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Establishment of Power Division Theorem

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Abstract:
Estimating actual Sharing of loads and network losses by each generator is a difficult task in a Power System. Still scholars are performing intensive research to identify the perfect algorithms/methods. Many techniques were presented and some are coming up in order to evaluate fair and transparent tariff in the deregulation environment. The task is solved in this work by establishing a general power division theorem based on Kirchhoff's Current Law (KCL). Complex power sharing is estimated. The statement of the theorem is proved fundamentally and examples are given to illustrate the application with simple network and actual IEEE-5 Bus test system. In general, the result of system having `n' generators and `m' loads is displayed in a complex power distribution matrix n×m at particular operating point. The results are validated by verifying complex power balance.

Key words- Power Division Theorem, complex power distribution matrix, complex power sharing, sharing by Kirchhoff's Current Law, power system network.

1. INTRODUCTION

Many Independent Power Producers (IPPs) and various utilities are entering in to open access environment of transmission services. It is required to fix the cost for transmission services which needs to find the contribution of generators to the system load and network losses at particular operating state [5]. Active power flow tracing based on the concept of extended incidence matrix for determining share of generator on load and line is explained [1].The method of using imaginary current for tracing active and reactive power is proposed [10].Contribution factor of individual generators .to line and extraction factor of individual load are calculated by using a Graph Theory[3,9].Generator participating factor is proposed for changes in the operating state[6]. Fixed cost allocation for transmission and optimization in tracing of power flow are explained [7]. The variations in power flow through a particular line due to changes in nodal generation/demand are determined by means of sensitivity analysis[3]. The transmission usage/ supplement change is allocated to individual loads and losses are allocated to generators in the case of Downstream looking algorithm. The transmission usage/ supplement change is allocated to individual generators and losses are allocated to the load in the case of Upstream looking algorithm [11]. The application of Bialek's algorithm is explained for Power tracing and loss allocation [2].

Proportional Sharing principle is the basic approach in many of the power tracing methods. In the sharing principle, share of a in feed line at a node to every out going line from the node is in same ratio of the flow in the in feed line to total flow of all in feed lines [6,11].There is no strict derivation available so far for estimating the actual individual contributions of generators to each load [1]. The incidence matrix method is applicable to loss-less network and hence R, L, C of transmission network is modeled as equivalent constant load [1].

Complex power division theorem is established in this work based on current sharing by Kirchhoff's Current Law(KCL). Each source connected to a node circulate current through each load at the node. The magnitude of the circulated current depends on the internal voltage of each source and the node voltage. The node voltage is established based on the current (Charge flow) through the admittance of each load for Current (charge)balance at the node (KCL). Change in any load admittance varies the node voltage and switch over to next operating state with a different set of each source current sharing. The balance is present at any instant. Hence complex power sharing is estimated instead of separate calculation on active and reactive power sharing.

The concept is also same for a network. The sum of currents entering into a network is equal to the sum of the currents leaving from the network. The network loss is also a load to all the sources. Based on the above explanations, the statement of power division theorem, derivation and examples are presented.

II. STATEMENT OF THE THEOREM.

If there are `n' sources supply power to `m' loads at a node or through a network, the i^{th} source shares j^{th} load `Soj' in the same ratio of its current `i_i' to the sum of source currents `I_A', where S represents complex power $S = V^* I = P – j Q$, P,Q active and reactive power and V^*complex conjunctive of voltage.

Therefore, the power supplied by i^{th} source to j^{th} load

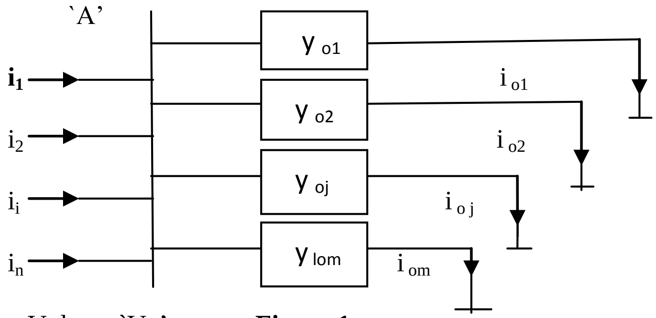
$$S_{ij} = S_{oj}\,\frac{i_i}{I_A} \qquad (1)$$

Where $I_A = \sum_1^n i_i = \sum_1^m i_{oj}$, i_{ojj} - j^{th} load current

III. DERIVATION

Consider a node `A'

