ISSN XXXX XXXX © 2017 IJESC

Research Article Volume 7 Issue No.12

Fixed Covering Polynomial for Friendship Graph and its Algorithm

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Abstract:
The fixed Cover Polynomial of a graph G of order n has been already introduced in [3]. It is defined as the polynomial $\mathcal{C}$ (G, x) = $\sum_{i=\gamma_c(G)}^{|V(G)|} C(G,i)x^i$, where C (G, i) is the number of fixed vertex covering sets of G of size i and γ(G) is the fixed covering number of G. In this paper, we found the fixed covering sets and fixed covering polynomial of the Friendship graphs F_n. Also we exhibited the fixed covering polynomial of the graph $K_n \circ K_1$ with an illustration. An introduction to obtain algorithm for the fixed covering polynomial is initiated.

Keywords: Fixed Vertex Covering, Fixed Vertex covering number, Fixed Vertex covering Polynomial, Fixed Centre.

I. INTRODUCTION

Let G = (V, E) be a finite, undirected, nontrivial and connected simple graph. For any vertex v ∈ V, the open neighborhood of v is the set N(v) = {u ∈ V / uv ∈ E} and the closed neighborhood of v is the set N[v] = N(v) ∪ {v}. For a set S ⊂ V, the open neighborhood of S is N(S) = $\bigcup_{v \in V}$ N (v) and the closed neighborhood of S is N [S] = N (S) ∪ S. A set K ⊆ V is a vertex covering of G, if every edge uv ∈ E is adjacent to at least one vertex in K. The vertex covering number β(G) is the minimum cardinality of the vertex covering sets in G. A vertex covering set of cardinality β(G) is called a β-set. The eccentricity of a vertex u ∈ V is denoted by e (u) and is defined by e (u) = max {d (u, v) | for any v ∈ V}. Let the centre of G be denoted by C (G) and is defined as the set of all vertices u ∈ V such that e (u) is minimum. Let M = C(G); a set F ⊂ V is a fixed covering of G, if every edge uv ∈ E is adjacent to atleast one vertex in F and F ∩ M ≠ ϕ. The fixed covering number f (G) is the minimum cardinality of the fixed covering sets in G. A fixed covering set with cardinality f (G) is called f- set. Let F (G, i) be the family of all fixed covering sets in G, with cardinality i. Let f (G, i) = | F (G, i) |. The polynomial F (G, x) = $\sum_{i=f(G)}^{|v(G)|}$ f (G, i) x^i is defined as the fixed cover polynomial of G. In [14] some properties of the fixed vertex cover polynomials of some standard graphs such as K_n, P_n, C_n, $K_{m,n}$ etc have been studied.

II. FIXED COVER POLYNOMIAL

2.1. Definition
A graph G is said to be complete if any pair of vertices u, v ∈ V are adjacent in G. A complete graph with n vertices is denoted by K_n.

2.2. Definition

A path is a connected graph with two pendant vertices and all other vertices are of degree two. A path with n vertices is denoted by P_n.

2.3. Definition

A closed path is called a cycle; that is a path which is originating and ending with the same vertex. A cycle with n vertices is denoted by C_n.

2.4. Definition

The composition of K_1 and K_n is called one corona, that is connecting one pendant vertex to each vertex of a complete graph with n vertices is called one corona.

2.5. Definition

The Friendship graph F_n is a Planar undirected graph with 2n+1 vertices and 3n edges obtained by joining n copies of cycle graph C_3 with a common vertex.

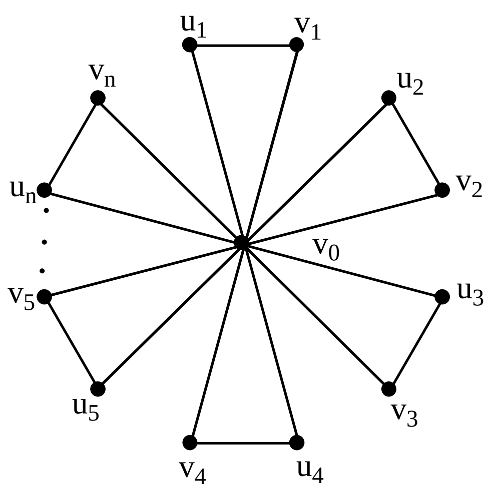
1.1. Theorem

The fixed covering polynomial of the Friendship graph F_n having 2n+1 vertices is

$$\mathcal{C}(G,x) = \sum_{r=0}^{n}\left(\sum_{i=0}^{n-r} nc_i \left\{(n-i)c_r\right\}\right)x^{n+r+1}.$$

Proof:

The Friendship graph F_n with 2n+1 vertices is represented in figure 1 as follows.



