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On Supra Regular Generalized Star Star b- Closed Continuous and Irresolute Functions and $rg^{**}b^{\mu}$-Spaces in Supra Topological Spaces

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Abstract:
In this paper supra regular generalized star star b- closed continuous functions are introduced in supra topological spaces and compared with existing continuous functions in supra topological spaces. Further $rg^{**}b^{\mu}$- irresolute functions and applications on $rg^{**}b^{\mu}$-closed sets are also discussed.

Keywords: $rg^{**}b^{\mu}$ closed set, $rg^{**}b^{\mu}$- continuous, $rg^{**}b^{\mu}$- irresolute.

I. INTRODUCTION:

In 1970,levine[5],introduced the concept of generalized closed set and discussed the properties of sets, closed and open , maps, compactness, normal and separation axioms. In 1983 Mashhour et al [7] introduced supra topological spaces and studied S-continuous maps and S*-continuous maps. In 2008, Devi et al [3] introduced and studied a class of sets called supra α-open and a class of maps called s α –continuous between topological spaces, respectively. In 2010, Sayed and Noiri [9] introduced and studied a class of sets called supra b-open and a class of maps called supra b-continuous. Ravi et al [8] introduced and studied a class of sets called supra g-closed and a class of maps called supra g-continuous respectively. In 2013, Indirani and Sindhu [4] introduced a new class of set called, Regular generalized star b-closed set in topological spaces. In 2014, Banupriya and Indirani [1], introduced Regular generalized star star b-closed sets in topological spaces. In 2015, Chinnapparaj, Sathishmohan, Rajendran, Indirani [2], introduced a new class of sets called, Supra regular generalized star b-closed set. In 2016, Ludi Jancy Jenifer and Indirani [6] introduced Supra regular generalized star star b- closed set in Supra topological spaces. In this paper we introduce supra regular generalized star star b- closed continuous and study some of its properties in supra topological spaces. Further $rg^{**}b^{\mu}$- irresolute functions are also discussed.

II. PRELIMINARIES

Definition 2.1:[7] Let X be a non empty set. The subfamily $\mu \subseteq \wp(X)$ where $\wp(X)$ is the power set of X is said to be a supra topology on X if $X \in \mu$, $\emptyset \in \mu$ and μ is closed under arbitrary unions. The pair (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X,μ). Complements of supra open sets are called supra closed sets.

Definition 2.2[7]: Let A be a subset of (X, μ). Then the supra closure of A is denoted by
$cl^{\mu}(A) = \cap \{ B / B$ is a supra closed set and $A \subseteq B\}$.

Definition 2.3[7]: Let A be a subset of (X, μ). Then the supra interior of A is denoted by
$int^{\mu}(A) = \cup \{ B / B$ is a supra open set and $A \supseteq B\}$.

Definition 2.4[7]: Let (X, μ) be a topological space and μ be a supra topology on X. μ is supra topology associated with τ if $\tau \subseteq \mu$.

Definition 2.5: A function $f:(X,\mu)\to(Y,\sigma)$ is called $rg^{**}b^{\mu}$-continuous if $f^{-1}(V)$ is $rg^{**}b^{\mu}$-closed set of (X,μ) for every closed set V of (Y,σ).

III. $rg^{**}b^{\mu}$*- Continuous*

Theorem 3.1: Let $f:(X,\mu)\to(Y,\sigma)$ be a map.Then the following conditions are equivalent:
(i)f is $rg^{**}b^{\mu}$-continuous.
(ii)The inverse image of each supra open set in Y is $rg^{**}b^{\mu}$-open in X.
(iii) $f(rg^{**}bcl^{\mu}(A)) \subseteq cl^{\mu}(f(A))$ for each subset A of X.
(iv)For each subset B of Y, $rg^{**}bcl^{\mu}(f^{-1}(B)) \subseteq f^{-1}(cl^{\mu}(B))$.

Proof:

(i)⇒(ii)
Let G be an supra open set in Y. Then Y\G is supra closed in Y.By hypothesis,
$f^{-1}(Y\setminus G)= X\setminus f^{-1}(G)$ is $rg^{**}b^{\mu}$-closed in X.Therefore $X\setminus f^{-1}(G)$ is $rg^{**}b^{\mu}$-closed in X.Hence $f^{-1}(G)$ is $rg^{**}b^{\mu}$-open in X.

(ii)⇒(i)
Let G be an supra closed set in Y. Then Y\G is supra open in Y.By hypothesis,
$f^{-1}(Y\setminus G)=X\setminus f^{-1}(G)$ is $rg^{**}b^{\mu}$-open in X.Therefore $X\setminus f^{-1}(G)$ is $rg^{**}b^{\mu}$-open in X.Hence $f^{-1}(G)$ is $rg^{**}b^{\mu}$-closed in X.Hence f is $rg^{**}b^{\mu}$-continuous.

