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Bio-Computational Analysis of Blood Flow through Two Phase Artery

Sapna Ratan Shah[1], Anuradha[2] and Anamika[3]
Associate Professor[1], Research Scholar[2, 3]
Department of School of Computational and Integrative Sciences[1, 3], Department of Mathematics[2]
Jawaharlal Nehru University, New Delhi, India[1, 3]
Harcourt Butler Technical University, Kanpur, India[2]

Abstract:
The effect of Bingham plastic fluid flow in the stenosed artery studied in this paper. A two layer model is taken. Pulsatile behavior of the fluid in the lumen stenosed artery is studied. Perturbation technique is use to solve the expression for velocity profile, wall shear stress and resistance to flow. Results are presented in the graphical form. It is found that the velocity is decreases when radius increases and resistance to flow is increases with time

Keywords: Stenosed artery, non-Newtonian, Bingham Plastic fluid, Flow rate, velocity profile.

I. INTRODUCTION

Atherosclerosis is a type of arteriosclerosis. It involves deposit of fatty substances, cholesterol, cellular waste products, calcium and fibrin (a clothing material in the blood) in the inner lining of an artery. The build up that result is called plaque. Plaque may partially or totally block the blood flow through an artery. It involves deposit of fatty substances, cholesterol, cellular waste products, calcium and fibrin (a clothing material in the blood) in the inner lining of an artery. The build up that result is called plaque. Plaque may partially or totally block the blood flow through an artery. If either of these occurs and blocks the entire artery, a heart attack or stroke may result. Usually high-grade stenosis with acute coronary changes results in sudden cardiac arrest (or death) that strikes 300000–400000 persons annually around the globe. Because of this, this is the interesting area of research. In this series [7, 9, 14, 18] find the results of blood flow in the stenosed artery. There are large number of researcher who works on the pulsetile blood flow through stenosed artery. Biswas, D. and Ali, M., (2014) [1] describe the two-layered mathematical model for blood flow inside an asymmetric stenosed artery with slip velocity. D. C. Sanyal, K. Das and S. Debnath, (2007) [3] studied the effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration. Devajyoti Biswas and Rezia Begum Laskar, (2011) [4] discussed the steady flow of blood through a stenosed artery: a non-newtonian fluid model. G. C. Hazarika, Barnali Sharma, (2014) [6] has been found the result for the study of two layered mathematical model for blood flow through tapering asymmetric stenosed artery with velocity slip at the interface under the effect of transverse magnetic field Joshi, P., Pathak, A. and Joshi, B.K. (2009) [8] study the two-layered model of blood flow through composite stenosed artery. Kumar, S., Diwakar, C., (2013) [9] has been calculated the resistance to flow for a small artery with the effect of multiple stenoses and post stenotic dilatation. Ponnalagasamy, R., (2012) [12] studied about the mathematical model of pulsatile flow of non-newtonian fluid in tubes of varying cross-sections and its implications to blood flow. Verma, S.R., (2014) [18] discussed about the mathematical modeling of bingham plastic model of blood flow through stenotic vessel

II. FORMULATION OF THE PROBLEM

Let us consider an axially symmetric, laminar, pulsatile and fully developed flow of blood through a circular artery having a stenosis (Fig. 1). Cylindrical polar coordinate($r^*, \emptyset^*, z^*$), with the pole located on the axis of the artery have been used to analyze the problem.