The vertex set V of G = F_n is V = $\{v_0, u_i, v_i | i=1,\ldots,n\}$. Divided the vertex set V into two sets S_1 and S_2 such that $S_1 = \{v_0, v_i | i = 1\ldots n\}$ and $S_2 = \{u_i | i = 1,2, \ldots, n\}$, where v_0 is the centre of G.

The fixed vertex covering sets with minimum cardinality n + 1 vertices are

$C(G, n + 1) = \{ S_1; S_1 - \{v_i\} \cup \{u_i\}/ i = 1, 2, \ldots n ;$
$S_1 - \{v_i, v_j\} \cup\{u_i, u_j\}/ i \neq j–1, 2, \ldots,n;$
$S_1 - \{v_i, v_j, v_k\} \cup\{u_i, u_j, u_k\}/ i \neq j \neq k =1,2, 3, \ldots,n;$
$\ldots\ldots\ldots\ldots\ldots\ldots ;$
$\{v_0, v_i\} \cup\{S_2 - u_j\}/ i = 1, 2, \ldots ;\{v_0\} \cup S_2\}.$

Hence, the total number of fixed vertex covering sets with cardinality n + 1 is

$$\gamma_c(G) = |C(G, n + 1)| = nC_0 + nC_1 + nC_2 + \ldots + nC_n = \sum_{i=0}^{n} nC_i$$

The fixed vertex covering sets with cardinality n + 2 are

$C(G, n + 2) = \{ S_1 \cup \{u_i\}/ i = 1, 2, \ldots n ;$
$S_1 - \{v_i\} \cup\{u_i, u_j\}/ i \neq j–1, 2, \ldots,n;$
$S_1 - \{v_i, v_j\} \cup\{u_i, u_j, u_k\}/ i \neq j \neq k ;1,2, \ldots,n;$
$S_1 - \{v_i, v_j, v_k\} \cup\{u_i, u_j, u_k, u_m\}/ i \neq j \neq k ;$
$\ldots\ldots\ldots ;$
$\{v_0, v_i\} \cup S_2;/ i = 1, 2, \ldots n\}.$

Hence, the total number of sets with cardinality n + 2 is

$$|C(G, n + 2)+ = nC_0 \{ nC_1\}+ nC_1\{(n–1)C_1\} + nC_2 \{(n–2)C_1\} + nC_3 \{(n–3)C_1\}+ \ldots + nC_{n-1}[\{n–(n–1)\}C_1] = \sum_{i=0}^{n-1} nC_i \left\{(n-i)C_1\right\}.$$

The fixed vertex covering sets of G with cardinality n + 3 are

$C(G, n + 3) = \{S_1 \cup\{u_i, u_j\}/ i,j = 1, \ldots n ;$
$S_1 - \{v_i\} \cup\{u_i, u_j, u_k\}/ i \neq j\neq k;$
$S_1 - \{v_i, v_j\} \cup\{u_i, u_j, u_k, u_l\}/ i \neq j \neq k \neq l;$
$S_1 - \{v_i, v_j, v_k\} \cup\{u_i, u_j, u_k, u_l, u_m\}/ i \neq j \neq k \neq l \neq m;$
$\ldots\ldots\ldots ;$
$\{u_0, u_i, u_j\} \cup S_2/ i = 1, 2, \ldots n \}.$

Therefore,

$$|C(G, n + 3)| = nC_0 \{ nC_2\}+ nC_1\{(n–1)C_2\} + nC_2 \{(n–2)C_2\} + nC_3 \{(n–3)C_2\}+ \ldots + nC_{n-2}[\{n–(n–2)\}C_2] = \sum_{i=0}^{n-2} nC_i \left[(n-i)C_2\right].$$

Continuing in this way, we obtain

$C(G, n + n) = \{S_1 \cup\{ S_2 - u:\}/i =1, \ldots n;$
$S_1 -\{v_i\}\cup S_2/i = 1, \ldots n.$

$$|C(G, n + n)| = nC_1+ nC_1 = \sum_{i=0}^{1} nC_i \left[(n-i)C_{n-1}\right]$$

Also, C (G, 2n + 1) = 1.