(i) ⇒(iii)

Let A be a subset of X.Since A $\subseteq f^{-1}$ (f(A)) and f(A) $\subseteq cl^{\mu}$ (f(A)) we have A $\subseteq f^{-1}$ (f(A)) $\subseteq f^{-1}$ (cl^{μ} (f(A))).Therefore by assumption, $f^{-1}(cl^{\mu}$ (f(A))) is $rg^{**}b^{\mu}$-closed set of X.Hence $rg^{**}bcl^{\mu}(A) \subseteq f^{-1}(cl^{\mu}$ (f(A))).

Thus, f($rg^{**}bcl^{\mu}(A)) \subseteq f(f^{-1}$ (cl^{μ} (f(A)))) $\subseteq cl^{\mu}$ (f(A)).

(iii)⇒(iv)

Let B be a subset of Y and f^{-1}(A)=B. So by assumption, f($rg^{**}bcl^{\mu}(A)) = f(rg^{**}bcl^{\mu}(f^{-1}$(B))).

Therefore, $rg^{**}bcl^{\mu}(f^{-1}$ (B)) $\subseteq f^{-1}(f(rg^{**}bcl^{\mu}(f^{-1}(B)))) \subseteq f^{-1}(cl^{\mu}(B))$.

(iv)⇒(i)

Let B be closed set in Y. Then by assumption, $rg^{**}bcl^{\mu}(f^{-1}$(B)) $\subseteq f^{-1}(cl^{\mu}(B))= f^{-1}(B)$.Therefore $f^{-1}(B)$ is $rg^{**}b^{\mu}$-closed set of X. Hence f is $rg^{**}b^{\mu}$-continuous.

Theorem 3.2: Let f : X → Y be a function where X and Y are supra topological spaces. Then the following are equivalent:

1) f is $rg^{**}b^{\mu}$-continuous.

2) For each point x∈X and each supra open set V in Y with f(x)∈V, there is a $rg^{**}b^{\mu}$-open set U in X such that x ∈ U and f(U)⊆V.

Proof: Obvious.

Theorem 3.3:

(i) Every supra continuous map is $rg^{**}b^{\mu}$-continuous.

(ii) Every supra semi- continuous map is $rg^{**}b^{\mu}$ -continuous.

(ii) Every supra $\alpha -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(iii) Every supra pre $-$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(iv) Every supra regular- continuous map is $rg^{**}b^{\mu}$ -continuous.

(v) Every supra $b -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(vi) Every supra $g -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(vii) Every supra $sg -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(viii) Every supra $gs -$continuous map is $rg^{**}b^{\mu}$-continuous.

(ix) Every supra $g\alpha -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(x) Every supra $\alpha g -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(xi) Every supra $gp -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(xii) Every supra $gr -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(xiii) Every supra $g^{*} -$continuous map is $rg^{**}b^{\mu}$-continuous.

(xiv) Every supra $g^{*}s -$continuous map is $rg^{**}b^{\mu}$-continuous.

(xv) Every supra $g^{\#} -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

(xvi) Every supra $g^{\#}s -$continuous map is $rg^{**}b^{\mu}$-continuous.

(xvii) Every supra $rg^{*}b -$ continuous map is $rg^{**}b^{\mu}$-continuous.

(xviii) Every supra $g^{*}b -$continuous map is $rg^{**}b^{\mu}$-continuous.

(xix) Every supra $g\alpha b -$continuous map is $rg^{**}b^{\mu}$-continuous.

(xx) Every supra s$gb -$ continuous map is $rg^{**}b^{\mu}$ -continuous.

Proof:

(i) Let f:(X,μ)→(Y,σ) be a supra continuous map.Let V be a supra closed set in (Y,σ).Since f is supra continuous,$f^{-1}(V)$ is supra closed in (X,μ).Also every supra closed set is $rg^{**}b^{\mu} -$closed set, $f^{-1}(V)$ is $rg^{**}b^{\mu} -$closed set in (X,μ).Therefore f is $rg^{**}b^{\mu}$-continuous map.

Proof is obvious for others.

Remark 3.4: The converse of the above theorems need not be true as shown in the examples below.

Example 3.5:

(i). Let X=Y={a,b,c} with μ_1={X,∅,{a,b} ,{b,c}} and μ_2 ={Y,∅,{c,b},{a,c}}.Let f:(X, μ_1) → (Y, μ_2) be an identity map. Hence f is $rg^{**}b^{\mu}$-continuous but not supra continuous. Since for a supra closed set {b} in Y, f^{-1}({b})={ b} is $rg^{**}b^{\mu}$-closed but not supra closed in (X,μ_1).