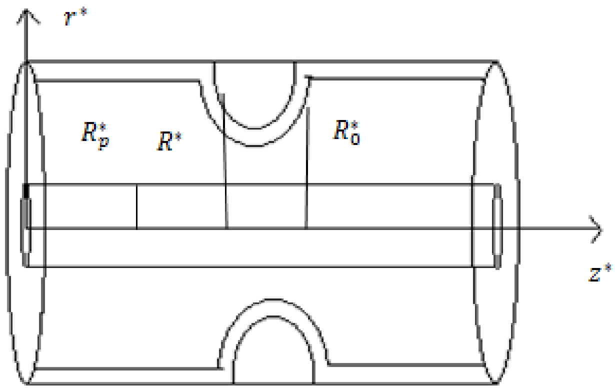

Figure.1. Diagram of multiphase blood flow in a stenosed artery

The momentum equation is given by

$$\rho \frac{\partial u^*}{\partial t^*} = -\frac{\partial p^*}{\partial z^*} - \frac{1}{r^*}\frac{\partial (r^*\tau^*)}{\partial r^*} \quad (1)$$

The Harshes Bulkly equation describing the non-Newtonian behavior of blood may be written as

$$-\frac{\partial u^*}{\partial r^*} = \frac{1}{\mu}(\tau - \tau_0), \qquad \tau^* > \tau_y \quad (2)$$

$$-\frac{\partial u^*}{\partial r^*} = 0, \qquad \tau^* \le \tau_y \quad (3)$$

The theoretical analysis takes care of the two-phase flow of blood, the peripheral plasma layer is considered to be Newtonian, while the core region that is supposed to contain all the erythrocytes contained in the blood inside the artery is treated as non-Newtonian. The mathematical model that is developed here is formulated by the following set of equations:

$$\tau^* = -\mu\frac{\partial u^*}{\partial r^*}, \qquad if\ R_0^*(z^*,t^*) < r^* < R^*(z^*,t^*), \qquad (4)$$

$$-\frac{\partial u^*}{\partial r^*} = \frac{1}{\mu}(\tau - \tau_0), \quad if\ R_p^*(z^*,t^*) < r^* < R_0^*(z^*,t^*), \qquad (5)$$

$$-\frac{\partial u^*}{\partial r^*} = 0, \qquad if\ 0 < r^* < R_p^*(z^*,t^*) \qquad (6)$$

Along with the boundary conditions

$u^* = 0$ at $r^* = R^*(z^*,t^*)$, (7)

τ^* is finite at $r^* = 0$. (8)

These equations are to be supplemented by the condition of continuity of u^* and r^* at the interfaces $r^* = R_0^*(z^*,t^*)$ and $r^* = R_p^*(z^*,t^*)$.

The pressure gradient which is function of z^* and t^*, is represented as

$$\frac{\partial}{\partial z^*}p^*(z^*,t^*) = -q^*(z^*)f(t^*)$$

with $q^*(z^*) = -\frac{\partial}{\partial z^*}p^*(z^*,0), f(t^*) = 1 + Asin(\omega t^*)$.

For the analysis presented in the sequel, we use the following non-dimensional variables

$$z = \frac{z^*}{a}, r = \frac{r^*}{a}, R(z,t) = \frac{R^*(z^*,t^*)}{a}, R_0(z,t) = \frac{R_0^*(z^*,t^*)}{a}, \quad R_p(z,t) = \frac{R_p^*(z^*,t^*)}{a}, \tau = \frac{2\tau^*}{q_0 a}, \quad \theta = \frac{2\tau_y}{q_0 a}, u = \frac{u^*}{\frac{q_0 a^2}{4\mu}}, t = t^*\omega, Q(z,t) = \frac{Q^*(z,t)}{\frac{\pi q_0 a^4}{8\mu}}, d = \frac{d^*}{a}, \ \delta = \frac{\delta^*}{a}, L_0 = \frac{L_0^*}{a}, L = \frac{L^*}{a}, \alpha^2 = \frac{\alpha^2\omega}{\frac{\mu}{\rho}}, q(z) = \frac{q^*(z^*)}{q_0} \qquad (9)$$

where q_0 is the constant pressure gradient (which is negative). In terms of these non-dimensional variables, eq. (1) reads

$$\alpha^2\frac{\partial u}{\partial t} = 4q(z)f(t) - 2\frac{1}{r}\frac{\partial(r\tau)}{\partial r}, \qquad 0 < r < R(z,t), \qquad (10)$$

while the equations (2) to (6) take the forms

$$-\frac{\partial u}{\partial r} = 2\tau, \qquad R_0(z,t) < r < R(z,t), \qquad (11)$$

$$-\frac{\partial u}{\partial r} = 2(\tau - \theta), \qquad R_p(z,t) < r < R_0(z,t), \qquad (12)$$

$$-\frac{\partial u}{\partial r} = 0, \qquad 0 < r < R_p(z,t), \qquad (13)$$

u=0 at r=R, τ is finite at r=0.