Therefore, the required polynomial is

$$\mathcal{C}(G, x) = \left[\sum_{i=0}^{n} nC_i \left\{(n-i)C_0\right\}\right] x^{n+1} + \sum_{i=0}^{n-1} nC_i \left\{(n-i)C_1\right\}\Big] x^{n+2} + \sum_{i=0}^{n-2} nC_i \left\{(n-i)C_2\right\}\Big] x^{n+3} + \ldots + \sum_{i=0}^{n-(n-1)} nC_i \left\{(n-i)C_{n-1}\right\}\Big] x^{2n} + \sum_{i=0}^{0} nC_i \left\{(n-i)C_n\right\}\Big] x^{2n+1.}$$

That is

$$\mathcal{C}(G, x) = \sum_{r=0}^{n}\left[\sum_{i=0}^{n-r} nC_i \left\{(n-i)C_r\right\}\right] x^{n+r+1}.$$

Algorithm:

Accept: n, i, r
Define: $S_1[i]; S_2[i]; i = 1, 2, \ldots, n.$
Assign the vertices $S_1[i] = \{ v_0, v_1, v_2, \ldots, v_i\}$ and
$S_2[i] = \{ u_1, u_2, \ldots, u_i \}$
for i = 1 to n

$$C[i] = \sum_{k=0}^{n-i} nC_k [(n-k)C_k];$$

$$\mathcal{C}(G, x) = \sum_{r=0}^{n}\left[\sum_{i=0}^{n-r} nC_i \left\{(n-i)C_r\right\}\right] x^{n+r+1}$$

2.2. Corollary

The coefficient of fixed vertex cover polynomial of F_n are log concave.

Proof

Clearly, the coefficients of $\mathcal{C}(F_n, x)$ satisfies the property $a_i^2 \geq a_{i-1}.a_{i+1} \; \forall$ i =1, 2, …n.

Therefore, they are log concave.

2.3. Theorem

The fixed vertex covering polynomial of the graph G = $K_n \circ K_1$ is

$$\mathcal{C}(G, x) = \sum_{i=0}^{n}\left[nC_i + n\{(n-1)C_i\right] x^{n+i}.$$

Proof

The graph G is given in figure 1.2 as below.

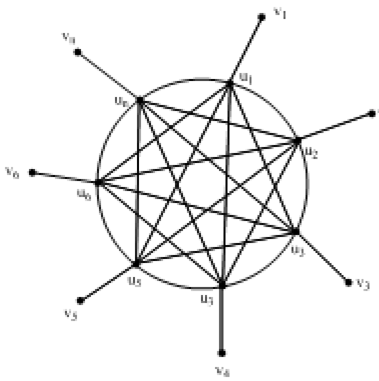


Figure.1.2

Let the vertex set be V = { u_i, v_i/i = 1,…,n}.

Now the vertex set V can be partitioned in to two sets S_1 and S_2 such that S_1={u_i / i = 1, …, n} and S_2={v_i / i = 1, 2, …, n}. Since the vertex set of S_1 form a complete subgraph of G and an element of S_1 is a centre of G.

The minimum fixed vertex covering sets with cardinality n are

C(G, n) = { S_1 : S_1– {u_i}∪ {v_i}/ i = 1,2,…,n}.

Hence, |C(G, n)| = 1 + n.

The fixed vertex covering sets with cardinality n +1 are

C(G, n + 1) = { S_1∪ {v_i}/ i = 1,2,…,n;. S_1∪ {u_i}∪ {v_i, v_j}/ i ≠ j = 1,2,…,n}.

Therefore, |C(G, n + 1) |= n + n(n – 1).