(ii) Let X=Y={a,b,c,d} with μ_1={X,∅,{b}, {c}, {b,c}, {a,b,c}, {a,c,d}} and μ_2 ={Y, ∅ ,{b},{b,c},{b,d},{c,d},{b,c,d}}.Let f:(X, μ_1) → (Y, μ_2)be defined as f(a)=c,f(b)=b,f(c)=d,f(d)=a. Hence f is $rg^{**}b^{\mu}$-continuous but not supra semi-continuous. Since for a supra closed set {a,d} in Y,f^{-1}({a,d})={c,d} is $rg^{**}b^{\mu}$-closed but not supra semi-closed in (X,μ_1).

(iii). Let X=Y={a,b,c,d} with μ_1 ={X, ∅ ,{b}, {d},{b,d}{b,c,d},{a,c,d}}and μ_2={Y,∅,{c},{b,c},{a,c},{a,b},{a,b,c}} .Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=d,f(b)=b,f(c)=c,f(d)=a. Hence f is $rg^{**}b^{\mu}$ -continuous but not supra-α-continuous. Since for a supra closed set {a,b,d} in Y,f^{-1}({a,b,d})={a,b,d} is $rg^{**}b^{\mu}$-closed but not supra α-closed in (X,μ_1).

(iv)Let X=Y={a,b,c,d} with μ_1 ={X, ∅ ,{b}, {c}, {b,c}, {a,b,c}, {a,b,d}} and μ_2={Y, ∅ , {b}, {d}, {b,d}, {a,c,d}, {b,a,d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=b,f(b)=d,f(c)=a,f(d)=c.

Hence f is $rg^{**}b^{\mu}$-continuous but not supra pre-continuous. Since for a supra closed set {a,c,d} in Y,f^{-1}({a,c,d})={b,c,d} is $rg^{**}b^{\mu}$-closed but not supra pre-closed in (X,μ_1).

(v). Let X=Y={a,b,c,d} with μ_1 ={X, $\emptyset$,{a}, {b},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d}, {a,b,c}, {a,c,d}, {b,c,d},{a,b,d}}and μ_2={Y,$\emptyset$,{a},{a,b},{b,c},{c,a},{a,b,c}}. Let f:(X, μ_1) →(Y, μ_2) be defined as f(a) =a, f(b) =b, f(c) =c, f(d)=d. Hence f is $rg^{**}b^{\mu}$ -continuous but not supra r-continuous. Since for a supra closed set {d} in Y,f^{-1} ({d}) = {d} is $rg^{**}b^{\mu}$-closed but not supra r-closed in (X,μ_1).

(vi). Let X=Y={a,b,c,d} with μ_1={X,$\emptyset$,{a}, {b}, {a,b} {a,b,d},{b,c,d}} and μ_2={Y,$\emptyset$,{a},{a,b},{a,d},{b,c},{a,b, d},{a,b,c}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=c,f(b)=b,f(c)=d,f(d)=a.
Hence f is $rg^{**}b^{\mu}$-continuous but not supra b-continuous. Since for a supra closed set {b,c} in Y,f^{-1}({b,c})={b,c} is $rg^{**}b^{\mu}$-closed but not supra b-closed in (X,μ_1).

(vii). Let X=Y={a,b,c,d} with μ_1 ={X,$\emptyset$,{c}, {d}, {c,d} {a,b,c}, {a,c,d}}and μ_2 ={Y, $\emptyset$,{c},{c,d},{b,c},{b,d},{b,c ,d}}. Let f:(X, μ_1) →(Y, μ_2) be defined as f(a)=a, f(b)= b,f (c)=c,f(d)=d. Hence f is $rg^{**}b^{\mu}$-continuous but not supra g-continuous. Since for a supra closed set {a,d} in Y,f^{-1}({a,d})={a,d} is $rg^{**}b^{\mu}$-closed but not supra g-closed in (X,μ_1).

(viii). Let X=Y={a,b,c,d} with μ_1 ={X, $\emptyset$,{a} ,{b}, {a,b},{a,d},{b,d},{a,b,d}}and
μ_2={Y,$\emptyset$,{a},{a,c},{a,d},{c,d},{a,c, d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=d,f(b)=c,f(c)=b,f(d)=a. Hence f is $rg^{**}b^{\mu}$-continuous but not supra sg-continuous. Since for a supra closed set {b,c,d} in Y, f^{-1} ({b,c,d})={a,b,c} is $rg^{**}b^{\mu}$-closed but not supra sg-closed in (X,μ_1).

