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Domination Parameters of $(P_n)^k$ and its Realization

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Abstract:
The domination parameters of a graph G of order n has been already introduced. It is defined as $D \subseteq V(G)$ is a dominating set of G, if every vertex $v \in V - D$ is adjacent to atleast one vertex in D. in this paper, we have established various domination parameters of k^{th} power of Path, and square of the Centipede graph, also we have studied the relation between this parameters and illustrated with an examples.

Key words: Graph, Domination, power graph, Centipede graph

I. INTRODUCTION: 1.0

A graph $G = (V, E)$, where V is a finite set of elements, called vertices and E is a set of unordered pairs of distinct vertices of G called edges. The degree of a vertex v in G is the number of edges incident on it. Every pair of its vertices are adjacent in G is said to be complete, the complete graph on'n'vertices is denoted by K_n. Let u and v be the vertices of a graph G,a $u - v$ walk of G is an alternating sequences $u = u_o, e, u, e_2, u_2, \ldots . u_{n-1}, e_n, v_n = v$ of vertices and edges beginning with vertex u and ending with vertex v such that $e_i = u_{i-1}u_i$ for all $i = 1,2, \ldots\ldots, n$. The number of edges in a walk is called its length. A walk in which all the vertices are distance in called a path. A path on'n' vertices is denoted by P_n. A closed path is called a cycle, a cycle on 'n' vertices is denoted by C_n. Let $G = (V, E)$ be a simple connected graph, for any vertex $v \in V$, the open neighborhood is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighborhood of v is the set $N[v] = N(v) \cup \{u\}$. For a set $S \subset V$, the open neighborhood of S is $N(S) = \cup N(v)$, $v \in s$ and the closed neighborhood of S is $N[s] = N(S) \cup S$.

Definition 1.1

A set $D \subseteq V$ is a dominating set of G if every vertex $v \in V - D$ is adjacent to at least one vertex of D. We call a dominating set D is a minimal if there is no dominating set $D' \subseteq V(G)$ with $D' \subset D$ and $D' \neq D$. Further we call a dominating set D is minimum if these is no dominating set $D \subseteq V(G)$ with $|D'| < |D|$. The cardinality of a minimum dominating set is called the domination number denoted by $\gamma(G)$ and the minimum dominating set D of G is also called a γ- set.

Definition 1.2:

A dominating set D is said to be a total dominating set if every vertex in V is adjacent to some vertex in D. The total domination number of G denoted by γ_t (G) is the minimum cardinality of a total dominating set.

Definition 1.3:

A dominating set D of a graph G is an independent dominating set, if the induced sub graph <D> has no edges. The independent domination number γ_i (G) is the minimum cardinality of a independent dominating set.

Definition 1.4:

A dominating Set D is said to be connected dominating set, if the induced subgraph <D> is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set.

Definition 1.5

A dominating Set D of a graph G is said to be a paired dominating set if the induced sub graph <D> contains atleast one perfect matching, paired domination number γ_p (G) is the minimum cardinality of a paired dominating set.

Definition 1.6

A dominating Set D of G is a split dominating set if the induced sub graph <V – D> disconnected Split domination number γ_s (G) is the minimum cardinality of a split dominating set.

Definition 1.7

A dominating Set D of G is a non split dominating set, if the induced sub graph <V – D> is connected. Non split domination number γ_{ns} (G) is the minimum cardinality of a non split dominating set.

Definition 1.8

Let D be a γ- set of G. A dominating set D^1 contained in V – D is called an inverse dominating set of G with respect to D. The inverse domination number γ' (G) is the minimum cardinality of all inverse dominating set of G, the vertices of γ' (G) is called γ'- set.

Definition 1.9

A dominating set D of a graph G is called a global dominating set, if D is also a dominating set of $\overline{G}$. The global domination number γ_g (G) in the minimum cardinality of a global dominating set.

Definition 1.10

A dominating set D is called a perfect dominating set, if every vertex in V – D in adjacent to exactly one vertex in D. The perfect domination number $\gamma_{pr}(G)$ is the minimum cardinality of a perfect dominating set.

Definition: 1.11

If D = $\{x\}$ is a dominating set of G, then x is called a dominating vertex of G. A vertex
$v \in V(G)$ is said to be a γ - required vertex of G, if v lies in every γ- set of G.

Definition: 1.12

Let x be any real value, then its upper sealing of x is denoted as x and is defined
$\lceil x \rceil$ =

$$\begin{cases} x & \text{if } x \text{ is an integer} \\ k, & \text{where k is an integer lies in the interval } x < k < x+1 \end{cases}$$

the lower sealing of x is denoted as $\lambda x \mu$ and is defined by
$\lfloor x \rfloor$ =

$$\begin{cases} x & \text{if } x \text{ is an integer} \\ k, & \text{where k is an integer lies in the interval } x-1 < k < x \end{cases}$$

Definition :1.13

n –Centipede graph is a tree on $2n$ vertices obtained by joining the bottom of n –copies of the path graph P_2 laid in a row with edges and is denoted by $\mathbb{C}_n$.

Definition ;1.14

The k^{th} power of a graph G is a graph with the same set of vertices of G and an edge between two vertices if there is a path of length almost k between them. G^2 is called the square of G, G^3 is called the cube of G etc.

Lemma 2.1

Let G be a connected graph with $\delta(G) \geq 2$, them $\gamma(G) + \gamma'(G) = n$ if and only if G = P_4 or C_4.

Lemma 2.2

Let G be a connected graph with $\delta = 1$ and $\Delta = n$ then $\gamma(G) + \gamma'(G) = n + 1$ if and only if $G = k_{1,n}$.

Lemma 2.3

For any tree with $n \geq 2$ with more than two pendent vertices then there exists a vertex $v \in V$ such that $\gamma(T - v) = \gamma(T)$.

Lemma 2.4

For any path P_n , $\gamma(p_n) \leq \gamma'(p_n)$ $\forall n \geq 3$.