The fixed vertex covering sets with cardinality n + 2 are

C(G, n +2) = { S_1∪ {v_i, v_j}/ i ≠ ;

S_1– {u_i}∪ {v_i, u_j, u_k}/ i ≠ j≠ k;

Therefore, |C(G, n + 2)| = nC_2 + n [(n–1) C_2]

The covering sets with cardinality n + 3 are

C (G, n + 3) = {S1∪ {u_i, v_j, v_k } i ≠ j≠ k;

S_1– {u_i}∪ {v_i, u_j, v_k, v_l }/ i ≠ j ≠ k ≠ l }.

Therefore, |C(G, n+3)| = nC_3 + n [(n–1) C_3].

Proceeding this way we get

C(G, 2n–1) = { S_1∪ S_2 – { v_i}; S_1 – { u_i} ∪S_2/i} = 1, 2, …n.

Therefore, |C(G, 2n–1)| = nC_{n-1} + n [(n–1) C_{n-1}] and C(G, 2n) = S_1∪ S_2, hence, |C(G, 2n)| = 1.

In general,

|C(G, n + i)| = nC_i + n[n–1)C_i] for all i = 0, I, …, n

Therefore, the fixed vertex covering polynomial is

$$\mathcal{C}(G, x) = \sum_{i=0}^{n}\left\{nC_i + n\left[(n-1)C_i\right]\right\} x^{n+i} \text{ for all i = 0,1, 2,…n.}$$

2.4. Corollary

The coefficients of x– n[C (G, x)] are log concave.

Algorithm

Define Dim of S_1 and S_2

Accept S_1, S_2, i , n.

Define nC_r.

For i = 1 to n,

S_1[i] ; S_2[i].

For i = 1 to n,

$\mathcal{C}$(G, n+i) = nC_i + n [(n–1) C_i]

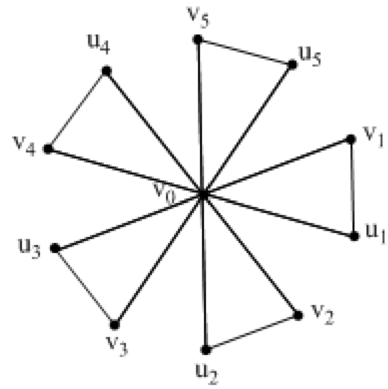
Display $C(G, x) = \sum_{i=0}^{n}\left\{nC_i + n\left[(n-1)C_i\right]\right\} x^{n+i}$

Example: 2.5

V = {v_0,v_i, u_i/i = 1,2, …,5}

S_1 = {v_0, v_i/i = 1, 2, …,5 }

S_2 = {u_i/i = 1, 2, …,5 }



By theorem 2.3,

C(G, 6) = {{v_0,v_2, v_3, v_4, v_5, u_1}; {v_0, v_1, v_3, v_4, v_5, u_2};
{v_0, v_1, v_2, v_4, v_5, u_3};{v_0, v_1, v_2, v_3, v_5, u_4};
{v_0, v_1, v_2, v_3, v_4, u_5};
{v_0, v_3, v_4, v_5, u_1, u_4}; {v_0, v_2, v_4, v_5, u_1, u_3};
{v_0, v_2, v_3, v_5, u_1, u_4};{v_0, v_2, v_3, v_4, u_1, u_5};
{v_0, v_1, v_4, v_5, u_2, u_3};{v_0, v_1, v_3, v_5, u_2, u_4};
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{v_0, v_1, v_2, u_3, u_4, u_5}; {v_0, v_1, v_3, u_2, u_4, u_5};
{v_0, v_1, v_4, u_2, u_3, u_5}; {v_0, v_1, v_5, u_2, u_3, u_4}; {v_0, v_1, v_3, u_1, u_4, u_5}; {v_0,v_2, v_4, u_1, u_3, u_5};
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{v_0, v_3, u_1, u_2, u_4, u_5}; {v_0, v_4, u_1, u_2, u_3, u_5};
{v_0, v_5, u_1, u_2, u_3, u_4};{v_0, u_1, u_2, u_3, u_4, u_5}}.

|C(G, 6) | = 32

By theorem 2.3,

$$|C(G, 6)| = \sum_{i=0}^{n} nC_i = \sum_{i=0}^{5} 5C_i$$

$= 5C_0 + 5C_1 + 5C_2 + 5C_3 + 5C_4 + 5C_5$

= 1+ 5 + 10 + 10 + 5 + 1

= 32.