Also u and τ have to be continuous at $r = R_0(z,t)$ and $r = R_p(z,t)$. The geometry of the stenosis in non-dimensional form is given by

$$R(z,t) = \begin{cases} 1 - A_1(t)\left[L_0^{(m-1)}(z-d) - (z-d)^m\right], & if\ d \le z \le d + L_0, \\ 1, & \text{otherwise} \end{cases}$$

With

$$A_1(t) = \frac{\delta\left[1 - e^{(-t/T)}\right]m^{\frac{m}{m-1}}}{aL_0^m(m-1)}, \qquad m \ne 1$$

here δ denotes the maximum height of the stenosis. The maximum height being attained at $z = d + L_0/m^{1/(m-1)}$. The volumetric flow rate is given by

$$Q(z,t) = 4\int_0^{R(z,t)} ru(z,r,t)dr$$

III. ANALYTICAL SOLUTION OF THE PROBLEM

Considering the Womersley parameter to be very small, the velocity u, shear stress τ as well as R_0 and R_p can be expressed in the following form

$$u(z, r, t) = u_0(z, r, t) + \alpha^2 u_1(z, r, t) + \ldots \qquad (14)$$

$$r(z, r, t) = r_0(z, r, t) + \alpha^2 u_1(z, r, t) + \ldots \qquad (15)$$

$$R_0(z, r, t) = R_{00}(z, r, t) + \alpha^2 R_{10}(z, r, t) + \ldots \qquad (16)$$

$$R_p(z, r, t) = R_{0p}(z, r, t) + \alpha^2 R_{1p}(z, r, t) + \ldots \qquad (17)$$

Using (14) and (15) in (10). we have

$$\frac{\partial}{\partial r}(r\tau_0) = 2rq(z)f(t) \qquad (18)$$

$$\frac{\partial u_0}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \qquad (19)$$

Integrating (18) and using the boundary condition , we have

$$\tau_0 = q(z)f(t)R_p, \qquad 0 \le r \le R_p. \qquad (20)$$

In the regions $R_p \le r \le R_0$ and $R_0 \le r \le R$, the continuity of τ_0 at R_{0p} and R_{00} yield

$$\tau_0 = q(z)f(t)r \qquad (21)$$

Introducing (14) and (15) into equations (11) to (13) and equating like powers of α we obtain

$$-\frac{\partial u_0}{\partial r} = 2\tau_0, \quad -\frac{\partial u_1}{\partial r} = 2\tau_1, \qquad \text{if } R_0 \le r \le R. \quad (22)$$

$$-\frac{\partial u_0}{\partial r} = 2(\tau_0 - \theta), -\frac{\partial u_1}{\partial r} = 2\tau_1, \qquad \text{if } R_p \le r \le R_0. \quad (23)$$

$$\frac{\partial u_0}{\partial r} = 0, \quad \frac{\partial u_1}{\partial r} = 0, \qquad \text{if } 0 \le r \le R_p. \quad (24)$$

The boundary condition for u_0 and u_1 are:

$u_0 = 0$, $u_1 = 0$ at r = R (25)

u_0 , u_1 are continuous at R_{00} and R_{0p}.