Proof :

Since P_n is a path with n vertices then $\gamma(P_n)$

$$= \begin{cases} \gamma'(P_n) - 1 & \text{if } n = 3k \ \ \forall \ k = 1, 2 \ldots \\ \gamma'(P_n) & \text{otherwise} \end{cases}$$

Therefore, $\gamma(P_n) \leq \gamma'(P_n)$. $\forall n \geq 2$

Note: Let G be a path of length n then

$$\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil \qquad \forall \ n > 3$$

$$\gamma'(P_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$$

Lemma : 2.5

Let G be a cycle of length four then $\gamma(G) = \gamma'(G) = \gamma_t(G) = \gamma_c(G) = \gamma_p(G) = \gamma_s(G) = \gamma_g(G) = \gamma_{ns}(G) = \gamma_i(G) = 2$.

Proof:

Let v_1, v_2, v_3 and v_4 are the vertices of C_4, each vertex v_i connected with v_{i+1}, i = 1, 2, 3 and v_4 is connected with v_1 in G. Hence v_1, v_3 and v_2v_4 are the edges in $\overline{G}$. Let D =$\{v_1, v_2\}$ be the vertices of G. Clearly D satisfies the conditions for total domination, connected domination, paired domination, global domination and non split domination.

Therefore, $\gamma(G) = \gamma_t(G) = \gamma_c(G) = \gamma_p(G) = \gamma_g(G) = \gamma_{ns}(G) = 2$.

Let $D' = \{v_3, v_4\}$ satisfies the condition for the inverse domination, therefore,
$\gamma'(G) = 2$. Let $D_1 = \{v_1, v_3\}$ satisfies the condition for independent domination and split domination, therefore, $\gamma_i(G) = \gamma_s(G) = 2$.

Hence,

$\gamma(G) = \gamma_t(G) = \gamma_p(G) = \gamma_g(G) = \gamma_{ns}(G) = \gamma'(G) = \gamma_s(G) = \gamma_i(G) = 2$

Lemma : 2.6

K_n is a complete graph with $'n'$ vertices. Let $G = (K_n)^k$ then

$$\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_{ns}(G) = 1 \text{ and}$$

$$\gamma_t(G) = \gamma_p(G) = 2.$$

Proof:

Source K_n is a complete graph $(K_n)^k = K_n$ for all $k = 1,2, \ldots$ then the result follows immediately.

Lemma:2.7

Let $G = (K_{1,n})^k$ then

$\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_{ns}(G) = 1$ and $\gamma_t(G) = \gamma_p(G) = 2$ for all $k > 1$

Proof:

$K_{1,n}$ is a star graph with $n + 1$ vertices
Then for every elements $u, v \in k_{1,n}, d(u,v) \leq 2$.
for an $u, v \in k_{1,n}$

Which implies $\left(K_{1,n}\right)^k = K_{n+1}$ for all $k \geq 2$
Let $G = \left(K_{1,n}\right)^k$ for all $k > 1$

Then G is a complete graph with $n+1$ vertices.

Now the result follows from lemma 2.6

Lemma: 2.8
Let G be any complete bipartite graph with m, n vertices, let $G = \left(K_{m,n}\right)^2$

$$\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_{ns}(G) = 1 \quad \text{and}$$

$$\gamma_t(G) = \gamma_p(G) = 2$$

Result:

If $G = \left(K_{m,n}\right)^k$ then

$$\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_{ns}(G) = 1$$

$$\gamma_t(G) = \gamma_p(G) = 2 \qquad \text{for all } k > 1$$

Lemma: 2.9

Let G be any bipartite graph with $dim\ (G) = k$ then

$$\gamma(G^k) = \gamma'(G^k) = \gamma_i(G^k) = \gamma_{ns}(G^k) = 1$$

and

$$\gamma_t(G^k) = \gamma_p(G^k) = 2$$

Theorem 2.10

P_n be any path on $'n'$vertices. Let $G = (P_n)^k$; $k > 1$ then

(i) $\gamma(G) = \gamma_g(G) = \gamma_i(G) = \gamma_{ns}(G) = \left\lceil \frac{n}{2k+1} \right\rceil$

(ii) $\gamma_t(G) = \begin{cases} 2\left\lceil \frac{n}{3k+1} \right\rceil + 1 \ if\ n = r(mod\ 3k+1), \\ 2\left\lceil \frac{n}{3k+1} \right\rceil \ otherwise \end{cases}$
$1 \leq r \leq k$

(iii) $\gamma_{ct}(G) = \left\lceil \frac{n-1}{k} \right\rceil$

(iv) $\gamma'(G) = \begin{cases} \left\lceil \frac{n}{2k+1} \right\rceil + 1 & if\ n \not\equiv 0(mod\ 2k+1) \\ \left\lceil \frac{n}{2k+1} \right\rceil & otherwise \end{cases}$

(v) $\gamma_p(G) = \begin{cases} \left\lceil \frac{n}{k+1} \right\rceil & if\ n \equiv 0(mod\ 2k+2) \\ 2\left\lceil \frac{n}{2K+1} \right\rceil & otherwise \end{cases}$

(vi) $\gamma_s(G) = k + \left\lceil \frac{n-3k}{2k+1} \right\rceil$ for all $k > 1$

Proof:

(i) Case (i) if $n \equiv 0(mod\ 2k+1)$

P_n is a path on $'n'$ vertices, are denoted by $v_1, v_2, v_3, \ldots . v_n$ and each vertex v_i is connected with v_{i-1} and v_{i+1} for $i = 2,3, \ldots n-1$.

