(21) External Dedendum (Ed)
\[ Ed = \frac{0.900}{\text{diametral pitch}} \]
\[ Ed = 2.286 \text{ mm} \]

(22) Effective spline Length (Le)
\[ Le = \left(122.43^3 \times 1 - \left(\frac{0.4^4}{122.43^4}\right)\right) / (10 \times 25.4)^2 \]
\[ Le = 28.5 \text{ mm} \]

(23) Shear stress (\(\tau\))
\[ \tau = \frac{16 \times T_s}{\pi \times D_p^2 / 3 \times 1150 \times 10^3} \times 3.141593 \times (10 \times 25.4)^2 \times 28.5 \]
\[ \tau = 4.165 \text{ N/mm}^2 \]

(24) Static Tooth load or beam strength or endurance strength calculation based on Lewis formula.
\[ Ws = \sigma e \times b \times pc \times y \]
\[ Ws = \sigma e \times b \times \pi \times m \times x \times y \]
\[ \sigma e = 1.75 \times B.H.N \text{ (in MPa)} \times (1 \text{ N/mm}^2 = 1 \text{ MPa}) \]
\[ \sigma e = 581 \text{ MPa} \]

### Calculations

- \( b = 28.448 \)
- \( m = 2.5 \)
- \( y = 0.6 \)
- \( \pi = 3.141593 \)
- \( Ws = 581 \times 28.448 \times 2.5 \times 3.141593 \times 0.6 \)
- \( Ws = 77887.73 \text{ MPa} \)

Now as per Buckingham suggest formula for on pulsating load for Dynamic load Calculation
\[ \text{Pulsating loads} = Ws \geq 1.35 \text{ Wd} \]
\[ \text{Wd} = \frac{77887.73}{1.35} \]
\[ \text{Wd} = 57694.62 \text{ MPa} \]

(25) Flex spline Deflection
\[ d = pc - p_l \]
\[ d = 132.08 - 127.00 \]
\[ d = 5.08 \text{ mm} \]
\[ d = \frac{5.08}{2.54} = (d/2) \]

(26) Flex spline Deflection stress
\[ Sf = \frac{3 \times E \times d \times Bt}{D_b^2} \]
\[ Sf = (3 \times 207 \times 10^3 \times 5.08 \times 0.6) / (121.83^2) \]
\[ Sf = 127.53 \text{ N/mm}^2 \]

(27) Flex spline Load Stress
\[ St = \frac{T}{D_b \times A_t} \]
\[ St = \frac{1150000}{121.83 \times 32.1} \]
\[ St = 490.1 \text{ N/mm}^2 \]

(28) Flex spline Tooth shear stress
\[ S_s = \frac{T}{R_b \times \pi \times \frac{X}{T}} \]
\[ S_s = (1150000 \times 0.1 \times 127^3 \times 2 \times \pi \times 60.92 \times 0.6) \]
\[ S_s = 82.20 \text{ N/mm}^2 \]

(29) Flex spline Bell Face Shear Stress
\[ S_{sb} = \frac{T}{R_b \times \pi \times \frac{X}{T}} \]
\[ S_{sb} = (1150000 \times 0.1 \times 127^3 \times 2 \times \pi \times 60.92 \times 0.6) \]
\[ S_{sb} = 82.20 \text{ N/mm}^2 \]

(30) Flex Spline Torsion Stress
\[ S_{st} = \frac{2 \times T}{\pi \times (r_1^4 - r_0^4)} \]
\[ S_{st} = 2 \times 1150000 \times 65 \times 3 \]
\[ S_{st} = 9.190 \text{ N/mm}^2 \]

(31) Torsional Deflection of Flex spline
\[ V = \frac{r \times T}{2 \times G_p} \times (L^2 - C) \]
\[ V = \frac{r \times T}{2 \times G_p} \times (L^2 - 25) \]
\[ V = \frac{60.92 \times (1150000 \times 321)}{0.07874 \times 82 \times 10^3 \times 0.0787 \times \pi \times 60.92^3 \times 0.6} \times (L^2 - 25) \]
\[ V = 0.025 \text{ mm} \]

That is maximum twist is 0.025 mm in 32.1 mm or an average twist is approximately 0.000785 mm/mm.
Table 2. Mechanical Properties of the steel for 30crMnSiA.