Sources Loads



Voltage `V_A' **Figure 1**

Total admittance $Y_A = \sum_1^m y_{oj}$

Total current at node `A'$=I_A = \sum_1^n i_i = \sum_1^m i_{oj} = V_A Y_A$ (2)

Total Complex power at node `A' = $S_A = V_A^* I_A$

Complex power balance $S_A = V_A^* \sum_1^n i_i = V_A^* \sum_1^m i_{oj}$

Current in j^{th} load $i_{oj} = V_A y_{oj}$ (3)

From (2)&(3) $\boldsymbol{i_{o,j}} = \mathbf{I_A} \frac{\mathbf{y_{oj}}}{\mathbf{Y_A}}$ (4)

Multiply (4) with V_A^* results $\boldsymbol{V_A^*}\ \boldsymbol{i_{oj}} = \boldsymbol{V_A^*}\mathbf{I_A} \frac{\mathbf{y_{oj}}}{\mathbf{Y_A}}$

$$\boldsymbol{S_{oj}} = \mathbf{S_A}\frac{\mathbf{y_{oj}}}{\mathbf{Y_A}} = \mathbf{S_A}\frac{\mathbf{i_{oj}}}{\mathbf{I_A}} = \boldsymbol{V_A^*}\mathbf{I_A}\frac{\mathbf{i_{oj}}}{\mathbf{I_A}} \quad \mathbf{(5)}$$

$$= \boldsymbol{V_A^*}\ \mathbf{i_{oj}} \frac{\sum_1^n i_i}{\mathbf{I_A}} \quad (I_A \text{ is replaced with } \sum_1^n i_i)$$

$$= S_{oj} \frac{(i_1 + i_2 + ..i_i + \cdots i_n)}{\mathbf{I_A}} \quad \mathbf{(6)}$$

From (6) $\boldsymbol{S_{i,j} = S_{oj}} \frac{i_i}{I_A}$ **same as (1)**

Equation (1) is applicable for a Node and Network.

Multiply (1) by V_A^* results $\boldsymbol{S_{ij}} = \boldsymbol{S_{oj}} \frac{S_i}{S_A}$ **(7)**

Equation **(7)** is the Proportional Power Sharing Principle which is applicable at a Node only when complex powers are considered in in-feed (Sources) and out going lines (Loads).

The Sharing Principle is invalid for a Network since voltage at load & source points are different. The total input power is not equal to total load (output) power in a Network. Network loss plus total load is the total input.

The division principle (1) is applicable for both. The reason is current balance is considered. Treat the node` A' as a network. The Sum of currents entering in to a network is equal to sum of currents leaving from the network. Consider the network admittance matrix Y and express all the currents in terms of elements of Y, source and load voltages. Adding all the currents results the same.

$$\sum_1^n i_i = \sum_1^m i_{oj} = I_A \quad (8)$$

Where i_i, i=1 to n entering currents, i_{oj}, j=1 to m leaving currents. This fundamental concept is also same at any node inside the Network.

Load at j^{th} out going line $S_{oj} = v_{oj}^*\ i_{oj}$ (9)

$$= v_{oj}^*\ i_{oj} \frac{\mathbf{I_A}}{\mathbf{I_A}} = S_{oj} \frac{\sum_1^n i_i}{\mathbf{I_A}} \quad (10)$$

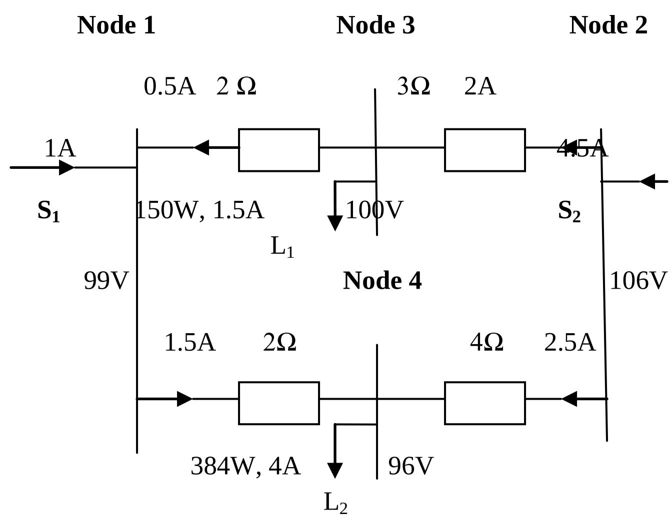
From (10) Power from i^{ith} Source to j^{th} Load

$$S_{ij} = S_{oj} \frac{i_i}{\mathbf{I_A}} \quad \textbf{same as (1)}$$

IV. EXAMPLES

Example 1

FIGURE 1



Power supplied by Source S_1 = 99×1= 99 W
Power Supplied by Source S_2 = 106×4.5= 477 W
Total Power supplied to the Network = 99+477=576 W
Total Load = L1 + L_2 = 150 + 384 = 534 W
Network loss by $i^2R = .5^2\times2+1.5^2\times2+2^2\times3+2.5^2\times4$ =42 W
Network Loss = 576 - 534 = 42 W