from (21), (22) and (25) we have

$u_0 = q(z)f(t)(R^2 - r^2)$ $\quad R_0 \le r \le R$ (26)

using (25) in (21) and (23), one can find

$$u_0 = [q(z)f(t)(R_{00}^2 - r^2) - 2\theta(R_{00} - r)] + q(z)f(t)(R^2 - R_{00}^2) \qquad R_p \le r \le R_0 \qquad (27)$$

Now from (21), (24), (25) and (27)

$$u_0 = q(z)f(t)\left(R_{00}^2 - R_{0p}^2\right) + q(z)f(t)(R^2 - R_{00}^2) \quad 0 \le r \le R_p \qquad (28)$$

Neglecting the squares and higher power of α in (17) and using (20), one obtains

$$r|_{\tau_0=\theta} = R_{0p} = \frac{\theta}{q(z)f(t)}. \qquad (29)$$

Again, making use of the regularity condition that τ_1 is finite at r = 0, equation (28) along with (19) gives

$$\tau_1 = -\left[\frac{q(z)f'(t)}{2}\left(R_{00}^2 - R_{0p}^2\right) - \theta\left(R_{00} - R_{0p}\right)\right]R_{0p} - q(z)f'(t)(R^2 - R_{00}^2)\frac{R_{0p}}{2} \quad 0 \le r \le R_p \tag{30}$$

The continuity of τ_1 at r = R_{0p} yields

$$\tau_1 = -\left[\frac{q(z)f'(t)}{2}\left(R_{00}^2\frac{r}{2} - \frac{r^3}{4}\right) - 2\theta(R_{00}\frac{R_{0p}}{2} - \frac{R_{0p}^2}{3})\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{r}{4} + \frac{A_2}{r} \quad R_p \le r \le R_0 \tag{31}$$

The expression for A_2 is given in the Appendix.
Similarly, since τ_1 is continuous at R_0, we have

$$\tau_1 = -\frac{1}{2}q(z)f'(t)\left(R^2\frac{r}{2} - \frac{r^3}{4}\right) + \frac{A_3}{r} \quad R_0 \le r \le R, \tag{32}$$

Where A_3 stands for a quantity whose expression is presented in the Appendix.

Using (25), the equations (22)-(24) give rise to

$$u_1 = -q(z)f'(t)\left[\frac{R^2}{4}(R^2 - r^2) - \frac{(R^4 - r^4)}{16}\right] - A_3\log\left(\frac{r}{R}\right) \quad R_0 \le r \le R$$

$$u_1 = X(r) \quad R_p \le r \le R_0$$

$$u_1 = X(R_{0p}) \quad 0 \le r \le R_p$$

Where,

$$A_2 = -\left[\frac{q(z)f'(t)}{8}R_{0p}\left(R_{00}^2\frac{R_{0p}}{2} - \frac{R_{0p}^3}{4}\right) - 2\theta(R_{00}\frac{R_{0p}}{2} - \frac{R_{0p}^2}{3})\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{R_{0p}}{2}$$

$$A_3 = \left[q(z)f'(t)\frac{R_{00}}{8} - \frac{\theta}{3}\right]R_{00}^3 + A_2$$

$$X(r) = -2\left[q(z)f'(t)\left(\frac{R_{00}^2}{8}(r^2 - R_{00}^2) - \frac{1}{32}(r^4 - R_{00}^4)\right) - 2\theta(\frac{R_{00}}{4}(r^2 - R_{00}^2) - \frac{1}{9}(r^3 - R_{00}^3))\right] - q(z)f'(t)(R^2 - R_{00}^2)\frac{1}{2}(r^2 - R_{00}^2) - A_2\log(\frac{r}{R_{00}})$$

The expression for velocity in the peripheral and core layers can now be calculated by using the equations (14), (26)-(28) and (30).