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cs number of teeth</td>
<td>52</td>
<td>Nos.</td>
</tr>
<tr>
<td>2</td>
<td>Fs number of teeth</td>
<td>50</td>
<td>Nos.</td>
</tr>
<tr>
<td>3</td>
<td>Pressure Angle $\alpha$</td>
<td>30°</td>
<td>Degre e</td>
</tr>
<tr>
<td>4</td>
<td>Cs pitch circle diameter</td>
<td>132.08</td>
<td>mm.</td>
</tr>
<tr>
<td>5</td>
<td>Fs pitch circle diameter</td>
<td>127.00</td>
<td>mm.</td>
</tr>
<tr>
<td>6</td>
<td>Addendum</td>
<td>1.27</td>
<td>mm.</td>
</tr>
<tr>
<td>7</td>
<td>Thickness of Fs</td>
<td>3.99</td>
<td>mm.</td>
</tr>
<tr>
<td>8</td>
<td>Thickness of Cs</td>
<td>3.99</td>
<td>mm.</td>
</tr>
<tr>
<td>9</td>
<td>Effective length Teeth for Cs</td>
<td>32.131</td>
<td>mm.</td>
</tr>
<tr>
<td>10</td>
<td>Effective length teeth for Fs</td>
<td>28.5</td>
<td>mm.</td>
</tr>
<tr>
<td>11</td>
<td>Inside diameter of Fs</td>
<td>121.23</td>
<td>mm.</td>
</tr>
<tr>
<td>12</td>
<td>Working Diameter of wave generator</td>
<td>121.03</td>
<td>mm.</td>
</tr>
<tr>
<td>13</td>
<td>Major axis of cam (a)</td>
<td>63.1</td>
<td>mm.</td>
</tr>
<tr>
<td>14</td>
<td>Minor axis of cam (b)</td>
<td>57.93</td>
<td>mm.</td>
</tr>
<tr>
<td>15</td>
<td>Length of wave generator</td>
<td>32.1</td>
<td>mm.</td>
</tr>
</tbody>
</table>

(32) Length of wave generator (Lw)

$Lw = \text{tooth length of flex spline} + 3.6\text{mm for circular spline tooth comp onset}$

(33) Flexspline Deflection Force (F)

$$F = \frac{0.56 \times d \times Lw \times Bt^3 \times X}{Rb^3}$$

From harmonic drive design standards,

$F = (0.56 \times 5.08 \times 32.1 \times 0.6^3 \times 207 \times 10^3) \div (60.92^3)$

$F = 18.06 \text{ N.}$

$F = 18.06 \div 32.1 \text{ (Length of wave generator)}$

$F = 0.563 \text{ N/mm}$

(34) Length of Supporting Arc (L)

$$L = \pi \times D1 \times \left(\frac{\alpha}{360}\right)$$

$L = \pi \times 121.23 \times \left(\frac{45^\circ}{360}\right)$

$L = 47.61 \text{ mm}$

The average oil pressure distribute over an arc of $30^\circ$ before major axis and $15^\circ$ behind it.

(35) Average Pressure (P)

$$P = \frac{F}{A}$$

$P = 0.56 \div (47.61 \times 0.0394)$

$P = 0.30 \text{ N/mm}^2$

The distributed pressure could higher than that of concentrated force so assume 1 N/mm² pressure.

(36) Maximum Pressure (Pmax)

$$P_{\text{max}} = P_{\text{average}} \times \frac{\pi}{2}, \text{sineoidal pressure distribution}$$

$P_{\text{max}} = 1 \times (\pi/2)$

$P_{\text{max}} = 1.571 \text{ N/mm}^2.$

$P_{\text{max}} = 1.571 \text{ N/mm}^2.$

(37) Tooth Separating Forces (Ft)

From Harmonic drive Design Standard,

$$Ft = \frac{1150000}{132.08 \times 25.4} \times (\tan 30^\circ)$$
Ft = 197.90941 N.
or, = 6.17 N/mm.

(38) Pressure to support Separating Load
separating load of 6.17 N/mm is disctributed over the 30° arc and symmetric to major axis and sunsoidal pressure distribution

(39) Length of Supporting Arc (L)
\[ L = \pi \times \frac{D}{2} \times \left( \frac{\alpha}{360} \right) \]
\[ L = \pi \times 121.23 \times \left( \frac{30^\circ}{360^\circ} \right) \]
\[ L = 31.74 \text{ mm.} \]

(40) Average Pressure (P)
\[ P = \frac{F}{A} \]
\[ P = 6.17 \div (31.74 \times 0.0394) \]
P = 4.94 N/mm².