(ix).Let X=Y={a,b,c,d} with μ_1={X,$\emptyset$,{d}, {a,b}, {b,d} {a,b,d}} and μ_2={Y,$\emptyset$,{a} ,{b}, {a,b},{a,b,d},{a,c,d}} .Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a, f(b)=b, f(c)=c,f(d)=d. Hence f is $rg^{**}b^{\mu}$-continuous but not supra gs-continuous. Since for a supra closed set {b} in Y,f^{-1}({b})={b} is $rg^{**}b^{\mu}$-closed but not supra gs-closed in (X,μ_1).

(x). Let X=Y={a,b,c,d} with μ_1={X,$\emptyset$,{a} ,{a,b}, {b,d}, {a,b,d}} and μ_2={Y,$\emptyset$,{d} ,{b,c}, {c,d},{b,d},{b,c,d}}.Let f:(X, μ_1) →(Y, μ_2) be defined as f(a)=a, f(b)=b, f(c)=c,f(d)=d. Hence f is $rg^{**}b^{\mu}$-continuous but not supra g α -continuous. Since for a supra closed set {a} in Y, f^{-1} ({a})={a} is $rg^{**}b^{\mu}$ -closed but not supra -gα −closed in (X,μ_1).

(xi).Let X=Y={a,b,c,d} with μ_1={X,$\emptyset$,{a}, {d}, {a,b} {b,d}, {a,d}, {a,b,d} }and μ_2={Y,$\emptyset$,{d},{a,c}, {a,d} ,{c,d},{a,c ,d}} Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a,f (b)=b,f(c)=c,f(d)=d. Hence f is $rg^{**}b^{\mu}$-continuous but not supra$-\alpha g$-continuous. Since for a supra closed set {b,d} in Y,f^{-1}({b,d})={b,d} is $rg^{**}b^{\mu}$-closed but not supra -αg-closed in (X,μ_1).

(xii). Let X=Y={a,b,c,d} with μ_1 ={X,$\emptyset$,{a} ,{a,c}, {a,d} {c,d},{a,c,d}} and μ_2 ={Y,$\emptyset$, {c}, {d}, {c,d},{a,b,c},{b, c,d},{a,b,d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-gp$ -continuous. Since for a supra closed set {a} in Y,f^{-1}({a})={a} is $rg^{**}b^{\mu}$-closed but not supra -gp-closed in (X,μ_1).

(xiii). Let X=Y={a,b,c,d} with μ_1 ={X, $\emptyset$,{b} ,{d} , {a,b},{b,d},{a,d},{a,b,d}} and μ_2= {Y,$\emptyset$, {c}, {a,c}, {a,d}, {c,d}, {a,c ,d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=b,f(b)=d,f(c)=a,f(d)=c.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-gr$ -continuous. Since for a supra closed set {b} in Y,f^{-1}({b})={a} is $rg^{**}b^{\mu}$-closed but not supra -gr-closed in (X,μ_1).

(xiv). Let X=Y={a,b,c,d} with μ_1={X,$\emptyset$,{a}, {d}, {a,d}, {a,b,c}, {a,c,d}} and μ_2 ={Y, $\emptyset$,{a},{d}, {a,d} ,{a,b,c}, {a,b,d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a, f(b)=b, f(c)=c,f(d)=d.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-g^{*}$ -continuous. Since for a supra closed set {c} in Y,f^{-1}({c})={c} is $rg^{**}b^{\mu}$-closed but not supra -g^{*}-closed in (X,μ_1).

(xv) Let X=Y={a,b,c,d} with μ_1 ={X,$\emptyset$,{c}, {d}, {c,d}, {a,b,c},{a,b,d}} and μ_2 ={Y, $\emptyset$,{a}, {c}, {d},{a,c},{a,d},{b,c},{b,d} ,{c,d}, {a,b,c}, {a,c,d}, {b,c,d}, {a,b,d}}. Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-g^{*}s$ -continuous. Since for a supra closed set {b,c,d} in Y, f^{-1}({b,c,d})={b,c,d} is $rg^{**}b^{\mu}$-closed but not supra -$g^{*}s$-closed in (X,μ_1).

(xvi). Let X=Y={a,b,c,d} with μ_1={X,$\emptyset$, {b}, {a,b}, {b,d}, {a,d},{a,b,d}} and
μ_2={Y,$\emptyset$,{a},{a,b},{b,d},{a,d},{a,b,d} }.
Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-g^{\#}$ -continuous. Since for a supra closed set {b,c,d} in Y,f^{-1}({b,c,d})={b,c,d} is $rg^{**}b^{\mu}$-closed but not supra -$g^{\#}$-closed in (X,μ_1).