Contribution of each Source to each Load by Power Division Theorem (Equation 1)

Source S_1 to L_1 = $S_{11} = \boldsymbol{S_{o1}} \frac{\mathbf{i_1}}{\boldsymbol{I_A}}$ = $150\times\frac{1}{5.5}$ = 27.27 W

Source S_1 to $L_2 = S_{12} = \boldsymbol{S_{o2}} \frac{\mathbf{i_1}}{\boldsymbol{I_A}}$ = $384\times\frac{1}{5.5}$ = 69.82 W

Source S_2 to $L_1 = S_{21} = \boldsymbol{S_{o1}} \frac{\mathbf{i_2}}{\boldsymbol{I_A}}$ = $150\times\frac{4.5}{5.5}$ = 122.73 W

Source S_2 to $L_2 = S_{22} = \boldsymbol{S_{o2}} \frac{\mathbf{i_2}}{\boldsymbol{I_A}}$ = $384\times\frac{4.5}{5.5}$ = 314.18 W

Power Distribution Matrix
(Watts)
Loads

		L_1	L_2	Total
Sources	S_1	27.27	69.82	97.19
	S_2	122.73	314.18	436.91
Total		150.00	384.00	534.00

Contribution of S_1 to total system Load = 97.19 W
Contribution of S_2 to total system Load = 436.91 W

Contribution of S_1 to Network loss = 99-97.19=1.81 W
Contribution of S_2 to Network loss = 477- 436.91 W
=40.09 W

Network loss is the same (1.81+40.09=42 W).
Note: It seems that there is no current towards the Load L_1 (at node 3) from the source S_1 (at node 1). This is not meant that there is no power from S_1 to L_1. There should be a back power flow from S_1 to L_1. S_1 is performing the task of charge(current) balance at node 3 for maintaining its voltage at 100. More charges are coming from S_2 to the node 3 and excess charges are transferred to S_1 which contribute 0.5A from node 3 to node 1.

Example 2

FIGURE 2
Standard IEEE 5 bus System

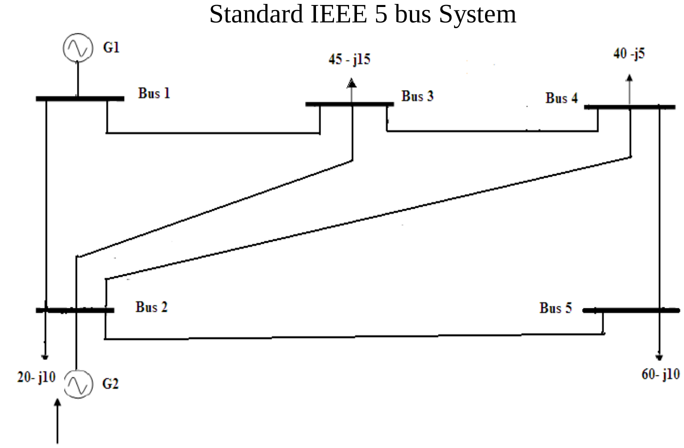


TABLE 2.1
BUS DATA

Bus No	Assumed Bus Voltages	Generation		Load	
		MW	MVAR	MW	MVAR
1	1.06 + j 0.0	0	30	0	0
2	1.0 + j 0.0	40	0	20	10
3	1.0 + j 0.0	0	0	45	15
4	1.0 + j 0.0	0	0	40	5
5	1.0 + j 0.0	0	0	60	10

TABLE 2.2
LINE DATA
Base MVA=100

Line p q	Z=R+jX Line Impedance		Line Charging
	R per Unit	X per Unit	B per unit
1– 2	. 0.02	0.06	0.0 + j 0.030
1– 3	0.08	0.24	0.0 + j 0.025
2 - 3	0.06	0.25	0.0 + j 0.020
2 - 4	0.06	0.18	0.0 + j 0.020
2 - 5	0.04	0.12	0.0 + j 0.015
3 - 4	0.01	0.03	0.0 + j 0.010
4 - 5	0.08	0.24	0.0 + j 0.025