(41) Maximum Pressure (Pmax)
\[ P_{\text{max}} = P_{\text{ave}} \times 2, \text{sunsoidal pressure distribution} \]
Pmax = 4.94 x (π/2)
\[ P_{\text{max}} = 7.751 \text{ N/mm}^2. \]

(42) Pressure profile to develop output torque
Pressure profile in which \[ P = K x \phi \]
from the minor axis \( \phi = 0 \) and major axis \( \phi = \pi/2 \). Will maintain the flex spline position when it’s run and generate necessary force as per requirement for output torque development.

(43) Wave generator Surface (La)
\[ L_a = \frac{2d}{D} \times r \sin 2 \phi \]

(44) R = D/d, from harmonic drive Design standard
\[ T = K x \phi^2 x L w \times \pi \]
\[ T = K x 60.62^2 x 32.1 \times \pi \]
K = (115000) ÷ (60.62² x 32.1 x π)
K = 3.10 N/mm².

(46) Pressure at Major Axis
\[ P = K x \phi \]
P = 3.10 x (π/2)
P = 4.88 N/mm².

(47) Flex spline harmonic shape
\[ r_{\phi} = \frac{d}{2} x (1 - \cos 2 \phi) \]
\[ r_{\phi} = 55.99 \text{ mm.} \]
r_{\phi} = 55.59 + [(4.88÷2) x (1-cos 2 \phi)]

(48) LH = 5 x 300 x 8
(assuming 300 working day in year)
LH = 5 x 300 x 8 = 12000 hours.
Basic dynamic equivalent load

(49) Life of the bearing in Revolution
\[ L = 60 \times N \times LH \]
L = 60 x 1500 x 12000
L = 1080 x 10⁶ Rev.

(50) The Basic dynamic equivalent radial load
\[ W = X_1 \times V \times F_r + Y_1 \times F_a \]
X₁ = 0.56, Y₁ = 1
W = (0.56 x 1 x 1150) + (1 x 300)
W = 644 + 300
W = 944 N

(51) We know that basic dynamic load rating.
\[ C = \left( \frac{W \times L}{10^6} \right)^{1/K} \]
(Taking K=3 for ball bearing)
C = 944 x 10.2598
C = 9685.3016 N
C = 9.69 kN \approx 10 kN

(52) From R.S. Khurmi Table 27.6 at page 1014 select bearing 305 Number
at 305 Number bearing
Static load rating \( C_0 \) = 10.4 kN
Dynamic load rating \( C \) = 16.6 kN
\[ \frac{C}{C_0} = \frac{10400}{1150} \]
= 0.110576
Now base on Eq. No. (5) answer taking \( X_1 \) and \( Y_1 \) factor new value

(53) Taking \( X_1 \) and \( Y_1 \) factor value
X = 0.56
Y = 1.4

(54) We have Dynamic equivalent load
\[ W = (0.56 \times 1 \times 1150) + (1.4 \times 300) \]
W = 644 + 420
W = 1064 N

(55) Basic dynamic load rating
\[ C = \left( \frac{W \times L}{10^6} \right)^{1/K} \]
(Taking K=3 for ball bearing)
C = 1064 x 10.2598
C = 10916.45N
C = 10.92 kN \approx 11 kN
on table 27.6, 205 Number bearing to be selected

(56) Size calculation of bearing
dm = mean diameter of bearing (pitch diameter for ball)
dm = 107.5 mm.

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(57) db diameter of ball.
\[ db = 0.3 \times (120 - 95) \]
\[ db = 0.375 \text{ mm.} \]

(58) S radial thickness of race
\[ S = 0.15 \times (D - d) \]
\[ S = 0.375 \text{ mm.} \]

(59) \( r_1 \) radius of corner
\[ r_1 = 0.5 \times r_p \]
\[ r_1 = 3.8625 \text{ mm.} \]

(60) \( r_p \) radius of ball groove in race
\[ r_p = 0.515 \times db \]
\[ r_p = 3.8625 \text{ mm.} \]

(61) Z number of balls in bearing.
\[ Z = 2.9 \times \left( \frac{(D+d)+(D-d)}{2} \right) \]
\[ Z = 24.94 \text{ Nos., } \approx 25 \text{ Nos.} \]

This formula used for every degree which are lie between \( \Phi = 0 \) and \( \Phi = 180 \) degree.

