



# A Review Paper on Mathematical Modeling of Magneto-Rheological Damper

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## Abstract:

To predict the behaviour of MR dampers under certain magnetic fields, it becomes essential to model the device with an appropriate approach. Among various existing models, Bouc-Wen model will be chosen to characterize the nonlinear hysteretic behavior of MR damper. This paper provides brief description of mathematical modeling of Magneto-Rheological damper using Bouc-Wen model and MATLAB application. To study and analyze MR fluid flow inside the cylinder, certain fluid flow models are preferred. Hysteresis behavior of MR damper during loading and unloading cycles is obtained only through proper and systematic modeling of MR dampers. Model parameters are identified by parameter identification procedure. Numerical modeling of MR damper is required to study the dynamics of MR damper.

**Keywords:** MR damper, Bouc-Wen model, Hysteresis loop, MATLAB Tool

## I. INTRODUCTION

To predict the behavior of MR dampers under certain magnetic field or excitations, it is necessary to model the device with an appropriate approach [2]. The main challenging problem regarding MR damper numerical modeling is the accurate inclusions of the characteristic non linear nature of these devices in to the model. The model must be able to simulate the non linear behavior of MR damper but at the same time it has to be simple as possible to allow their effective implementation in control systems [2]. To evaluate the performance of MR dampers in vibration control applications and to take full advantage of the unique features of these devices, a model is needed to accurately describe the behaviour of the MR damper [4]. The Bouc–Wen hysteresis model is one of the most widely accepted approaches to simulate an extensive variety of softening/hardening smoothly varying hysteretic behaviour that can also include hysteresis pinching and stiffness/strength degradation [2].

### Quasi static analysis of fluid flow:

To analyze the fluid flow, quasi static axis symmetric model is preferred [1]. The Navier- stokes equation in cylindrical coordinates is used. But constants  $C_1$  and  $C_2$  are integral constants determined by boundary conditions. For the purpose of fluid flow study, pre yield region and post yield region are considered. The fluid region is evaluated separately as region I, II and III because of different nature of fluid. The volume flow rate is determined once velocity profiles at each of three regions are known. The two unknowns 'a' and 'b' of volume flow rate equation are identified by Newton-Raphson method [1]. In the region of inactive magnetic field the regions particularly adjacent to winding where  $\tau_y = 0$ , MR fluid behaves/ exhibits Newtonian like behavior. The procedure to determine pressure

drop due to viscous effects is described systematically through following steps

- i. Calculation of shear stress across annular gap
- ii. The constants  $C_1$  and  $C_2$  are determined by applying boundary conditions
- iii. Than total volume flow rate through the flow channel is calculated.

### Modeling hysteretic behavior of MR damper:

There are two models one is quasi static flow models and the other is dynamic models [1]. It is observed from the literature [1] that quasi static flow model successfully can be adapted to design MR dampers. But the above said quasi static models unable to capture the dynamic operational behavior of damper. However a dynamic hysteresis model is needed to simulate the hysteresis phenomenon of MR dampers. In the model given by Guo and Hu [1], three major terms are identified i.e., one is preload force of the pressurized nitrogen gas in accumulator; second term is viscous force of the damper and lastly observed hysteretic behavior. The parameters ( $f_0$ ,  $c_b$ ,  $f_y$ ,  $K$  and  $\dot{x}_0$ ) given in Guo and Hu model are determined by experimental data by least – square curve fitting method. Each term of the model has an effect on the shape of the curve. The other parameters like ' $f_0$ ' slides the curve up or down maintaining the shape of the curve. Whereas ' $f_y$ ' controls dynamic force range,  $C_b$  controls the slope of the whole curve,  $k$  &  $\dot{x}_0$  controls the slope of the curve and span of the low velocity hysteresis loop respectively, parameter 'm' controls the span of high velocity hysteresis loop [1]. Modeling techniques in general are of two type's i.e, parametric and non- parametric models. Parametric models mean assembly of linear and non- linear springs, dashpots and other physical elements in order to define accurate device behavior. Non-parametric models are framed based on analytical expressions that describe the characteristics of MR damper [2]. Bouc-Wen

and Bingham models are most widely accepted phenomenological models”.