(xvii). Let X=Y={a,b,c,d} with μ_1 ={X, $\emptyset$,{a} ,{a,b}, {b,d},{a,d},{a,b,d}} and
μ_2={Y,$\emptyset$,{b},{a,b},{a,d},{b,d},{a,b, d}}.
Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a, f(b)=b, f(c)=c,f(d)=d.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-g^{\#}s$ -continuous. Since for a supra closed set {a,c,d} in Y, f^{-1}({a,c,d})={a,c,d} is $rg^{**}b^{\mu}$-closed but not supra -$g^{\#}s$-closed in (X,μ_1).

(xviii). Let X=Y={a,b,c,d} with μ_1={X,Ø,{b}, {c}, {b,c}, {a,c,d},{b,c,d}}and μ_2={Y,Ø,{b},{a,c},{a,b}, {b,c},{a,b,c}} Let f:(X, μ_1) →(Y, μ_2) be defined as f(a)=d, f(b)=a, f(c)=c, f(d)=b. Hence f is $rg^{**}b^{\mu}$-continuous but not supra−$rg^{*}b$-continuous. Since for a supra closed set {c,d} in Y,f^{-1}({c,d})={c,a} is $rg^{**}b^{\mu}$-closed but not supra -$rg^{*}b$-closed in (X,μ_1).

(xix). Let X=Y={a,b,c,d} with μ_1= {X ,Ø, {a}, {d}, {a,d}, {a,b,c},{b,c,d}} and μ_2 ={Y, Ø ,{b},{c},{b,c}, {a,b,c}, {a,c,d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a ,f(b)=b, f(c)=d,f(d)=c.
Hence f is $rg^{**}b^{\mu}$-continuous but not supra-$g^{*}b$-continuous. Since for a supra closed set {a,c,d} in Y,f^{-1}({a,c,d})={a,c,d} is $rg^{**}b^{\mu}$-closed but not supra -$g^{*}b$-closed in (X,μ_1).

(xx). Let X=Y={a,b,c,d} with μ_1={X,Ø,{a}, {c}, {a,c}, {a,b,c}, {a,b,d}}and μ_2 ={Y, Ø ,{b},{c},{b,c},{a,c,d},{a,b, d}}. Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a,f(b)=b,f(c)=c,f(d)=d.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-g\alpha b$ -continuous. Since for a supra closed set {a,c,d} in Y,f^{-1}({a,c,d})={a,c,d} is $rg^{**}b^{\mu}$ -closed but not supra -$g\alpha b$-closed in (X,μ_1).

(xxi). Let X=Y={a,b,c,d} with μ_1={X,Ø,{d} ,{a,b}, {b,d}, {a,d},{a,b,d}} and μ_2={Y,Ø,{b},{c},{b,c},{a,c,d},{a, b,d}}. Let f:(X, μ_1) →(Y, μ_2) be defined as f(a)=a, f(b)=b, f(c)=d, f(d)=c.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-sgb$ -continuous. Since for a supra closed set {a,c,d} in Y,f^{-1}({a,c,d})={a,c,d} is $rg^{**}b^{\mu}$ -closed but not supra -sgb-closed in (X,μ_1).

Remark 3.6: $rg^{**}b^{\mu}$-continuity is independent from rg^{μ}-continuity and gpr^{μ}-continuity

Example 3.7:

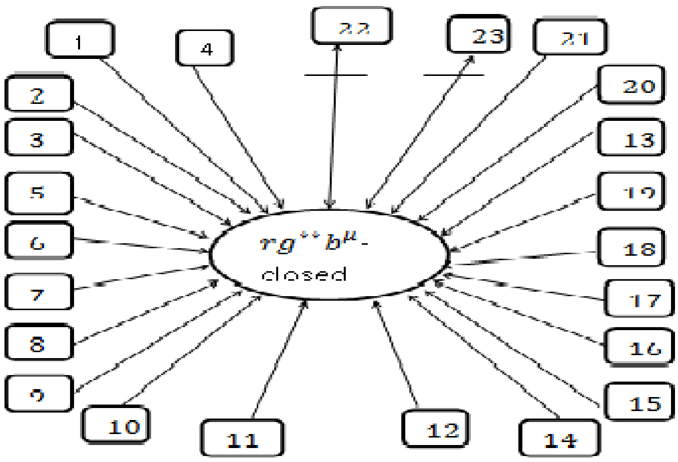
LetX=Y={a,b,c,d,e}with μ_1={X,Ø,{e}, {d}, {b,c}, {c,d}, {e,d},{b,c,e},{c,d,e},{b,c,d},{a,b,c,e}, {a,c,d,e},{b,c,d,e}}and μ_2={Y,Ø,{c},{d},{c,d},{a,b},{b,c},{a,b,c}, {a,b,d},{c,b,d}, {a,b,c,d},{a,b,d,e},{c,b,d,e,}}.Let f:(X, μ_1) →(Y, μ_2) be defined as f(a)=a,f(b)=b,f(c)=c, f(d)=d,f(e)=e.
Hence f is $rg^{**}b^{\mu}$ -continuous but not supra $-gpr$ -continuous. Since for a supra closed set {e} in Y,f^{-1}({e})={e} is $rg^{**}b^{\mu}$ -closed but not supra -gpr-closed in (X,μ_1) and f^{-1}({c,e})={c,e} is supra gpr-closed but not $rg^{**}b^{\mu}$-closed in (X,μ_1).