H) Designing of Circular Spline Second stage.

[17][18][19][20][24][25]

Data given for 30:1 Ratio Second Stage Harmonic Drive

Number of teeth of circular spline (N) - 62
Diametral pitch (Dp) - 0.393700787 teeth/mm (10 teeth/inch)
Pressure Angle = 30° degree
Torque (Ts) = 1150 Nm

(62) Pitch Circle Diameter (p)
\[ p = \frac{\text{Number of teeth of circular spline}}{62} \]
\[ p = 0.393700787 \text{ mm.} \]

(63) Circular Pitch (c)
\[ c = \frac{\pi \times \text{Diametral Pitch}}{\pi} \]
\[ c = 7.9796 \text{ mm/teeth} \]

(64) Addendum (hf)
\[ hf = \frac{0.5}{\text{diametral pitch}} \]
\[ hf = 1.27 \text{ mm.} \]

(65) Base circle diameter (Bd)
\[ Bd = 157.5 \times \cos 30^\circ \]
\[ Bd = 136.4 \text{ mm.} \]

(66) Circular Tooth Thickness (Tt)
\[ Tt = \frac{2 \times \text{diametral pitch}}{3.142} \]
\[ Tt = 3.99 \text{ mm.} \]

(67) Major internal diameter (Mdi)
\[ Mdi = \frac{\text{Number of teeth + 1.8}}{\text{pitch circle diameter}} \]
\[ Mdi = 162.052 \text{ mm.} \]

(68) Minor external diameter (Mde)
\[ Mde = \frac{\text{pitch circle diameter}}{62 - 1.8} \]
\[ Mde = 0.393700787 \]
\[ Mde = 152.91 \text{ mm.} \]

(69) Internal Dedendum (Id)
\[ Id = \frac{\text{diametral pitch}}{0.900} \]
\[ Id = 0.393700787 \]
\[ Id = 2.286 \text{ mm.} \]

(70) External Dedendum (Ed)
\[ Ed = \frac{\text{diametral pitch}}{0.900} \]
\[ Ed = 0.393700787 \]
\[ Ed = 2.286 \text{ mm.} \]

(71) Effective spline Length (Le)
\[ Le = 31.25 \times \frac{1}{\left( \frac{0.900}{152.91} \right)^2} \]
\[ Le = 55.5 \text{ mm.} \]

(72) Shear stress (\( \tau \))
\[ \tau = \frac{1.64}{157.5} \]
\[ \tau = 1.64 \text{ N/mm}^2 \]

(73) Static Tooth load or beam strength or endurance strength calculation based on Lewis formula.
\[ Ws = \sigma_e \times b \times \pi \times m \times x \times y \]
\[ \sigma_e = 1.75 \times \text{B.H.N. (in MPa)} \]
\[ \sigma_e = 362.25 \text{ MPa} \]
\[ b = 55.41447 \]
\[ m = 2.5 \]
\[ y = 0.6 \]
\[ \pi = 3.141593 \]
\[ Ws = 362.25 \times 55.41447 \times 2.5 \times 3.141593 \times 0.6 \]
\[ Ws = 94595.9967 \text{ MPa} \]

Now as per Buckingham suggest formula for on pulsating load for Dynamic load Calculation
Pulsating loads = \( Ws \geq 1.35 \times Wd \)
\[ Wd = \frac{94595.9967}{1.35} \]
\[ Wd = 70071.10869 \text{ MPa} \]

Figure. 3. 3D Model of circular spline second stage

I) Designing of Flex Spline Second Stage.

[17][18][19][20][24][25]

Data given for 30:1 Ratio Second Stage Harmonic Drive
Number of teeth of flex spline (N) - 60
Diametral pitch (Dp) – 0.393700787 teeth/mm (10 teeth/inch)
Pressure Angle = 30° degree
Torque (Ts) = 1150 Nm
Root Diameter = p – 2 x hf = 152.4 – (2x2.286) = 147.83 mm.
Inside Diameter = 146.63 mm.
Bell Thickness = (147.83 – 139.85) ÷ 2 = 0.6 mm.
Mean Bell Diameter = 146.63 + 0.6 = 147.23 mm.
Mean Bed Radius = 147.23 ÷ 2 = 73.62 mm.