Let $G = (P_n)^k$, $k > 1$ is the k^{th} power of P_n. by definition each vertex

$v_i \in G$, $i = 1,2, \ldots, n$ is connected with all v_j such that $d\left(v_i, v_j\right) \leq k$

For all $v_i, v_j \in P_n$, that is $\delta(G) = k, \Delta(G) = 2k$

Now for every element $v_i \in G, N(v_i) = \left\{v_j / d\left(v_i, v_j\right) \leq k, for\ all\ vj \in Pn.\right.$

Collecting all the vertices $v_{k+1}, v_{3k+2}, v_{5k+3, \ldots\ldots}$ of G as

$$D = \left\{v_{(2i-1)k+i} / \quad i = 1,2,3, \ldots and\ (2i-1)k + i \leq n \right\}$$

is the required dominating set with minimum cardinality

Therefore, $|D| = \frac{n}{3k+2-(k+1)}$

$|D| = \frac{n}{2k+1}$ if n is a multiple of $2k+1$

$$\gamma(G) = \frac{n}{2k+1} \qquad \text{if } n \equiv 0(mod\ 2k+1)$$

Case (ii)

If $n \not\equiv 0(mod\ 2k+1)$

Divide the vertices of G into $m = \left\lfloor \frac{n}{2k+1} \right\rfloor$ number of sub sets such-that each partition containing $2k+1$ vertices as follows

$$G_1 = \{v_1, v_2, v_3, \ldots . v_{2k+1}\}; G_2 = \{v_{2k+2}, \ldots, v_{4k+2}\}$$
$$G_3 = \{v_{4k+3}, v_{4k+4}, \ldots v_{6k+3}\}, \ldots, G_m = \left\{v_{2(m-1)k+m}, \ldots v_{2mk+m}\right\}$$

Now the last partition containing $'t'$ vertices where $1 \leq t \leq 2k$

Let $G_{m+1} = \left\{v_{2mk+m+t} / m = \left\lfloor \frac{n}{2k+1} \right\rfloor ; 1 \leq t \leq 2k\right\}$

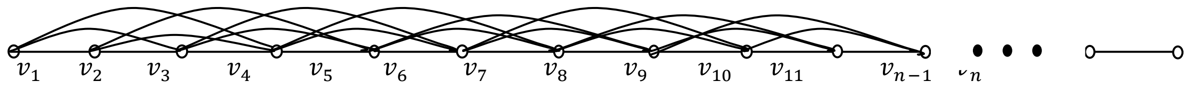
Now $D = \left\{v_{(2i-1)k+i} / i = 1,2, \ldots and\ (2i-1)k + i < n\right\} \cup \{v_j\} / v_j \in G_{m+1}\ j$ and $j = \frac{k}{2}$

is the required minimum dominating set of G.

Therefore,
$|D| = \left\lfloor \frac{n}{2k+1} \right\rfloor + 1 = \left\lceil \frac{n}{2k+1} \right\rceil$ $\left[since \left\lceil \frac{n}{2k+1} \right\rceil = \left\lfloor \frac{n}{2k+1} \right\rfloor + 1\right.$

Foe example, without loss of generality we choose $k = 3$

Then, $G = (P_n)^3$ is given in figure 1 as below

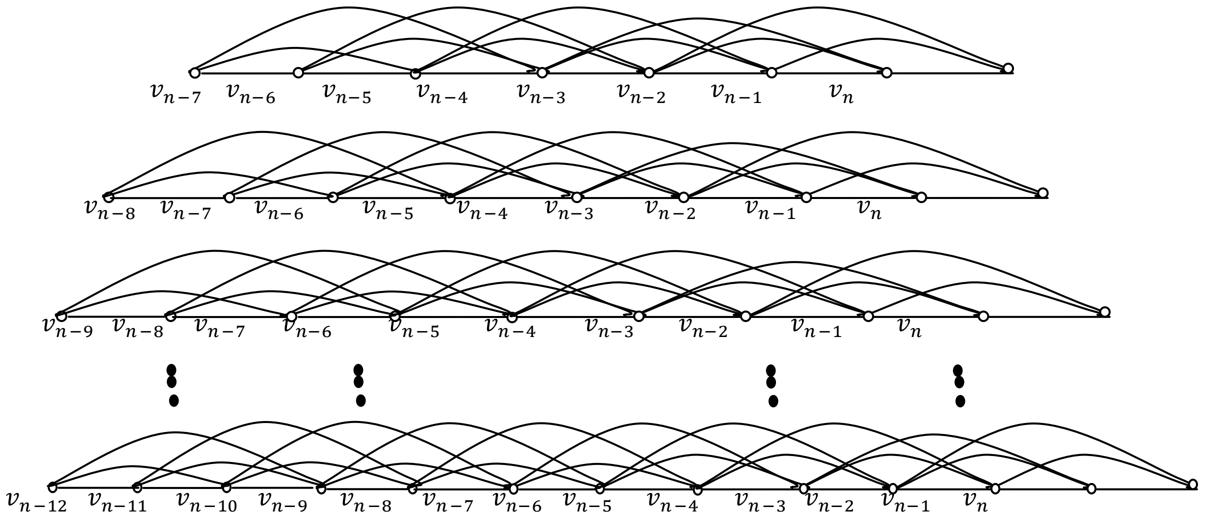


(Figure:1.)

$D = \{v_{k+1}, v_{3k+2}, v_{5k+8}, \dots\} is$

$D = \{v_4, v_{11}, v_{18}, \dots . v_{n-3}\}$ is the required dominating set of $G = (P_n)^3$.

If $n \not\equiv 0(mod\ 2k+1) \Rightarrow n \not\equiv 0(mod\ 7)$ then the last two combined partition of G is any one of the graphs given if figure 2 as below.



(Figure: 2.)

If $n \not\equiv 0(mod\ 2k+1)$, then the last partition G_{m+1}, where $m = \left\lfloor\frac{n}{2k+1}\right\rfloor$ containing the vertices as

$$G_{m+1} = \{v_{n-i} / \ i = 0,1,2, \dots, k-1\}$$

Therefore, the required dominating set of G is

$D = \{v_{(2i-1)k+i} / \quad i = 1,2, \dots and\ (2i-1)\ k + i < n) \cup \{v_j\} / j = n - \frac{k+1}{2}\}$

Therefore, $|D| = \left\lfloor\frac{n}{2k+1}\right\rfloor + 1$

$= \left\lceil\frac{n}{2k+1}\right\rceil$ Since $n \not\equiv 0(mod\ 2k+1)$

$\Rightarrow \gamma(G) = \left\lceil\frac{n}{2k+1}\right\rceil$ for all $k > 1$ … (1)

Cleary, the induced sub graph $\langle D\rangle$ of G has no edges in G. which shows that the vertices of D in G are independent set in G. That is D is the independent dominating set of G.