**Bouc-Wen Model Formulation:**

Rheological structure is presented in below figure (1), [5]. MR damper model, considering the appearance of the hysteresis and damping force saturation, is the Bouc-Wen model. Accordingly, in structures showing hysteresis, the restoring force  $Q(x, \dot{x})$  is the sum of two components, without the hysteresis  $g(x, \dot{x})$  and with  $h(x)$  [5].

$$Q(x, \dot{x}) = g(x, \dot{x}) + h(x) + \dots \dots \dots (1)$$

The component  $h(x)$  is defined by an equation, which refers to the displacement  $x$  and  $z$  (evolutional variable), through which the position of the equation is dependent on whether  $n$  is an even or odd number [26]. For an odd  $n$ , the equation is

$$\dot{z} = -\alpha|\dot{x}|z^n - \beta\dot{x}|z|^n + A\dot{x} \dots \dots \dots (2)$$

however for an even number

$$\dot{z} = -\alpha|\dot{x}|z^{n-1} - \beta\dot{x}z^n + A\dot{x} \dots \dots \dots (3)$$

The damping force in the Bouc-Wen model can be written as

$$F = c_o\dot{x} + k_o(x - x_o) + \alpha(z) \dots \dots \dots (4)$$

where the evolutional variable  $z$  is described by the equation

$$\dot{z} = -\gamma|\dot{x}||z||z|^{n-1} - \beta\dot{x}|z|^n + A\dot{x} \dots \dots \dots (5)$$

Where

$\beta, \gamma, A$  – parameters representing the control of the linearity during unloading and the smoothness of the transition from the pre-yield to post-yield area

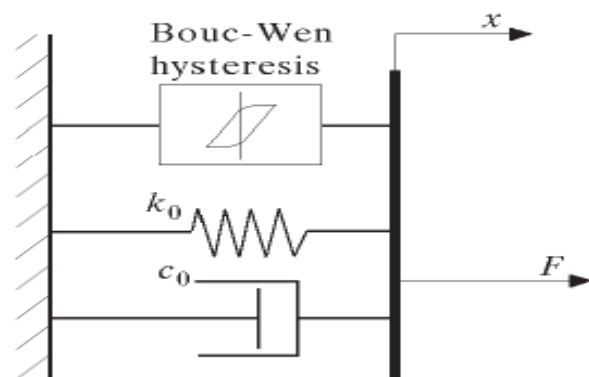
$\alpha$  – parameter representing the stiffness for the damping force component associated with the Evolution variable  $z$

$k_o$  – parameter representing the stiffness of the spring associated with the nominal damper due to the accumulator

$c_o$  – parameter representing viscous damping

$c_1$  – parameter representing the dashpot included in the model to produce the roll-off at low Velocities

$x_o$  – parameter representing the initial displacement of the spring with the stiffness  $k_o$ .



**Figure.1. Simple Bouc–Wen model (Spencer et al., 1997).**

Bouc-Wen hysteresis model is widely accepted approach. This model can able to simulate an extensive variety of softening/hardening smoothly varying hysteresis behavior. The Bouc-Wen model considered [2] has three components, a spring, dashpot and Bouc-Wen block in parallel configuration as shown in above figure (1). The nonlinearity of the system is located in the Bouc-Wen block, which is capable to capture the behaviour of MR dampers [2].  $F(t) = c_o\dot{x} + k_o(x - \dot{x}_o) + \alpha z$ . Where,  $C_o$  is Viscous coefficient,  $K_o$  is Stiffness coefficient and  $z$  refers to evolutionary variable. The characteristic / shape parameters of the model are  $C_o, K_o, \alpha, \beta, \gamma, \eta$  and  $A$ . These characteristic parameters are the function of current applied to MR damper, the

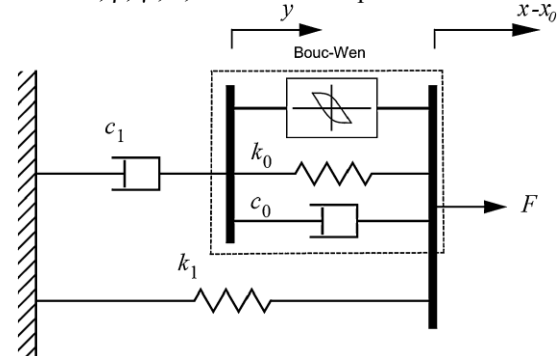
amplitude and frequency of vibration. However the non-linear shape of hysteresis curve can be adjusted by changing the values of the Bouc-Wen block parameters allowing controlling the linearity in the unloading and the smoothness of the transition from pre-yield to post yield region [2]. The Bouc-Wen block is a useful general tool to model whatever hysteresis phenomenon. Mathematically, it is based on an evolutionary variable, called ‘ $z$ ’, which evolves with a differential law, described in below Eq. (7) [7].