C(G,7) = {v_0, v_1, v_2,v_3, v_4, v_5, u_1} ; {v_0, v_1, v_2,v_3, v_4, v_5, u_2};
{v_0, v_1, v_2,v_3,v_4, v_5, u_3}; {v_0, v_1, v_2, v_3, v_4, v_5, u_4}; {v_0, v_1, v_2, v_3, v_4, v_5, u_5} ;
{v_0, v_2, v_3, v_4, v_5, u_1, u_4};{v_0, v_2, v_3, v_4, v_5, u_1, u_3};
{v_0, v_2, v_3, v_4, v_5, u_1,u_4};{v_0, v_2, v_3, v_4, v_5, u_1, u_5}; {v_0, v_1, v_3, v_4, v_5, u_1, u_2};{v_0, v_1, v_3, v_4, v_5, u_2, u_3};
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$\{v_0, v_2, u_1, u_2, u_3, u_4, u_5\}$; $\{v_0, v_1, u_1, u_2, u_3, u_4, u_5\}\}$

Therefore, $|C(G, 7)| = 80$

By Theorem 2.3,

$$|C(G, 7)| = \sum_{i=0}^{n-1} nC_i\{(n-i)C_1\}$$

$= 5C_0 + \{(5–0)C_1\} + 5C_1 \{(5–1)C_1\} + 5C_2 \{(5–2)C_1\} + 5C_3 \{(5–3)C_1\} + 5C_4 \{(5–4)C_1\}$
$= 5 + 20+30+ 20+ 5$
$= 80.$

$C(G,8 =\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_2\}$; $\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_3\}$;
$\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_4\}$; $\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_5\}$;
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$\{v_0, v_3, v_5, u_1, u_2, u_3, u_4, u_5\}$;$\{v_0, v_4, v_5, u_1, u_2, u_3, u_4, u_5\}\}$