Therefore, $\gamma_i(G) = \left\lceil\frac{n}{2k+1}\right\rceil$ … (2)

In (i) is $D = \{v_{k+1}, v_{3k+2}, v_{5k+3}, \dots, v_{n-k-1}\}$ or,

$D = \{v_{k+1}, v_{3k+2}, v_{5k+3}, \dots v_j\} \quad n - r < j < n$

Is the dominating set of G for $|n| = 2k+2$, or otherwise respectively, since k is the power of P_n each vertex $v_i \in G$ is adjacent with all vertices whose distance atmost k in P_n., then $G - \langle D\rangle$ having a $v_1 v_n$ path as $v_1, v_2, \dots v_k, v_{k+2} \dots, v_{3k+1}, v_{3k+3}, \dots . v_{j-1}, v_{j+1}, \dots . v_n$

It is clear that $\langle G - D\rangle$ is connected,

Therefore, D is also a non split dominating set of G,

Hence, $\gamma_{ns}(G) = \left\lceil\frac{n}{2k+1}\right\rceil$ … (3)

By (i) $D = \{v_{(2i-1)ik+i} / \ i = 1,2, .. and \quad (2i-1)k + i \leq n\}$

is the dominating set of G.

That is every element $v_i \in D$ is adjacent with v_j

such that $d(v_i, v_j) \leq k\ for\ all\ v_i, v_j \in P_n$,

That is every element $v_i, v_j \in G$ is not adjacent in G if $d(v_i, v_j) > k$

$$\Rightarrow N(v_i) \cap N(v_j) = \varphi\ for\ all\ v_i, v_j \in D$$

That is $N(v_i)$ is connected with v_j and $N(v_j)$ is connected with v_i in $\bar{G}$.

$\Rightarrow$ the elements of D are the dominating set of $\bar{G}$

$\Rightarrow D$ is the global dominating set of G

$\Rightarrow \gamma(G) = \gamma_g(G)$ … (4)

Hence, $\gamma(G) = \gamma_i(G) = \gamma_{ns}(G) = \gamma_g(G) = \left\lceil \frac{n}{2k+1} \right\rceil$

(ii) Case (i)

If $n \equiv 0\ (mod\ 3k+1)$ divide the vertices of the graph G into m partitions $G_1, G_2, \dots, G_m$ such that $n = m(3k+1)$ and each partition $G_i, i = 1,2,\dots,m$ consists of $3k+1$ vertices of the form $G_i = \{v_{(3i-3)k+i}, \dots, v_{(3i-2)k+i}, \dots, v_{(3i-1)k+i}, \dots, v_{(3k+1)i},$ $i = 1,2,\dots,m\}$ Select the vertices $v_{(3i-2)k+i}, v_{(3i-1)k+i}$ form each partition G_i, $i = 1,2,\dots,m$ is the required total dominating set of G.

That is, $D_{\vdash}(G) = \{v_{(3i-2)k+i}, v_{(3i-1)k+i} / \ i = 1,2,\dots,m, n = m(3k+1)\}$ is the total dominating set of G with minimum cardinality.

Therefore, $|D_{\vdash}(G)| = 2m$

$$= 2\left(\frac{n}{3k+1}\right)$$

$$= 2\left\lceil \frac{n}{3k+1} \right\rceil$$

$$\left[\because n = m(3k+1) \Rightarrow \frac{n}{3k+1} is\ an\ integer\right]$$

Case (ii)

$$n \equiv r\ (mod\ 3k+1) \quad \text{here } 1 \le r \le k$$

by Case (i) divided the vertices of G in to m sets $G_1, G_2, \dots, G_m$ as mentioned above $G_i = \{v_{(3i-3)k+i}, \dots, v_{(3i-2)k+i}, \dots, v_{(3i-1)k+i}, \dots, v_{(3k+1)i},$ $i = 1,2,\dots,m\}$

Now the first $m-1$ partition containing exactly $3k+1$ vertices and the last partition G_m contains $n-[(m-1(3k+1)$ vertices, that is Gm contains $3k+1+r$ vertices where $1 \le r \le k$., some $r \le k$. The vertex $V_{[n-\eta_2]}$ is adjacent with all elements $v_{n-t,\ \ t=1,\dots r}$ and $v_{(3m-1)k+i} \in G_m$.

Therefore, $D_{\vdash}(G) = \{v_{(3i-2)k+i}, v_{(3i-1)k+i}; i = 1,2,\dots,m\} \cup \left\{v_{\left[n-\frac{r}{2}\right]}\right\}$ is the required minimum total dominating set of G.

$$\Rightarrow |D_t(G)| = 2\left\lfloor \frac{n}{3k+1} \right\rfloor + 1$$

$$= 2\left\lceil \frac{n}{3k+1} \right\rceil + 1$$

$$\left[\because n - r \equiv 0\ mod\,(3k+1) \Rightarrow \left\lfloor \frac{n}{3k+1} \right\rfloor = \left\lceil \frac{n}{3k+1} \right\rceil\right]$$

Case (iii)

If $n \equiv r\ (mod\ 3k+1)$ where $k < r < 3k$.

Divide $V(G)$ into m sets $G_1, G_2, \dots, G_m$ as in case (i) each set consists $3k+1$ vertices, then each G_i, $i = 1,2,\dots,\overline{m-1}$ contains $3k+1$ vertices and G_m containing r vertices, where $k < r < 3k$.