The force produced by the Bouc-Wen block is defined as:

$$F_{Bouc-Wen} = \alpha z \dots \dots \dots (6)$$

$$\dot{z} = -\gamma \cdot (|\dot{x} - \dot{y}|) \cdot |z||z|^{n-1} + (\dot{x} - \dot{y}) \cdot (A - \beta|z|^n) \dots \dots \dots (7)$$

where  $\alpha, \beta, \gamma, n, A$  are constant parameters.



**Figure. 2. Spencer-Carlson model of the MR Damper [7]**

Using the above equations (6) and (7), the upper part of the model in Fig. 2 .is governed by the following equation

$$C_1\dot{y} = \alpha z + k_o(x - x_o - y) + c_o(\dot{x} - \dot{y}) \dots \dots \dots (8)$$

The above said equation (8) is rearranged as: [7]

$$\dot{y} = 1/(c_o + c_1)(\alpha z + k_o((x - x_o - y) + c_o\dot{x} \dots \dots \dots (9)$$

By combining equation (8) and (9), we get: [7]

$$F = c_1/(c_o + c_1) * (\alpha z + k_o((x - x_o - y) + c_o\dot{x} + k_1(x - x_o) \dots \dots \dots (10)$$

In particular there are five parameters in this equation:  $\alpha, c_o, c_1, k_o, k_1$  and the evolutionary variable ‘ $z$ ’. Since  $k_o, k_1$  represent the stiffness at high speed and the effect of the nitrogen accumulator, their value should be independent of the current, while the remaining parameters are the ones which govern the system [7]. The procedure adopted to retrieve the parameters was divided in two steps. The first step was a global optimization applied to the system which allows all the parameters to be current dependent. The second step was a sub optimization aimed at reducing the number of parameters that depend on the current, thus simplifying the model [7]. In the work proposed in [46], Bouc-Wen model structure has been considered for a non linear least square based model fit of the experimental results. For the initial set of run the parameters were assumed free and are allowed to vary with independent variables, namely, input current ( $i_c$ ), amplitude of stroke ( $x_a$ ), and frequency ( $\omega$ ) of excitation. As shown in the above figure (1). Force  $u(t)$  provided by a MR damper is given by

$$u(t) = k_o x(t) + c_o \dot{x}(t) + \alpha z(t, x) \dots \dots \dots (11)$$

$$\dot{z} = -\gamma|\dot{x}||z||z|^{n-1} - \beta\dot{x}|z|^n + A\dot{x} \dots \dots \dots (12)$$

Where  $x$  is the displacement at the damper location;  $z$  is the evolutionary variable; and  $\gamma, \beta, n$ , and  $A$  are parameters controlling the linearity in the unloading and the smoothness of the transition from the preyield to the postyield region. The six parameters ( $c_o, k_o, \alpha, \gamma, \beta$ , and  $A$  are estimated for every single frequency of excitation at a particular amplitude and input

current on the basis of minimizing the error between the model predicted force ( $u$ ) and the force ( $Fe$ ) obtained in experiment [8]. Here optimum values for the six parameters have been obtained using "lsqcurvefit" algorithm available in MATLAB optimization tool-box for nonlinear curve fitting.

**Parameter Identification in Bouc-Wen Model:**

Here identification procedure is repeated for each set of experimental data and the different values of model parameters of the Bouc-Wen model are determined. The parameters that are current independent i.e, ( $A, \beta, \gamma$ ) are considered constant values during sinusoidal excitation and the average values are estimated. Whereas ' $\alpha$ ' and ' $C_o$ ' values are obtained by polynomial curve fitting of the average values of each set of frequencies and amplitudes for a specific operating current. The equation to find the value of ' $\alpha$ ' and ' $C_o$ ' is given below as

$$\alpha(I) = -9.66I^3 + 9.76I^2 + 1.12I + 0.1 \text{ (N/mm)} \text{-----(13)}$$

$$C_o(I) = 4.48I^4 - 4.74I^3 + 1.35I^2 + 0.05I + 0.01 \text{-----(14)}$$

Bouc-Wen model has additional capability to characterize the hysteretic behavior of the device [2]. Simple Bouc-Wen model is highly effective and most commonly used approach. The force generated by MR damper comprises of three components i.e, dashpot ' $C_o$ ', linear spring  $K_o$  and Bouc-Wen block ' $\alpha$ '. Bouc-Wen block ' $\alpha$ ' represents yield stress of MR fluid associated with evolutionary variable ' $z$ ' [3].