The volumetric flow rate can be computed from (18) by re-writing it in the form

$$Q(z,t) = 4\left(u(z,R_p,t)\frac{R_p^2}{2} + \int_{R_p}^{R_0} ru(z,r,t)dr + R0Rruz,r,tdr.\right) \tag{33}$$

Different expression for u(z, r, t) can to be used for the different regions.
The value of the wall shear stress τ_ω is a quantity that is of particular importance from the physiological point of view. It is given by

$$\tau_\omega = (\tau_0 + \alpha^2\tau_1)|_{r=R} = q(z)f(t)R + \alpha^2\left(-\frac{1}{2}q(z)f'(t)\left(\frac{R^3}{4}\right) + \frac{1}{R}A_3\right) \tag{34}$$

The value of R_{00} in (16) is found by using the continuity of u_0 at R_{00}. In doing so. We have used the Newton-Raphson method, by taking the non-dimensional velocity in the peripheral layer at R_{00} as its value in the steady case, i.e. 0.03. In the order to determine the value of R_{10}, we consider the equation

$\tau^2(R_{00} + \alpha^2 R_{10}) = \tau_0^2(R_{00})$.

The value of R_{10} can be obtained by expanding the left side of (25) in the Taylor's series about R_{00}.
It may be noted that if we write u = $u_0 + \alpha^2 u_1$ and use (26)-(28) and (30).

$q(z) = \frac{Q_s}{R^4} + \frac{16}{7}\left(\frac{\theta Q_s}{R^5}\right)^{\frac{1}{2}} + \frac{64\theta}{49R}$, where R = R(z, t).

while computing q(z), one may take Q_s = 1.0. After q(z) is determined, Q(z, t) can be calculated from .

IV. RESULTS AND DISCUSSION

The volumetric flow rate and the wall shear stress are the two important characteristics in the study of fluid flow through a stenosed artery. Using appropriate boundary conditions, analytical expressions for the velocity profile, volumetric flow rate and shear stress have been obtained. The expressions for volumetric flow rate and wall shear stress, given by (33) and (34) respectively have been numerically evaluated using MATLAB software for different values of relevant parameters. For the purpose of numerical computation of the quantities of interest, we have performed a thorough quantitative analysis, by taking the following values of the different parameters involved in the present study: a = 0.5mm, L = 30, L_0 = 10, d = 10, θ = 0.05, A = 0.7, δ = 0.1, α^2 = 0.049, m = 2.0, T = 1.0Fig. 1 Shows that variation of velocity of blood with radius of the blood vessel for different values of m. fig. shows that velocity of blood decreases with the increasing value of radius. It is also found that the velocity increases for the increasing values of m.

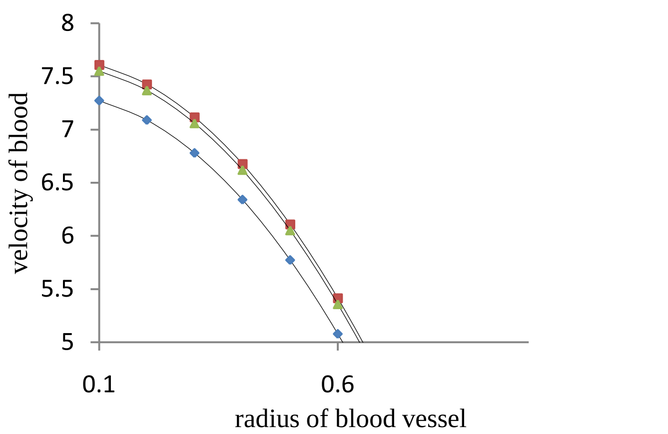

Figure. 1. variation of velocity of blood with respect to r with different values of m

Fig. 2 shows that the variation of volumetric flow rate with time for different values of amplitude A. it is found that the volumetric flow rate first decreases then increases and then decreases for a interval of time. It can be also shown that the volumetric flow is increases with increasing values of amplitude A.