Now $D_t(G) = \{v_{(3i-2)k+i}, v_{(3i-1)k+i} / \ \ i = 1,2,\dots,m-1\} \cup \{v_s, v_{\vdash}\}$;

$\{v_s, v_{\vdash}\} \in G_m$ and $n-2k \le s \le n-2k+2$; $n-k \le \vdash \le n-k+2$ is the required dominating set of G with minimum cardinality

$$|D_{\vdash}(G)| = 2\left\lceil \frac{n}{3k+1} \right\rceil$$

Hence, $$\gamma_{\vdash}(G) = \begin{cases} 2\left\lfloor \frac{n}{3k+1} \right\rfloor + 1 & if\ n \equiv r\ (mod\ 3k+1) \\ 2\left\lceil \frac{n}{3k+1} \right\rceil & otherwise \end{cases}$$

… (5)

(iii) Since, $d(v_i, v_j) \le k\ for\ all\ v_i, v_j \in G$ that is each vertex $v_i \in G$ is adjacent with v_{i-h} and v_{i+h}, $1 \le h \le k$ now collect all vertices $v_{k+1}, v_{2k+1}, v_{3k+1}, \dots$ is the dominating set of G also each v_{ik+1} is adjacent with $v_{(i-1)k+1}$ to $v_{(i+1)k+1}$, for all $i = 1,2,\dots$

Therefore, $D_{ct}(G) = \{v_{ki+1} / i = 1,2,3,\dots; ki+1 \le n\}$ is the required minimum connected dominating set of G

$$\therefore \gamma_{ct(G)} = \begin{cases} \left\lceil \frac{n - \overline{k+1}}{k} \right\rceil & if\ n - \overline{k+1} > k \\ \left\lceil \frac{n - \overline{k+1}}{k} \right\rceil + 1 & if\ n - \overline{k+1} = k \end{cases}$$

(iv) Case (i)

If $n \equiv 0\,(mod\ 2k+1)$

by (i) $D = \{v_{(2i-1)k+i} / \ i = 1,2,\dots and\ (2i-1)k + i < n\}$ is the minimum dominating set of G then

$D' = \{v_{(2i-1)k+i-1} / \quad i = 1,2,\dots;\ (2i-1)k + i - 1 < n\} \cup \{v_j\}/_{n-k<j\le n}$ is the one of the required inverse dominating set of G with minimum cardinality.

Therefore, $|D'(G)| = \frac{n}{2k+1} + 1$

$$= \left\lceil \frac{n}{2k+1} \right\rceil + 1$$

$$[\because n \equiv 0(mod\ 2k+1)]$$
$$\frac{n}{2k+1} = \left\lceil \frac{n}{2k+1} \right\rceil$$

Case (ii)

If $n \not\equiv 0\,(mod\ 2k+1)$

Let $n \equiv r\,(mod\ 2k+1)$ where $1 \le r \le 2k$

Divide the elements of G in to m sets $G_1, G_2, \dots, G_m$ such that each G_i, $i = 1,2,\dots,\overline{m-1}$ contains exactly $2k+1$ vertices and the last partition contains r vertices.

ie. $G_1 = \{v_1, v_2, \dots, v_{2k+1}\}, G_2 = \{v_{2k+2}, \dots, v_{4k+2}\}, \dots,$

$G_m = \{v_{n-r}, \dots, v_n\} / \ \ i \le r \le 2k$

Now collect all the vertices $v_{(2i-1)k+i-1}$, for all $i = 1,2,\dots,\overline{m-1}$ and $v_{n-\left[\frac{r}{2}\right]}$ is the required inverse dominating set of G.

That is, $D'(G) = \{v_{(2i-1)k+i-1;\ i=1,2,\dots,m}\} \cup \left\{v_{n-\left[\frac{r}{2}\right]}\right\}$; $1 \le r \le 2k$

$$\Rightarrow |D'(G)| = \left[\frac{n-r}{2k+1}\right] + 1$$

$$= \left[\frac{n}{2k+1}\right]$$

$$\left[\because r < 2k+1 ; \left[\frac{r}{2k+1}\right] = 1\right]$$

$$\Rightarrow \gamma'(G) = \begin{cases} \left[\frac{n}{2k+1}\right] + 1 & if\ n \equiv 0 mod(2k+1) \\ \left[\frac{n}{2k+1}\right] & otherwise \end{cases}$$

…(6)

(v) Case (i)

If $n \equiv 0\ (mod\ 2k+2)$ then divide the vertices if G into m subgraphs $G_1, G_2, \dots, G_m$ such that $n = m(2k+2)$ and each $G_i, i = 1,2,\dots,m$ containing $2k+2$ vertices as below

$$G_i = \{v_{2[i(k+1)-k]-1}, \dots, v_{(2i-1)k+i}, v_{(2i-1)k+i+1}, \dots v_{i(2k+2);\ i=1,2,} \dots m\}$$

all $2k+2$ elements in each partition of $G_i, i = 1,2,\dots,m$ are adjacent with the vertices $v_{(2i-1)k+i}, v_{(2i-1)k+i+1}$. Also each edge $v_{(2i-1)k+i}\ v_{(2i-1)k+i+1,} \in E(G)$ are independent in G and forms a perfect matching.

Therefore, $D_p(G) = \{v_{(2i-1)k+i,} v_{(2i-1)k+i+1;}\ i = 1,2,\dots,m\}$ …(A)

is the required paired dominating set of G with minimum cardinalily,

ie, $|D_p(G)| = 2\left(\frac{n}{m}\right)$

$$= 2\left(\frac{n}{2k+2}\right)$$

$$= \frac{n}{k+1} \quad \left[\because m = \frac{n}{2k+2}\ \begin{matrix} n \equiv 0\ (mod\ 2k+2) \\ is\ an\ integer \end{matrix}\right.$$

$$= \left[\frac{n}{k+1}\right] \qquad \Rightarrow \frac{n}{k+1} = \left[\frac{n}{k+1}\right]]$$

Case (ii)

If $n \not\equiv 0\ (mod\ 2k+2)$

Suppose $n \equiv r\ (mod\ 2k+2)$ where $1 \le r \le 2k+1$. By Case(i) of (v) last partition containing r vertices. Suppose $r = 1$, then $v_{n-1} \in G_{m-1}$ and $v_n \in G_m$ together with (A) is the required paired dominating set of G.