(74) Pitch Circle Diameter (p)
\[ p = \frac{\text{Number of teeth of flex spline}}{60} \]
p = 0.393700787
p = 152.4 mm.

(75) Circular pitch (c)
\[ c = \frac{\pi}{\text{Diametral Pitch}} \]
c = 7.9796 mm/teeth.

(76) Addendum circle (hf)
\[ hf = \frac{0.5 \times \text{diametral pitch}}{} \]
hf = 1.27 mm.

(77) Base circle diameter (Bd)
\[ Bd = 152.4 \times \cos 30° \]
Bd = 131.98 mm.

(78) Flexspline Tooth Thickness (Tt)
\[ Tt = \frac{\pi}{2 \times \text{diametral pitch}} \]
Tt = 3.99 mm.

(79) Major internal diameter (Mdi)
\[ Mdi = \frac{\text{Number of teeth} + 1.8}{\text{pitch circle diameter}} \]
Mdi = 156.972 mm.

(80) Minor external diameter (Mde)
\[ Mde = \frac{\text{Number of teeth} - 1.8}{\text{pitch circle diameter}} \]
Mde = 147.83 mm.

(81) Internal Dedendum (Id)
\[ Id = \frac{0.900}{\text{diametral pitch}} \]
Id = 2.286 mm.

(82) External Dedendum (Ed)
\[ Ed = \frac{0.900}{\text{diametral pitch}} \]
Ed = 2.286 mm.

(83) Effective spline Length (Le)
\[ Le = (147.83^2 \times \left[ 1 - \frac{0.4}{147.83^2} \right] / (10 \times 25.4)^2) \]
Le = 50.1 mm.

(84) Shear stress (τ)
\[ \tau = \frac{16 \times Ts}{\pi \times Dp^2 \times Le} \]
\[ \tau = \frac{16 \times 1150 \times 1000}{3.141593 \times (10 \times 25.4)^2 \times 50.1} \]
τ = 1.82 N/mm².

(85) Static Tooth load or beam strength or endurance strength calculation based on Lewis formula.
\[ Ws = \sigma_e \times b \times pc \times xy \]
Ws = 581 MPa (1 N/mm² = 1 MPa)
b = 50.0634
m = 2.5
y = 0.6
π = 3.141593
Wd = 137068.50 MPa

Now as per Buckingham suggest formula for on pulsating load for Dynamic load Calculation
Pulsating loads = Ws ≥ 1.35 Wd
Wd = 137068.50 ÷ 1.35
Wd = 101532.221 MPa

(86) Flex spline Deflection
\[ d = pc - pf \]
d = 157.48 – 152.4
d = 5.08

(87) Flex spline Deflection stress
\[ Sf = \frac{3E \times d \times Bt}{Db^2} \]
Sf = (3 x 207 x 10³ x 5.08 x 0.6) ÷ (147.23²)
Sf = 87.32 N/mm²

(88) Flex spline Load Stress
\[ St = \frac{T}{Db \times Ar} \]
St = 234.56 N/mm²

(89) Flex spline Tooth shear stress
\[ Ss = \frac{0.1 \times pc^2 \times Lw}{\pi} \]
Ss = (1150 x 10³) ÷ (0.1 x 152.4³ x 55.5)
Ss = 8.94 N/mm²

(90) Flex spline Bell Face Shear Stress
\[ Ssb = \frac{Rb \times A}{T} \]
A = 2 x π x Rb x Bt
Ssb = (1150000) ÷ (60.92 x 2 x π x 60.92 x 0.6)
Ssb = 56.283 N/mm²

(91) Flex spline Torsion Stress
\[ Ss = \frac{2 \times T \times r^3}{\pi (r_0^4 - r^4)} \]
Sst = 6.35 N/mm²

\( (92) \) Torsional Deflection of Flex spline

\[
V = \frac{6}{2 \times Glp} \left( L^2 - C \right)
\]

\[
V = \frac{6}{2 \times Glp} \left( L^2 - 25 \right)
\]

\[
V = \frac{6}{0.007874 \times 82 \times 10^{10} \times 0.007874 \times 8 \times 73.62^2 \times 0.5 \times 0.6}{0.0100} \text{mm}
\]

That is maximum twist is 0.010 inch in 55.5 mm or an average twist is approximately 0.0018 mm/mm.