Example 3.8:

LetX=Y={a,b,c,d,e}with
μ_1={X,Ø,{e},{d},{e,d},{c,d},{c,e},{b,c,d},{b,c,e},{c,d,e},{a ,b,c,d},{a,b,c,e},{b,c,d,e}} and μ_2={Y,Ø, {b}, {c}, {b,c}, {c,e},{a,c,e},{a,b,e},{c,b,e},{a,b,c,e}{c,b,d,e}}.
Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=a,f(b)=b, f(c)=c,f (d)=d ,f(e)=e. Hence f is $rg^{**}b^{\mu}$ -continuous but not supra−rg-continuous. Since for a supra closed set {a,d,e} in Y,f^{-1}({a,d,e})={a,d,e} is $rg^{**}b^{\mu}$-closed but not supra -rg-closed in (X,μ_1) and f^{-1}({c,d})={c,d} is supra rg-closed but not $rg^{**}b^{\mu}$-closed in (X,μ_1).

Table.1. From the above discussions we have the following implications:

1.supra continuous
2.s^{μ} - continuous
3.α^{μ} - continuous
4.supra pre-continuous
5.r^{μ} - continuous
6.b^{μ} - continuous
7.g^{μ} –continuous
8.sg^{μ} –continuous
9.gs^{μ} –continuous
10.gα^{μ} –continuous
11.αg^{μ} –continuous
12.gp^{μ} –continuous
13.gr^{μ} continuous
14.$g^{*\mu}$ –continuous
15.$g^{*}s^{\mu}$ –continuous
16.$g^{\#\mu}$ –continuous
17.$g^{\#}s^{\mu}$-continuous
18.r$g^{*}b^{\mu}$-continuous
19.$g^{*}b^{\mu}$-continuous
20.$g\alpha b^{\mu}$-continuous
21.sgb^{μ}-continuous
22.rg^{μ}-continuous
23.gpr^{μ}-continuous



Remark 3.9:
The composition of two $rg^{**}b^{\mu}$ -continuous functions need not to be a $rg^{**}b^{\mu}$-continuous function in general as seen from the following example:

Example 3.10:

Let X = Y = Z = {a, b, c,d}, μ_1= {X,ϕ, {b}, {a,b}, {b,d}, {a,d}, {a,b,d}},μ_2 = {Y,ϕ,{a}, {a,b},{a,d},{b,d},{a,b,d}}, and μ_3 ={Z,ϕ,{d}, {a,b},{b,d},{a,d},{a,b,d}}. Let f : (X, μ_1) → (Y, μ_2) and g : (Y, μ_2) → (Z, μ_3) be identity maps.

Then f and g are $rg^{**}b^{\mu}$-continuous but g ∘ f: (X, μ_1) →(Z, μ_3) is not $rg^{**}b^{\mu}$-continuous, since the subset {b, c} is supra closed in (Z, μ_3) but $(g \circ f)^{-1}$({b, c}) = {b, d} is not $rg^{**}b^{\mu}$-closed in (X, μ_1).

IV. $rg^{**}b^{\mu}$-Irresolute functions

Definition 4.1: A function f:(X,μ)→(Y,σ) is called $rg^{**}b^{\mu}$-irresolute if f^{-1}(V) is $rg^{**}b^{\mu}$-closed in X for every $rg^{**}b^{\mu}$-closed set V of Y.

Theorem 4.2: A function f : X → Y is $rg^{**}b^{\mu}$-irresolute if and only if f^{-1}(V) is $rg^{**}b^{\mu}$-open in X for every $rg^{**}b^{\mu}$-open set V in Y.

Proof: obvious.

Theorem 4.3: Every $rg^{**}b^{\mu}$-irresolute map is $rg^{**}b^{\mu}$-continuous but not conversely.

Proof: obvious.