Otherwise $r > 1$ then last partition G_m contains atleast two and almost $2k+1$ vertices of G.

That is $G_m = \{v_{n-r}, \dots, v_n\}$; now the vertices $v_s, v_t \in G_m$ where $s = n + [{}^{r}/_{2}]$; $t = s + r$ together with (A) forms a dominating set of G and the edges formed by this vertices are independent in G which is the required minimum paired dominating set of G.

Therefore; $D_p(G) = \{v_{(2i-1)k+i}, v_{(2i-1)k+i+1}; i = 1,2,\dots,m-1\} \cup \{v_s, v_t\}$; $v_s, v_t \in G_m$ $D_p(G) = 2\left[\frac{n-r}{2k+2}\right] + 2$

$$= 2(m-1) + 2$$

$$\left[\because \begin{matrix} n \equiv r(mod\ 2k+2) \\ n-r \equiv 0(mod\ 2k+2) \\ \frac{n-r}{2k+2} = m-1, 1 < r < 2k+2 \\ \left[\frac{r}{2k+2}\right] = 1 \Rightarrow m = \left[\frac{n}{2k+2}\right] \end{matrix}\right]$$

$$= 2m \quad = 2\left[\frac{n}{2k+2}\right]$$

$$\Rightarrow \gamma_p(G) = \begin{cases} \left[\frac{n}{k+1}\right] & if\ n \equiv 0\ (mod\ 2k+2) \\ 2\left[\frac{n}{2k+2}\right] & otherwise \end{cases}$$

… (7)

(vi) Case (i)

If $n = 3k$, then the vertices of G are $G = \{v_i;, = 1,2,\dots,3k\}$choose $D_s = \{v_i/i = k+1,\dots,2k\}$ clearly $\langle V(G) - D_s\rangle$ is disconnected in G, and D_s is the required split dominating set of $G \Rightarrow |D_s(G)| = k$.

Case (ii)

If $n > 3k$.

Let $m = n - 3k$ and the vertices of G are partitional into two sets G_1 and G_2 such that

$$G_1 = \{v_i/i = 1,2,\dots,3k\}: \quad G_2 = \{v_j/j = 3k+1,\dots,n\}$$

Where $|G_1| = 3k$ and $|G_2| = n - 3k$

by (i) $\gamma(G_2) = \left[\frac{n-3k}{2k+1}\right]$ and $\gamma(G_1) = k$, Therefore, $\gamma_s(G) = \gamma_s(G_1) + \gamma(G_2)$

$$= k + \left[\frac{n-3k}{2k+1}\right]$$

…(8)

Hence the proof.

Result 2.11

P_n is a sub graph of $(P_n)^k, k > 1, n > 3$ then

$$\gamma[(P_n)^k] \le \gamma(P_n)$$

Since $\gamma(P_n) = \left[\frac{n}{3}\right]$; $\gamma[(P_n)^k] = \left[\frac{n}{2k+1}\right]$ and $\left[\frac{n}{2k+1}\right] \le \left[\frac{n}{3}\right]$, for all $k > 1$

$$\Rightarrow \gamma[(P_n)^k] \le \gamma(P_n) \qquad \text{for all} \quad n > 3\ and\ k > 1$$

The following Table slows the domination parameters of $(P_n)^k$.

$$n = 21 \quad and \quad k = 1,\dots,10$$

$n=21$	P_n	$(P_n)^2$	$(P_n)^3$	$(P_n)^4$	$(P_n)^5$	$(P_n)^6$	$(P_n)^7$	$(P_n)^8$	$(P_n)^9$	$(P_n)^{10}$
γ	7	5	3	3	2	2	2	2	2	1
γ_i	7	5	3	3	2	2	2	2	2	1
γ_{ns}	19	5	3	3	2	2	2	2	2	1
γ_g	7	5	3	3	2	2	2	2	2	1
γ_t	11	6	5	4	4	3	2	2	2	2
γ_{ct}	19	9	6	4	3	3	2	2	2	2
γ'	8	5	4	3	2	2	2	2	2	2
γ_p	12	8	6	6	4	4	4	4	4	2
γ_s	7	5	5	5	6	7	7	8	9	10

(Table: 1.)

Theorem:2.12

Let G be a Centipede graph with $2n$ vertices, then its domination parameters are

$$\gamma(G)=\gamma_t(G)=\gamma_i(G)=\gamma_c(G)=\gamma_s(G)=\gamma_{ns}(G)=\gamma'(G)=\gamma_s(G)=n \quad \text{and}$$

$\gamma_P(G)=2\left\lceil\frac{n}{2}\right\rceil$.

Proof :

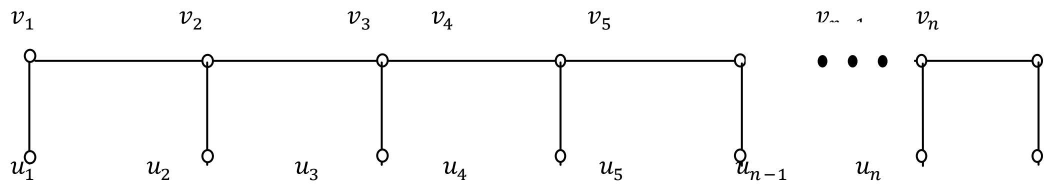


Figure (4)

Centipede graph with $2n$ vertices is represented in figure (4)

Let $V(G)=\{v_1,v_2,\dots,v_n,u_1,u_2,\dots,u_n\}$ be the vertex set of G. Choose $S_1=\{v_i/i=1,2,\dots,n\}$ and $S_2=\{u_i/i=1,2,\dots,n\}$.

Now $d(v_1)=d(v_n)=2;d(u_i)=1,i=1,2,\dots,n$ and $d(v_i)=3$ for all $i=2,3,\dots,\overline{n-1}$

Since every clement in S_2 is a pendent vertex of G, either $u_i\in S_2$ or $N(u_i)$ belongs to the dominating set of G, choose $D=\{v_i/i=$ *1,2,…,n* is one of the required minimum dominataing set of with cardinality n.