Therefore $|C(G, 8)| = 80$

By Theorem 2.3,

$$|C(G, 8)| = \sum_{i=0}^{n-2} nC_i\{(n-i)C_2\}$$

$= 5C_0 + \{(5–0)C_2\} + 5C_1 \{(5–1)C_2\} + 5C_2 \{(5–2)C_2\} + 5C_3 \{(5–3)C_2\}$
$= 10 +30+ 30+ 10$
$= 80$

The covering the sets with cardinality 9 are

$C(G,9) = \{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_2, u_3\}$;
$\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_2, u_4\}$; $\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_3, u_4\}$;
$\{v_0, v_1, v_2,v_3, v_4, v_5, u_2, u_3, u_4\}$; $\{v_0, v_1, v_2,v_3, v_4, v_5, u_2, u_4, u_5\}$;
$\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_3, u_5\}$; $\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_3, u_4\}$;
$\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_2, u_5\}$; $\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_2, u_4\}$;
$\{v_0, v_1, v_2,v_3, v_4, v_5, u_1, u_2, u_3\}$; $\{v_0, v_2, v_3,v_4, v_5, u_1, u_2, u_3, u_4\}$;
$\{v_0, v_2, v_3,v_4, v_5, u_1, u_2, u_3, u_5\}$; $\{v_0, v_2, v_3,v_4, v_5, u_1, u_2, u_4, u_5\}$; $\{v_0, v_2, v_3,v_4, v_5, u_1, u_3, u_4, u_5\}$; $\{v_0, v_1, v_3,v_4, v_5, u_2, u_1, u_3, u_4\}$;
$\{v_0, v_1, v_3,v_4, v_5, u_2, u_1, u_3, u_5\}$; $\{v_0, v_1, v_3,v_4, v_5, u_2, u_1, u_4, u_5\}$; $\{v_0, v_1, v_3,v_4, v_5, u_2, u_3, u_4, u_5\}$;$\{v_0, v_1, v_2,v_4, v_5, u_3, u_1, u_2, u_4\}$; $\{v_0, v_1, v_2,v_4, v_5, u_3, u_1, u_2, u_5\}$; $\{v_0, v_1, v_2,v_4, v_5, u_3, u_1, u_4, u_5\}$; $\{v_0, v_1, v_2,v_4, v_5, u_3, u_2, u_4, u_5\}$; $\{v_0, v_1, v_2,v_4, v_5, u_4, u_1, u_2, u_3\}$; $\{v_0, v_1, v_2,v_3, v_5, u_4, u_1, u_2, u_5\}$;$\{v_0, v_1, v_2,v_3, v_5, u_4, u_1, u_3, u_5\}$;
$\{v_0, v_1, v_2,v_3, v_5, u_4, u_2, u_3, u_5\}$; $\{v_0, v_1, v_2,v_3, v_4, u_5, u_1, u_2, u_3\}$; $\{v_0, v_1, v_2,v_3, v_4, u_5, u_1, u_2, u_4\}$;$\{v_0, v_1, v_2,v_3, v_4, u_5, u_1, u_3, u_4\}$; $\{v_0, v_1, v_2,v_3, v_4, u_5, u_2, u_3, u_4\}$;$\{v_0, v_1, v_2,v_3, u_1, u_2, u_3, u_4, u_5\}$;
$\{v_0,v_1,v_2,v_4,u_1, u_2, u_3, u_4, u_5\}$; $\{v_0,v_1,v_2,v_5,u_1, u_2, u_3, u_4, u_5\}$;
$\{v_0,v_1,v_3,v_4,u_1, u_2, u_3, u_4, u_5\}$;$\{v_0,v_1,v_3,v_5, u_1, u_2, u_3, u_4, u_5\}$;
$\{v_0,v_1,v_4,v_5, u_1, u_2, u_3, u_4, u_5\}$; $\{v_0,v_2,v_3,v_4, u_1, u_2, u_3, u_4, u_5\}$;
$\{v_0,v_2,v_3,v_5, u_1, u_2, u_3, u_4, u_5\}$; $\{v_0,v_2,v_4,v_5, u_1, u_2, u_3, u_4, u_5\}$;
$\{v_0,v_3,v_4,v_5, u_1, u_2, u_3, u_4, u_5\}$;

Therefore, $|C(G,9)| = 40$

by Theorem 2.4,

$$|C(G, 9)| = \sum_{i=0}^{n-3} nC_i\{(n-i)C_3\}$$

$= 5C_0 + [(5–0)C_3\} + 5C_1 \{(5–1)C_3\} + 5C_2 \{(5–2)C_3]$
$= 10 +20+ 10$
$= 40$

Covering sets with cardinality 10 are

$C(G,10) = \{v_0, v_2, v_3, v_4, v_5, u_1, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_3, v_4, v_5, u_1, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_2, v_4, v_5, u_1, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_2, v_3, v_5, u_1, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_2, v_3, v_4, u_1, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_2, v_3, v_4, v_5, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_2, v_4, v_5, u_1, u_2, u_3, u_4, u_5\}$
$\{v_0, v_1, v_2, v_3, v_4, v_5, u_1, u_2, u_4, u_5\}$
$\{v_0, v_1, v_2, v_3, v_4, v_5, u_1, u_2, u_3, u_5\}$
$\{v_0, v_1, v_2, v_3, v_4, v_5, u_1, u_2, u_3, u_4\}$

Hence, |C(G, 10)| = 10

By Theorem 2.3,

$$|C(G, 10)| = \sum_{i=0}^{1} nC_i \left[(n-i)C_{n-1}\right]$$

$$= 5C_0 + [(5-0)C_4\} + 5C_1 \{(5-1)C_4]$$

$$= 5+5$$

$$= 10.$$

And $S_1 \cup S_2$ is the clearly fixed vertex covering sets with cardinality1.

Therefore, the required polynomial by theorem 2.4 and as usual calculation is

$$\mathcal{C}(G,x) = 32x^6 + 80x^7 + 50x^8 + 40x^9 + 10x^{10} + x^{11}$$

$$= x^6 [\ 32 + 80x + 60\ x^2 + 40\ x^3 + 10x^4 + x^5$$

$$= x^6[\ x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32],$$

Which satisfies the property of log concave [some each $a_i^2 \geq a_{i-1}, a_i$ for all i].

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IV. ACKNOWLEDGEMENT

The first author is supported under the grants-in-aid to the minor project (No.F MRP-5653/15-SERO/UGC) by University Grants Commission, South East Regional Office, Hydrabad.