The non-linear hysteretic behavior of the device is controlled by Bouc-Wen component i.e, related with the restoring force of a non linear hysteretic system. According to Bouc-Wen formulation, the restoring force of non-linear hysteretic system  $Q(x, \dot{x}) = g(x, \dot{x}) + z(x)$ , where  $g(x, \dot{x})$  represents non- hysteretic component expressed by displacement ' $x$ ' and velocity  $\dot{x}$ . Another term  $z(x)$  represents Hysteretic component expressed by displacement and evolutionary variable  $z$ .

$$\dot{Z} = -\beta|\dot{x}|Z^n - \gamma \dot{x} z^n + A \dot{x} \text{-----(15);}$$

For  $n=1,3,5...$

$$\dot{Z} = -\beta|\dot{x}|Z^{n-1} - \gamma \dot{x} z^n + A \dot{x} \text{-----(16);}$$

For  $n=2,4,6...$

The above said equations are implemented in simple Bouc-Wen model with an equivalent formulation in which evolutionary variable ' $z$ ' is written as

$$z(t) = -\beta|\dot{x}(t)||z(t)||z(t)|^{n-1} - \gamma \dot{x}(t) |z(t)|^n + A \dot{x}(t) \text{-----(17)}$$

$A, \beta, \gamma$  and  $n$  are the parameters that describe shapes and sizes of hysteresis loop. Parameter ' $n$ ' controls the smoothness of transition from elastic to plastic response. The influence of individual parameters i.e,  $A, \beta, \gamma$  and  $n$  are detailed as below, Parameter ' $A$ ' defines scale and amplitudes of the hysteresis loop and also controls the slope of hysteresis loop. Increasing the value of ' $A$ ' will generate large hysteresis loop that implies an increase in energy dissipation. Parameter ' $n$ ' relates to smoothness of the transition from linear to non linear region. Parameter ' $\beta$ ' controls variation in the stiffness. Parameter ' $\gamma$ ' relates to area and flatness of hysteresis loop. ' $\alpha$ ' represents scale factor of hysteresis loop [3]. The mathematical model proposed by Spencer et.al, is applied in the work [4]. The hysteretic behaviour in the damper was described by the Bouc-Wen model. The phenomenological model is governed by the following equations [4]. The estimation of model parameters can be established based on several numerical approaches such as least square methods, fuzzy methods, neural networks, genetic algorithm etc. The least square method is a simple and standard

approach to reduce the difference between an experimental value and the fitted value provided by a dynamic model, and the application of this method is mostly appropriate for models that have linear variations in the parameters. There are several computation routines to solve nonlinear curve-fitting problems in the MATLAB optimization toolbox [3]. The force generated by MR damper is defined by the seven parameters of the Bouc-Wen block ( $A, \beta, \gamma, n, c_o, k_o, k_1, c_1$  and  $n$ ), or ten if the force offset  $f_o$  (or  $x_o$ ) is considered as a variable to be identified and the parameter vector is than defined as [3]

$$\Theta = [\alpha, \beta, A, n, k_o, k_1, c_o, c_1, x_o] \text{-----(18)}$$

According with curve fitting procedure, the model parameters  $\alpha, c_o$  and  $c_1$  can be described by [3],

$$\alpha(I) = -826.67I^3 + 905.14I^2 + 412.52I + 38.24 \text{----- (19)}$$

$$c_o(I) = -11.73I^3 + 10.51I^2 + 11.02I + 0.59 \text{----- (20)}$$

$$c_1(I) = -54.40I^3 + 57.03I^2 + 64.57I + 4.73 \text{----- (21)}$$

**Improved Bouc-Wen Model:**

**Mathematical expression of damping force and speed in Bingham model is:**

$F = f_c \text{sgn}(\dot{x}) + C_o \dot{x} + f_o$ , the model is easy to understand with simple form and clear concept. It can describe force and displacement relationship, energy dissipation ability of damper, but it cannot describe nonlinear performance among force, speed and input voltage of damper. Mathematical expression of damping force in improved Bouc-Wen model is

$$f = c_1 \dot{y} + k_1(x - x_o) \text{-----(22)}$$

$$\dot{y} = [az + c_o \dot{x} + k_o(x - y)] / (c_o + c_1) \text{-----(23)}$$

$$\dot{z} = -\gamma|\dot{x} - \dot{y}||z||z|^{n-1} - \beta(\dot{x} - \dot{y})|z|^n + A(\dot{x} - \dot{y}) \text{-----(24)}$$

The advantage of improved Bouc-Wen model are smooth, high calculation accuracy [6].