Hence, $\gamma(G)=n$

Also every elements of $V(G)$ is adjacent with some elements $v_i\in D$.

Therefore, D dominates all the elements of $V(G)$ and the induced sub graph $<D>$ is connected. Hence D is the total and connected dominating set of G with minimum cardinality. Now the induced sub graph $<V-D>$ is the null graph whose vertices are the elements of S_2, therefore, D is a split dominating set of G and its cardinality is $'n'$.

The elements of S_2 are independent and dominates the elements of $V-S_2$ is the required independent dominating set with cardinality $'n'$, the induced sub graph $\langle V-S_2\rangle$ is connected in G. Hence S_2 is the non split dominating set of G. Also S_2 is the inverse dominating set of S_1 and vice verse with cardinality $'n'$.

Every elements $u_i\in S_2$ is adjacent only with $v_i\in S_1$ in G. Therefore, u_i is adjacent with all vertices other than v_i in $\bar{G}$.

$\Rightarrow\{(u_i,u_j)\},i\neq j\in G$ dominates all elements of $\bar{G}$.

since $\{(u_i,u_j)/i\neq j\}\in S_2$ is the dominating set of $\bar{G}$.

Therefore, S_2 is the dominating set of G and $\bar{G}$
$\Rightarrow S_2$ is the global dominating set of G.

Similarly S_1 is also a global dominating set

$$\Rightarrow\gamma(G)=\gamma_t(G)=\gamma_c(G)\equiv\gamma_{ns}(G)=\gamma_i(G)=\gamma'(G)=\gamma_g(G)=n$$

If $|S_1|=2k$, then each pair $(v_{2i-1},v_{2i})/i=1,2,\dots,k$ are the non adjacent edges in G, gives the required perfect matching of G. Otherwise $|S_1|=2k+1$ then choose the piar of vertices $(v_{2i-1},v_{2i}),i=1,2,\dots,k$ and (v_n,u_n) then the independent edges

$\{v_{2i-1}v_{2i},v_nu_n/i=1,\dots,k\}$ forms a perfect matching of G.

If $|S_1|=2k$ then $|D_P|=2k=n$

If $|S_1|=2k+1$ then $|D_P|=2k+1+1=2(k+1)$

$=2\left\lceil\frac{n}{2}\right\rceil$

$\left[\because n=2k+1;\frac{n}{2}=k+\frac{1}{2};\left\lceil\frac{n}{2}\right\rceil=k+1\right]$

In both case $|D_P|=2\left\lceil\frac{n}{2}\right\rceil$

Hence, $\gamma_P(G)=2\left\lceil\frac{n}{2}\right\rceil$

Theorem:2.13

$\mathbb{C}_n$ be any centipede graph with $2n$ vertices. Let $G_2(\mathbb{C}_n)^2$ then

(i) $\gamma(G)=\gamma_i(G)=\gamma_{ns}(G)=\left\lceil\frac{n}{3}\right\rceil$

(ii) $\gamma'(G)=\begin{cases}\gamma(G)\,if & n\not\equiv 0(mod\ 3)\\ \gamma(G)+1\ if & n\equiv 0(mod\ 3)\end{cases}$

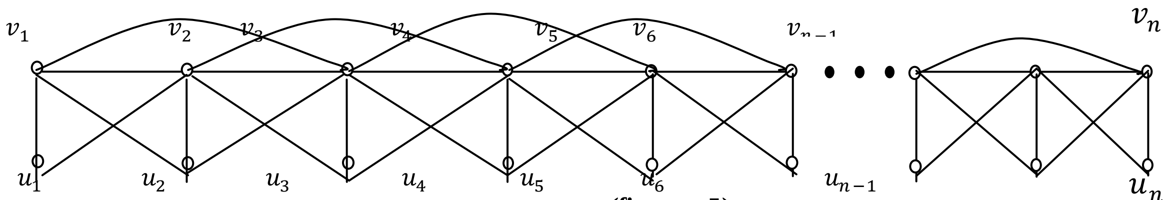
(iii) $\gamma_{\vdash}(G)=\begin{cases}2m\ if & n=5m-r:\quad r=0,1,2\\ 2m+1\ if & n=5m+r:\quad r=1,2\end{cases}$

(iv) $\gamma_s(G) = \begin{cases} \left\lceil \frac{n-1}{3} \right\rceil + 1 & if\ n \equiv 1\ (mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 & otherwise \end{cases}$

(v) $\gamma_p(G) = 2\left\lceil \frac{n}{4} \right\rceil$ for an n.

Proof:

Let $G = (\mathbb{C}_n)^2$ where, $\mathbb{C}_n$ is a Centipede graph with $2n$ vertices



(figure : 5)

In G each vertex $v_i \in S_1$ is adjacent with almost four vertices $v_j \in S_1$ and almost three vertices $u_j \in S_2$

Case (i)

If $n \equiv 0\ (mod 3)$

Let $D = \left\{ v_{3i-1} / i = 1,2, \dots, k; k = \frac{n}{3} \right\}$ …(i)

is the required dominating set of G with minimum cardinality.

case(ii)

If $n \equiv r(mod\ 3), r \neq 0$ then

$D = \{v_{3i-1}/i = 1,2, \dots, k; k = \left\lceil \frac{n}{3} \right\rceil\} \cup \{v_j\}$ …(ii)

where $j = n$ or $n - 1$accordingly as $r = 1$ or $r = 2$

In case (i) $|D| = \left\lceil \frac{n}{3} \right\rceil$ and $|D| = \left\lceil \frac{n}{3} \right\rceil + 1$ since by case (ii)

$$= \left\lceil \frac{n}{3} \right\rceil$$

Therefore, $|D| = \left\lceil \frac{n}{3} \right\rceil$ for all $n \geq 3$

Let $D = \left\{ v_{3i-1} / i = 1,2, \dots, k; k = \frac{n}{3} \right\}$ if $n \equiv 0(mod\ 3)$ (or)

$D = \{v_{3i-1}/i = 1,2, \dots, k; k = \left\lceil \frac{n}{3} \right\rceil\} \cup \{u_n\}$ if $n \equiv 1(mod\ 3)$ (or)

$D = \{v_{3i-1}/i = 1,2, \dots, k; k = \left\lceil \frac{n}{3} \right\rceil\} \cup \{v_n\}$ if $n \equiv 2(mod\ 3)$

Then all the elements of D are independent in G is the required independent dominating set of G with cardinality of $D = \left\lceil \frac{n}{3} \right\rceil$ for all $n \geq 3$ by equation (i) & (ii) the induceed sub graph $\langle V - D \rangle$ has a $v_1 v_{3i} v_{3i+1} v_n / i = 1,2, \dots$ path from v_1 to v_n.