Example 4.4:

Let X=Y={a,b,c,d} with

μ_1={X,∅,{a,c},{a,d},{b,c},{a,c,d},{a,b,c}}

and μ_2={Y, ∅,{b},{a,b}, {a,d},{b,c} ,{a,b,c},{a,b ,d}}.Let f:(X, μ_1) → (Y, μ_2) be defined as f(a)=d,f(b)=c,f(c)=b,f(d)=a. Hence f is $rg^{**}b^{\mu}$-continuous but not $rg^{**}b^{\mu}$-irresolute. Since for a supra closed set {b,c,d} in Y,f^{-1}({b,c,d})={a,b,c} is not $rg^{**}b^{\mu}$-closed in (X,μ_1).

Theorem 4.5: If f : X → Y and g : Y → Z are both $rg^{**}b^{\mu}$-irresolute, then g∘f : X → Z is also $rg^{**}b^{\mu}$-irresolute.

Proof: Obvious.

Theorem 4.6: Let X, Y and Z be any supra topological spaces. For any $rg^{**}b^{\mu}$-irresolute function f : X → Y and for any $rg^{**}b^{\mu}$-continuous function g : Y → Z, the composition $g \circ f$: X → Z is is $rg^{**}b^{\mu}$-continuous.

Proof: obvious.

Theorem 4.7: Let f: X → Y be a function, where X and Y are supra topological spaces. Then the following are equivalent:

1) f is $rg^{**}b^{\mu}$-irresolute.

2) For each point x∈X and each $rg^{**}b^{\mu}$-open set V in Y with f(x)∈V, there is a $rg^{**}b^{\mu}$ open set U in X such that x∈ U and f(U)⊆V.

Proof: obvious.

V. APPLICATIONS

As an application of $rg^{**}b^{\mu}$ closed sets, new spaces namely, $T^{\mu}_{rg^{**}b}$ –space, $_gT^{\mu}_{rg^{**}b}$ –space, $_{rg^{*}b}T^{\mu}_{rg^{**}b}$ – space, $_{gs}T^{\mu}_{rg^{**}b}$ -space, $_bT^{\mu}_{rg^{**}b}$ –space are introduced. We introduce the following definitions:

Definition 5.1:

(i) A supra topological space (X,μ) is called a $T^{\mu}_{rg^{**}b}$ –space if every $rg^{**}b^{\mu}$ closed set is supra closed.

(ii) A supra topological space (X,μ) is called a $_gT^{\mu}_{rg^{**}b}$ -space if every $rg^{**}b^{\mu}$ closed set is g^{μ} closed.

(iii) A supra topological space (X,μ) is called a, $_{rg^{*}b}T^{\mu}_{rg^{**}b}$ -space if every $rg^{**}b^{\mu}$ closed set is $rg^{*}b^{\mu}$ closed.

(iv) A supra topological space (X,μ) is called a $_{gs}T^{\mu}_{rg^{**}b}$ -space if every $rg^{**}b^{\mu}$ closed set is gs^{μ} closed.

(v) A supra topological space (X,μ) is called a $_bT^{\mu}_{rg^{**}b}$ -space if every $rg^{**}b^{\mu}$ closed set is b^{μ} closed.

Theorem 5.2:

(i) Every $T^{\mu}_{rg^{**}b}$ -space is $_gT^{\mu}_{rg^{**}b}$ –space.

(ii) Every $T^{\mu}_{rg^{**}b}$ -space is $_{rg^{*}b}T^{\mu}_{rg^{**}b}$ –space.

(iii) Every $T^{\mu}_{rg^{**}b}$ -space is $_{gs}T^{\mu}_{rg^{**}b}$ –space.

(iv) Every $T^{\mu}_{rg^{**}b}$ -space is $_bT^{\mu}_{rg^{**}b}$ –space.

Remark 5.3:

The converse of the above theorem need not be true as seen from the examples below.

Example 5.4:

(i)Let X={a,b,c,d}

and μ={X,∅,{a},{b},{a,b},{b,c,d},{a,c,d}}.

Every $rg^{**}b^{\mu}$ closed set is g^{μ} closed but {c},{d},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,c,d} are $rg^{**}b^{\mu}$ closed set but not supra closed. Therefore a $_gT^{\mu}_{rg^{**}b}$ –space need not be a $T^{\mu}_{rg^{**}b}$ –space.

(ii). Let X={a,b,c} and μ={X,∅,{a},{a,b},{b,c}}. Every $rg^{**}b^{\mu}$ closed set $rg^{*}b^{\mu}$ is closed but {b},{a,c} are $rg^{**}b^{\mu}$ closed set but not supra closed. Therefore a $_{rg^{*}b}T^{\mu}_{rg^{**}b}$ –space need not be a $T^{\mu}_{rg^{**}b}$ –space.