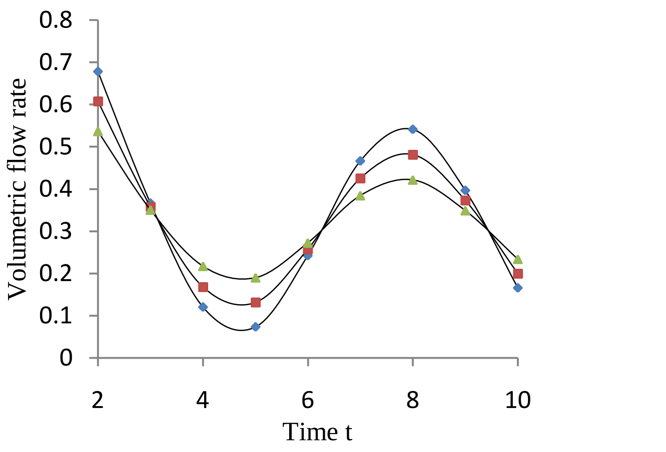

Figure.2. variation of volumetric flow rate with time for different values of A

Fig. 3 depicts the variation of wall shear stress with height of the stenosis for different values of time. It is shown in the figure that the wall shear stress increases with increasing values of height of the stenosis and it is also found that the wall shear stress increases with increasing values of time

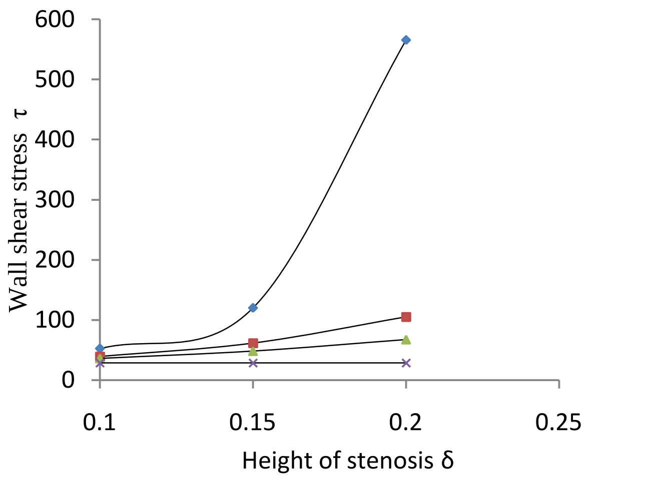

Figure. 3. variation of wall shear stress with δ for different values of time

Fig. 4 depicts the variation of wall shear stress with axis z for different values of time. It is found that the wall shear stress increases with the increasing values of time.

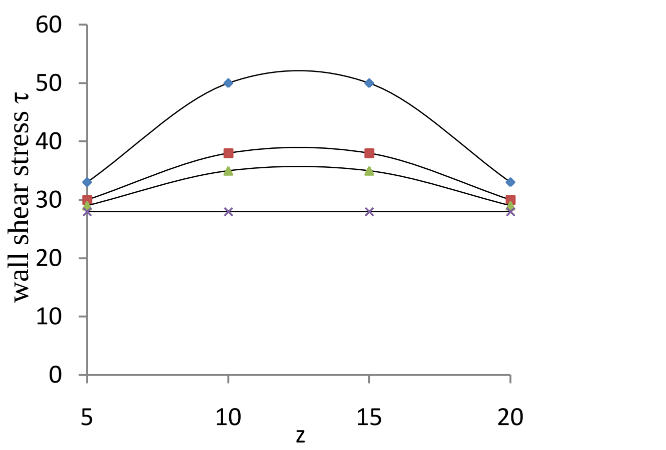

Figure.4. Variation of wall shear stress with z for different values of time

V. CONCLUSION

In the present paper blood is taken as Bingham plastic fluid. The numerical expression is found for the velocity, wall shear street and volumetric flow rate. In comparision of results we found that the Bingham plastic fluid has better understanding. It is found that wall shear stress and volumetric flow rate is strong parameter through all this study. It is also shown that the velocity in the core layer is heigher than the peripharel layer. So that the Bingham plastic fluid model gives better results in comparision to Newtonion fluid.

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