Figure 4. 3D Model of flex spline second stage

J) Designing of wave generator Second Stage. \[17\] \[18\] \[19\] \[20\]

The designing of wave generator are based on flexspline and circular spline diameter and pressure angle and minimum number of teeth in contact this all will becomes on parameter consideration for wave generator designing. Data given

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Cs number of teeth</td>
<td>62</td>
<td>Nos.</td>
</tr>
<tr>
<td>2</td>
<td>Fs number of teeth</td>
<td>60</td>
<td>Nos.</td>
</tr>
<tr>
<td>3</td>
<td>Pressure Angle ( \alpha )</td>
<td>30°</td>
<td>Degree</td>
</tr>
<tr>
<td>4</td>
<td>Cs pitch circle diameter</td>
<td>157.5</td>
<td>mm.</td>
</tr>
<tr>
<td>5</td>
<td>Fs pitch circle diameter</td>
<td>152.4</td>
<td>mm.</td>
</tr>
<tr>
<td>6</td>
<td>Addendum</td>
<td>1.27</td>
<td>mm.</td>
</tr>
<tr>
<td>7</td>
<td>Thickness of Fs</td>
<td>3.99</td>
<td>mm.</td>
</tr>
<tr>
<td>8</td>
<td>Thickness of Cs</td>
<td>3.99</td>
<td>mm.</td>
</tr>
<tr>
<td>9</td>
<td>Effective length Teeth for Cs</td>
<td>55.5</td>
<td>mm.</td>
</tr>
<tr>
<td>10</td>
<td>Effective length teeth for Fs</td>
<td>50.1</td>
<td>mm.</td>
</tr>
<tr>
<td>11</td>
<td>Inside diameter of Fs</td>
<td>146.6</td>
<td>mm.</td>
</tr>
<tr>
<td>12</td>
<td>Working Diameter of wave generator</td>
<td>146.4</td>
<td>mm.</td>
</tr>
<tr>
<td>13</td>
<td>Major axis of cam (a)</td>
<td>75.82</td>
<td>mm.</td>
</tr>
<tr>
<td>14</td>
<td>Minor axis of cam (b)</td>
<td>70.61</td>
<td>mm.</td>
</tr>
<tr>
<td>15</td>
<td>Length of wave generator</td>
<td>55.5</td>
<td>mm.</td>
</tr>
</tbody>
</table>

For centre distance is formulated based when elliptical shape cam are inserted inside of flexspline and flexspline take shape as like elliptical as cam on that time minor center displacement is occurs.

\[
Ac = 2.5 x \left( \frac{(62-60)}{2} \right)
\]

\[
Ac = 2.5 (10Dp)
\]

\[
Ac = 2.5 \text{ Centre Distance / Eccentricity}
\]

The Distance formula is applicable at 2.5mm center distance deflection

\[
\left( \frac{X}{a} \right)^2 + \left( \frac{Y}{b} \right)^2 = 1
\]

Semi major axis for wave generator is

\[
a = r_{fg} + A
\]

\[
a = 73.32 + 2.5
\]

\[
a = 75.82
\]

\[
b = 70.61
\]

\( (93) \) Length of wave generator (Lw)

\[
Lw = \text{tooth length of flex spline} + 5.4 \text{ mm for circular spline tooth component}
\]

\( (94) \) Flexspline Deflection Force (F)

\[
F = \frac{0.56 \times \pi \times Lw \times Bt^3 \times E}{Rb^5}
\]

From harmonic drive design standards,

\[
F = \frac{(0.56 \times 5.08 \times 55.5 \times 0.6^3 \times 207 \times 10^3)}{(73.62^3)}
\]

\[
F = 17.69 \text{ N/mm}^2
\]

\[
F = 17.69 \div 55.5 \text{ (Length of wave generator)}
\]

\[
F = 0.320 \text{ N/mm}
\]

\( (95) \) Length of Supporting Arc (L)

\[
L = \pi \times Di \times \left( \frac{\alpha}{360} \right)
\]

\[
L = \pi \times 146.63 \times \left( \frac{45^\circ}{360^\circ} \right)
\]

\[
L = 57.58 \text{ mm.}
\]

The average oil pressure distribute over an arc of 30° before major axis and 15° behind it.