TABLE 2.3
LOAD FLOW SOLUTIONS

Bus2 Voltage v_2	Bus 3 Voltage v_3	Bus 4 Voltage v_4	Bus 4 Voltage v_5
1.041-j0485	1.0109-j0.0985	1.0087-j0.0953	1.0037-j0.107

Slack Bus Voltage v_1 = 1.06 + j 0.0
Slack Bus Power S_1 = 1.297-j0.0898
Generator Bus 2 Power S_2 = 0.2-j0.20
Total Generation = S_1+S_2 = 1.497-j0.2898
Generator 1 current i_1 = 1.2236-j0.0847
Generator 2 current i_2 = 0.1828-j0.2006
Total Current in to the System network = I_A=i_1+ i_2
=1.4064-j0.2853

Complex Power Distribution Matrix by Power Divison Theorem (Equation 1) Sample Calculation

Generator G 1 to Load 1 = $S_{11} = \boldsymbol{S_{o1}} \frac{\boldsymbol{i_1}}{\boldsymbol{I_A}}$

$$S_{11}=(0.45-j0.15)\times\frac{1.2236-i0.0847}{1.4064-j0.2853}$$

= 0.3981-j0.0768

LOADS/(Columns) GENERATORS(Rows)	Bus 3 Load 1 S_{o1}	Bus 4 Load 2 S_{o2}	Bus 5 Load 3 S_{o3}
Bus1 Generator 1	0.3981-j0.0768	0.3445+j0.0023	0.5196-j0.177
Bus2 Generator 2	0.0519-j0.0732	0.0555-j0.0523	0.0804-j0.0823

Load 0.45-j0.15 0.4-j0.05 0.6-j0.1

(By adding column wise)
Contribution of Generator 1 to total system load (by adding first row) = 1.2622-j0.0922

Contribution of Generator 2 to total system load (by adding second row) = 0.1878-j0.2078
By adding the Generators' contributions = 1.45-j 0.3

Network loss = Total Generation – Total system Load
= S_1+S_2 – (1.45-j0.30)=**0.047+ j0.0102**

The network loss is verified by adding all line loss and the effect of line charging capacitances at all Busses.

Line 1- 2 = $(v_1 - v_2)^2 \times z_{12} = 0.0136 - j0.0407$

Line 1- 3 = $(v_1 - v_3)^2 \times z_{13} = 0.0133 - j0.0398$

Line 2- 3 = $(v_2 - v_3)^2 \times z_{23} = 0.0024 - j0.0101$

Line 2- 4 = $(v_2 - v_4)^2 \times z_{24} = 0.0053 - j0.0158$

Line 2- 5 = $(v_2 - v_5)^2 \times z_{25} = 0.0120 - j0.0360$

Line 3- 4 = $(v_3 - v_4)^2 \times z_{34} = (2.4446 - j7.3339) \times 10^{-4}$

Line 4- 5 = $(v_4 - v_5)^2 \times z_{45} = (2.1502 - j1.4506) \times 10^{-4}$

Total loss in all line = 0.047-j0.1438

TABLE 2.4

EFFECT OF LINE CHARGING CAPACITANCES

$B11 \times v_1^2$	$B22 \times v_2^2$	$B33 \times v_3^2$	$B44 \times v_4^2$	$B55 \times v_5^2$	Total
j0.0309	j0.0462	j0.0283	j0.0283	j0.0204	j0.154

Network loss=Total loss in all line+effect of capacitance

=0.047- j0.01438+j0.154 = **0.047+j0.0102**

Line charging capacitances inject reactive power j0.154

Contribution of Generator 1 to Network loss

$S_{1L} = S_1 - (S_{11} + S_{12} + S_{13}) = 0.0348 + j0.0025$

Contribution of Generator 2 to Network loss

$S_{2L} = S_2 - (S_{21} + S_{22} + S_{23}) = 0.0122 + j0.0078$

$S_{1L} + S_{2L} =$ **0.047+j0.0102**

In view of all above, actual power balance is achieved by Power Division Theorem.

V. CONCLUSION

Power Division Theorem is established in this work. Statement of the Theorem is given and proved based on the basic concept of KCL. It is applicable to Node, Network and Part of the Network, which solves the problem of finding the actual contribution of each source to each load and network loss in a System at particular operating state. Power tracing in the system network can be performed much faster with exact power balance. Two examples are given to illustrate the application of the Theorem.

Standard IEEE- 5 Bus system is considered as one of the example. The displayed results in a matrix are validated by verifying exact Power balance. Fair and transparent tariff can be evaluated in the Deregulation environment by applying the Theorem. The Theorem can be included like maximum power transfer theorem, in the topic of circuit theory for graduate course studies universally.

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