That is the induced sub graph $\langle V - D \rangle$ is connected in G. Therefore, D is required non split dominating set of G.

ie, $\gamma(G) = r_i(G) = \gamma_{ns}(G) = \left\lceil \frac{n}{3} \right\rceil$.

If $n \equiv 0\ (mod\ 3)$,

(i) is the dominating set of G.
Choose $D' = \{v_{3i+1}/i = 1,2, \dots, k; k = \left\lceil \frac{n}{2} \right\rceil\} \cup \{v_n\}$ is the required inverse dominating set of G.

Suppose $n \equiv r\ (mod\ 3), 1 \leq r \leq 2$ then $D' = \{v_{3i+1}/i = 1,2, \dots, 3i + 1 \leq n;\}$

is the inverse dominating set of G with minimum cardinality.

Therefore, $|D'| = |D|$

Hence, $\gamma'(G) = \begin{cases} \gamma(G) & \text{if } n \not\equiv 0\ (mod\ 3) \\ \gamma(G) + 1 & \text{if } n \equiv 0\ (mod\ 3) \end{cases}$.

Divide the vertices of G in to m sets $G_1, G_2, G_3, \dots, G_m$ Such that each partition

of G_i containing exactly ten vertices v_i and u_i of the form

$G_i = \{v_{5i+r}, u_{5i+r}/r = 1,2,3,4,5 and\ i = 1,2, \dots, m\}$.

Now $\{v_{5i+2}, u_{5i+4}\}$ is the required dominating set of G, if $G_m = \{v_{5i+r}, u_{5i+r}/\ r = 1,2,3,4,5 and\ i = m\}$.

Otherwise $\{v_{5i+2}, u_{5i+4}/\ i = 1, \dots, m\}$ is the required dominating set of G.

Therefore,

$$\gamma_t(G) = \begin{Bmatrix} 2m & if\ \ n = 5m - r, & r = 0,1,2 \\ 2m + 1 & if\ n = 5m + r; & r = 1,2 \\ & & \text{for all } m = 1,2, .. \end{Bmatrix}$$

Consider the elements $\{v_{2i}/i = 1,2, \dots, (n/2)\}$ of G; it dominates all elements of G and the induced sub graph formed by this vertices are connected in G, hence it is the required connected total dominating set with minimum cardinality $\left\lceil \frac{n-1}{2} \right\rceil$. Since by (i) $D_s(G) = D \cup \{v_3\}$ is a dominating set and the induced sub graph $\langle V(G) - D_s(G) \rangle$ is not connected. Therefore, $D_s(G)$ is the required split dominating set and its cardinality is

$\left\lceil \frac{n-1}{3} \right\rceil + 1\ \ if\ n \equiv 1(mod3)$ and $\left\lceil \frac{n}{3} \right\rceil + 1$ *otherwise*

If $n = 4m$ divide, the vertices G into m sets $G_1, G_2, \dots, G_m$ such that

$G_i = \{v_{4i-3}, v_{4i-2}, v_{4i-1}, v_{4i}/i = 1,2, \dots, m\}$then

$D_p(G) = \{v_{4i-2}, v_{4i-1} / i = 1,2, \dots, m\}$ is the set of vertices and the edges

$\{v_{4i-2}, v_{4i-1} / i = 1,2, \dots, m\}$ are independent in G its forms a perfect matching and its cardinality is $2m$, Therefore, $\gamma_p(G) = 2m$ if $n \equiv 0(mod 4)$

$$= 2\left\lceil\frac{n}{4}\right\rceil$$

If $n - 1 = 0(mod 4)$ then

$D_p(G) = \{v_{4i-2}, v_{4i-1} / i = 1,2, \dots, m\} \cup \{v_n, u_n\}$ is the required paired dominating set of G.

Therefore, $|D_p(G)| = 2\left\lceil\frac{n-1}{4}\right\rceil + 2$ $[\because n - 1$ is a multiple of 4, $n4=n-13+1; 2n4=2n-14+2$

$$= 2\left\lceil\frac{n}{4}\right\rceil$$

If $n \equiv 2,3 \ (mod\ 4)$

Choose $D_p(G) = \{v_{4i-2}, v_{4i-1} / i = 1,2, \dots, m\} \cup \{v_{n-1}, u_{n-1}\}$ now the edges formed by these vertices are the perfect marching of G, hence $D_p(G)$ is the required paired dominating set of G with minimum cardinality

$$\Rightarrow |D_p(G)| = 2\left\lceil\frac{n-r}{4}\right\rceil + 2$$

where $r = 2,3$

$$= 2\left\lceil\frac{n}{4}\right\rceil$$

In all cases

$\gamma_p(G) = 2\left\lceil\frac{n}{4}\right\rceil$ for all n.

Remark :2.14

Let G_1 be any Bar belled graph with $2n$ vertices.

Let $G = (G_1)^2$ then

(i) $\gamma(G) = \gamma'(G) = \gamma_i(G) = \gamma_{ns}(G) = 1$
(ii) $\gamma_{\vdash}(G) = \gamma_s(G) = \gamma_p(G) = \gamma_g(G) = 2$

Hence the Proof.

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