\( (96) \) Average Pressure (A)

\[
P = \frac{F}{A}
\]

\[
P = 0.320 \div (57.58 \times 0.0394)
\]

\[
P = 0.141 \text{ N/mm}^2
\]

The distributed pressure could higher than that of concentrated force so assume 1 N/mm² pressure.

\( (97) \) Maximum Pressure (Pmax)

\[
P_{max} = P_{average} \times \frac{\pi}{2}, \text{sunsoidal pressure distribution}
\]

\[
P_{max} = 1 \times (\pi/2)
\]

\[
P_{max} = 1.57 \text{ N/mm}^2
\]

\[
P_{max} = 1.6 \text{ N/mm}^2
\]

\( (98) \) Tooth Separating Forces (Ft)

From Harmonic drive Design Standard,

\[
1150 \times \frac{1000}{\tan 30^\circ}
\]

\[
Ft = 165.97 \text{ N}
\]

or, \( = 3.0 \text{ N/mm} \)

\( (99) \) Pressure to support Separating Load

separating load of 3.0 N/mm is distributed over the 30° arc and symmetric to major axis and sunsoidal pressure distribution

\[
L = \pi \times Di \times \left( \frac{\alpha}{360} \right)
\]

\[
L = \pi \times 146.63 \times \left( \frac{30^\circ}{360^\circ} \right)
\]

\( (100) \) Length of Supporting Arc (L)
L = 38.39 mm.

(101)  
\[ P = \frac{F}{A} \]
\[ = 3 \div (38.39 \times 0.0394) \]
\[ = 1.984 \text{ N/mm}^2 \]

(102)  
\[ \text{Maximum Pressure (Pmax)} \]
\[ P_{\text{max}} = 1.984 \times \left( \frac{\pi}{2} \right) \]
\[ P_{\text{max}} = 3.12 \text{ N/mm}^2 \]

(103)  
\[ \text{Pressure profile to develop output torque} \]
Pressure profile in which from the minor axis \( \Phi = 0 \) and major axis \( \Phi = \frac{\pi}{2} \). Will maintain the flex spline position when it’s run and generate necessary force as per requirement for output torque development.

(104)  
\[ \text{Wave generator Surface (La)} \]
\[ L_{a} = \frac{2a}{D} \times r \sin 2 \Phi \]

(105)  
\[ \text{R = D/d, from harmonic drive Design standard} \]

(106)  
\[ \text{T = K x r}^2 x L x \pi \]
\[ T = K \times 73.32^2 \times 55.5 \times \pi \]
\[ K = \frac{1150 \times 10^3}{73.32^2 \times 55.5 \times \pi} \]
\[ K = 1.23 \text{ N/mm}^2 \]

(107)  
\[ \text{Pressure at Major Axis (P)} \]
\[ P = K \times \Phi \]
\[ P = 1.23 \times (\pi/2) \]
\[ P = 1.93 \text{ N/mm}^2 \]

(108)  
\[ \text{Flex spline harmonic shape} \]
= (total deflection of flex spline) \div 2
= 5.08 / 2
=2.54 mm outside deflection at major axis
= inside deflection at minor axis = (137 / 149) x 2.54 = 2.34 mm,
Total Deflection = 2.54 + 2.34 = 4.88 mm,
\[ r \Phi = r_\Phi + \frac{d}{2} x (1 - \cos 2 \Phi) \]
\[ = 70.61 - 2.34 \]
\[ = 68.27 \text{ mm}, \]
\[ r_\Phi = 68.27 + [(4.88/2) \times (1 - \cos 2 \Phi)] \]
\[ r_\Phi = 65.83 \text{ mm.} \]
This formula used for every degree which are lie between to \( \Phi = 0 \) and \( \Phi = 180 \) degree.

K)  
\[ \text{Design calculation for Second stage bearing and wave plug}^{[8]} \]
Data given : -
Dynamic load on bearing \( F_{r} = 1150 \text{ N} \)
Axial load on bearing \( F_{a} = 300 \text{ N} \)
Rotation = 1500 RPM
Assume Average life = 5 year, at 8 hr.