**Numerical Analysis:**

Parametric models are mathematical models that require characterization of the parameters and parameter identification is needed to determine the corresponding values of parameters for a particular MR damper [2]. To implement identification procedure the back work is model parameters need to be identified and defined as  $\theta = [\theta_1, \dots, \theta_n]$  [2].

To compare experimental response with model predicted response the performance criterion or objective function ' $J$ ' is introduced i.e,  $J(\theta) = F_{mr}(\theta) - F_e$ . In the above said equation  $F_{mr}$  refers to model predicted force and  $F_e$  refers to force obtained in experimental procedure. Optimization algorithm is used to adjust model parameters in order to minimize the objective function [2]. To reduce the difference between experimental values and fitted value provided by a dynamic model, least square method is a simple and standard approach. [2].

**Importance of Least square method:**

Least square method application is mostly appropriate for models that are linear in parameters. However, the dynamic models for MR damper are non linear in the parameters and a non linear approach should be considered to estimate model parameters [2].

**MAT LAB application:**

Lsq curvefit MATLAB routine is used as least- square solver since it provides a convenient interface for data fitting problems

[2]. Relationship between Bingham parameters and operating current are defined by polynomial functions

$$i.e, f_c = -85.19i^3 + 75.67i^2 + 24.95i + 0.14 \text{ -----(25)}$$

$$C_o = -0.079i^3 - 0.133i^2 + 0.184i + 0.004 \text{ -----(26)}$$

The above said mathematical expressions as functions are implemented in MATLAB routine to obtain the predicted MR damper response [2]. Based on the data of the damping force versus the velocity of piston at various input voltages, the parameters ( $c_1$ ,  $k_o$ ,  $k_1$ ,  $n$ ,  $A$ ,  $\beta$  and  $\gamma$ ) of phenomenological model proposed by Spencer et al. [10] are identified by a Simulink program. In which, the parameter values of  $\alpha$  and  $c_o$  vary nonlinearly with command voltages. The functions of ' $\alpha$ ' and ' $c_o$ ' are given below [10]

$$\alpha = 2.3363 v^2 - 3.4209 v + 5000 \text{ -----(27)}$$

$$c_o = 0.1794 v^2 - 2.0484 v + 2700 \text{ -----(28)}$$

MATLAB and its Simulink modeling environment are used to generate a numerical model of MR damper that accurately simulated the dynamics of the MR damper as measured during the performance tests. The model proposed in research paper [7] was implemented in the software Simulink in order to automatically estimate the parameters on the basis of the experimental data. The Simulink Parameter Estimation Toolbox was used to calculate the optimum set of parameter which best fitted the experimental data to the model for the specific combination of stroke, frequency and applied current. The toolbox is able to tune the model parameters for all the 36 configurations analyzed [7]. In the first globally optimal step, each parameter can achieve a value which is a function of the supply current and of the mechanical excitation applied. After the second step, an interpolation of the current dependent parameters was performed, using the Curve Fitting Toolbox of Matlab [9], for an applied current ranging from 0 to 1 a, in accordance with experimental tests. The interpolation parameter is performed in order to obtain a good fitting of the data with the least number of parameter, bearing in mind that the more parameters in the model the longer the experimental tests to estimate them [7].

## II. CONCLUSION:

In this paper, review of mathematical modeling of Magneto-Rheological damper using Bouc-Wen model has been undertaken. MR Fluid flow inside the MR damper is analyzed through quasi static axis symmetric model. Bouc-Wen model can able to simulate hysteresis behavior and it seems to be the most extensive model for modeling a hysteretic system. By the application of Simulink program of MATLAB, it is possible to identify the model parameters. Least-square solver is commonly used tool for parameter identification and curve fitting. It is from the literature study we can arrive that, to understand the dynamics of Magneto-rheological damper numerical model is generated.

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