(iii).Let X={a,b,c,d} and μ ={X, ∅ ,{a},{d},{a, d},{a,b,c},{b,c,d}}. Every $rg^{**}b^{\mu}$ closed set gs^{μ} is closed but {b},{c},{a,b},{a,c} ,{b,c},{b,d} ,{c,d},{a,c,d},{a,b,d} are $rg^{**}b^{\mu}$ closed set are not supra closed. Therefore a $_{gs}T^{\mu}_{rg^{**}b}$ –space need not be a $T^{\mu}_{rg^{**}b}$ –space.

(iv). Let X={a,b,c} and μ ={X, ∅ ,{c},{a,b},{a,c}}.Every $rg^{**}b^{\mu}$ closed set b^{μ} is closed but {b},{a,c} are $rg^{**}b^{\mu}$ closed set but not supra closed. Therefore a $_bT^{\mu}_{rg^{**}b}$ –space need not be a $T^{\mu}_{rg^{**}b}$ –space.

Theorem 5.5:

Let (X,τ) be a supra topological space then (i)$O^{\mu}(\tau) \subset RG^{**}b^{\mu}O(\tau)$

(ii)A space (X, τ) is said to be a $T^{\mu}_{rg^{**}b}$ space if $O^{\mu}(\tau) = RG^{**}b^{\mu}O(\tau)$.

Proof:

(i) Let A be supra open set in X.
Then X-A is supra closed and hence it is $rg^{**}b^{\mu}$ closed.
Then A is $rg^{**}b^{\mu}$open in X.
Therefore $O^{\mu}(\tau) \subset RG^{**}b^{\mu}O(\tau)$.

(ii) Let A be a $T^{\mu}_{rg^{**}b}$ space.
Let A be a $rg^{**}b^{\mu}$open in X. Then X-A is $rg^{**}b^{\mu}$ closed in X. By hypothesis, X-A is supra closed and hence, A is supra open in X. Therefore, $O^{\mu}(\tau) \supset RG^{**}b^{\mu}O(\tau)$.

By (i) $O^{\mu}(\tau) \subset RG^{**}b^{\mu}O(\tau)$.
Hence $O^{\mu}(\tau) = RG^{**}b^{\mu}O(\tau)$.
Conversely, let $O^{\mu}(\tau) = RG^{**}b^{\mu}O(\tau)$.
Let A be $rg^{**}b^{\mu}$ closed set in X. Then X-A is $rg^{**}b^{\mu}$-open set and hence X-A is supra open.

Thus A is supra closed in X.

Therefore (X,τ) is said to be a $T^{\mu}_{rg^{**}b}$ space.

Theorem 5.6:

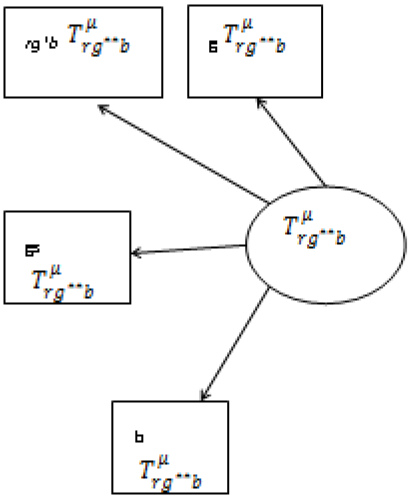
(a) Let (X,τ) be a supra topological space then
(i)$B^{\mu}O(\tau) \subset RG^{**}b^{\mu}O(\tau)$
(ii)A space (X,τ) is said to be a ${}_{b}T^{\mu}_{rg^{**}b}$ space if $B^{\mu}O(\tau) = RG^{**}b^{\mu}O(\tau)$.

(b) Let (X,τ) be a supra topological space then
(i)$B^{\mu}O(\tau) \subset RG^{**}b^{\mu}O(\tau)$
(ii)A space (X,τ) is said to be a ${}_{g}T^{\mu}_{rg^{**}b}$ space if $G^{\mu}O(\tau) = RG^{**}b^{\mu}O(\tau)$.

(c) Let (X,τ) be a supra topological space then
(i)$GS^{\mu}O(\tau) \subset RG^{**}b^{\mu}O(\tau)$
(ii)A space (X,τ) is said to be a ${}_{gs}T^{\mu}_{rg^{**}b}$ space if $GS^{\mu}O(\tau) = RG^{**}b^{\mu}O(\tau)$.

(d) Let (X,τ) be a supra topological space then
(i)$RG^{*}B^{\mu}O(\tau) \subset RG^{**}b^{\mu}O(\tau)$
(ii)A space (X,τ) is said to be a ${}_{rg*b}T^{\mu}_{rg^{**}b}$ space if $RG^{*}B^{\mu}O(\tau) = RG^{**}b^{\mu}O(\tau)$.

Proof: Obvious.



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