(109)  
\[ \text{LH = 5 x 300 x 8} \]
(Assuming 300 working day in year)
\[ \text{LH = 5 x 300 x 8 = 12000 hours.} \]
Basic dynamic equivalent load

(110)  
\[ \text{Life of the bearing in Revolution} \]
\[ L = 60 \times N \times LH \]
\[ L = 60 \times 1500 \times 12000 \]
\[ L = 1080 \times 10^6 \text{ Rev.} \]

(111)  
\[ \text{The Basic dynamic equivalent radial load} \]
\[ W = X_{1} \times V \times F_{r} + Y_{1} \times F_{a} \]
\[ X = 0.56 \quad \text{Y} = 1 \]
\[ W = (0.56 \times 1 \times 1150) + (1 \times 300) \]
\[ W = 644 + 300 \]
\[ W = 944 \text{ N} \]

(112)  
\[ \text{We know that basic dynamic load rating.} \]
\[ C = C = \left[ W \times L / 10^6 \right]^{\frac{1}{K}} \] (Taking K=3 for ball bearing)
\[ C = 944 \times 10.2598 \]
\[ C = 9685.3016 \text{ N} \]
\[ C = 9.69 \text{ kN} \approx 10 \text{ kN} \]

(113)  
\[ \text{From R.S. Khurmi Table 27.6 at page 1014 select bearing 305 Number} \]
at 305 Number bearing
Static load rating \( C_{0} = 10.4 \text{ kN} \)
Dynamic load rating \( C = 16.6 \text{ kN} \)
\[ = \frac{1150}{10400} \]
\[ = 0.110576 \]
Now base on Eq. No. (5) answer taking \( X_{1} \) and \( Y_{1} \) factor new value

(114)  
\[ \text{Taking X}_{1} \text{ and Y}_{1} \text{ factor value} \]
\[ X_{1} = 0.56 \]
\[ Y_{1} = 1.4 \]

(115)  
\[ \text{We have Dynamic equivalent load} \]
\[ W = (0.56 \times 1 \times 1150) + (1.4 \times 300) \]
\[ W = 644 + 420 \]
\[ W = 1064 \text{ N} \]

(116)  
\[ \text{Basic dynamic load rating} \]
\[ C = C = \left[ W \times L / 10^6 \right]^{\frac{1}{K}} \] (Taking K=3 for ball bearing)
\[ C = 1064 \times 10.2598 \]
\[ C = 10916.45 \text{ N} \]
\[ C = 10.92 \text{ kN} \approx 11 \text{ kN} \]
on table 27.6, 205 Number bearing to be selected

(117)  
\[ \text{Size calculation of bearing} \]
\[ d_{m} = \text{mean diameter of bearing (pitch diameter for ball)} \]
\[ d_{m} = (D+d) / 2 \]
\[ d_{m} = \Omega 132.5 \text{ mm.} \]

(118)  
\[ \text{db diameter of ball.} \]
\[ d_{b} = 0.3 \times (D - d) \]
\[ d_{b} = 0.3 \times (145 - 95) \]
\[ d_{b} = \Omega 7.5 \text{ mm.} \]

(119)  
\[ \text{S radial thickness of race} \]
\[ S = 0.15 \times (D - d) \]
\[ S = 0.15 \times (145 - 120) \]
\[ S = 3.75 \text{ mm.} \]

(120)  
\[ \text{r}_{1} \text{ radius of corner} \]
\[ r_{1} = 0.5 \times r_{p} \]
\[ r_{1} = 0.5 \times 3.8625 \]
\[ r_{1} = 1.94 \text{ mm.} \]
Based on study and research work I have calculated harmonic drive design parameter which are almost comes to near about standard design harmonic drive. All the equation as per harmonic standard design which fully satified the design criterio for designing harmonic drive. All calculation are full fill design parameter of harmonic drive geometry calculation and stress developed on drive which are proven by the equation. Further the analysis are pending for 100% surety of designing of harmonic drive. This paper is only calculation based it’s analysis are pending.

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Appendix –I

Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>p</td>
<td>Pitch circle diameter</td>
</tr>
<tr>
<td>c</td>
<td>Circular pitch</td>
</tr>
<tr>
<td>Dp</td>
<td>Diametral pitch</td>
</tr>
<tr>
<td>hf</td>
<td>Addendum Circle</td>
</tr>
<tr>
<td>Bd</td>
<td>Base circle diameter</td>
</tr>
<tr>
<td>α</td>
<td>Pressure angle</td>
</tr>
<tr>
<td>Tt</td>
<td>Circular teeth thickness</td>
</tr>
<tr>
<td>Mdi</td>
<td>Major internal diameter</td>
</tr>
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<td>Mde</td>
<td>Minor external diameter</td>
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<td>Internal dedendum</td>
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