ISSN XXXX XXXX © 2017 IJESC

IJESC

Research Article Volume 7 Issue No.7

Local Continuity versus Global Uniform Continuity

S.Sumathira
Assistant Professor
Department of Mathematics (SF)
Nehru Memorial College, Puthanampatti, Trichy, India

Abstract:
A function *f* is uniformly continuous if, roughly speaking, it is possible to guarantee that *f*(*x*) and *f*(*y*) be as close to each other as we please by requiring only that *x* and *y* are sufficiently close to each other; unlike ordinary continuity, the maximum distance between *f(x)* and *f(y)* cannot depend on *x* and *y* themselves. For instance, any isometry (distance-preserving map) between metric spaces is uniformly continuous. We have the following chain of inclusions for functions over a compact subset of the real line.
Continuously differentiable ⊆Lipschitz continuous ⊆ α-Hölder continuous ⊆ uniformly continuous = continuous

Keywords: Continuously differentiable, Lipschitz continuous, α-Hölder continuous, uniformly continuous and continuous

1.INTRODUCTION

Every uniformly continuous function between metric spaces is continuous. Uniform continuity, unlike continuity, relies on the ability to compare the sizes of neighbourhoods of distinct points of a given space. Instead, uniform continuity can be defined on a metric space.

2.MAIN RESULT

2.1 Definition: continuously differentiable

A function *f* is said to be **continuously differentiable** if the derivative *f*′(*x*) exists and is itself a continuous function.
For example2.1.1

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

2.2 Definition: Uniformly continuous

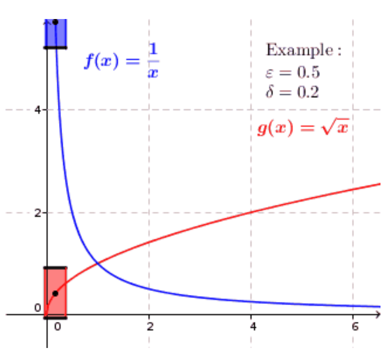
Given metric spaces (X, d_1) and (Y, d_2), a function $f : X \rightarrow Y$ is called **uniformly continuous** if for every real number $\varepsilon > 0$ there exists $\delta > 0$ such that for every $x, y \in X$ with $d_1(x, y) < \delta$, we have that $d_2(f(x), f(y)) < \varepsilon$.

2.2.2 Example

Define $f(x)=\frac{1}{x}$

$g(x)=\sqrt{x}$

Uniform continuity



2.3 Definition: Lipschitzs continuous

Let B⊆R, g: $B \rightarrow R$ is called Lipschitz continuous if there exists a positive real constant K such that, for all real x_1 and x_2 if$|g(x_1) - g(x_2)| \leq k|x_1 - x_2|$.

2.4 Definition: α-Holder continuous

A real or complex-valued function g on *d*-dimensional Euclidean space satisfies a **Hölder condition**, or is **Hölder continuous**, when there are no negative real constants *C*, α, such that

$$|g(x_1) - g(x_2)| \leq c\|x_1 - x_2\|^{\alpha}$$

The number α is called the *exponent* of the Hölder condition.
If α = 1, then the function satisfies a Lipschitz condition.
If α > 0, the condition implies the function is continuous.
If α = 0, the function need not be continuous, but it is bounded.

2.5 Definition: continuous

Let A⊆R, f: A→R is continuous if ε> 0 there exist $\delta > 0$ such that $|x - c| < \delta$
Implies $|f(x) - f(c)| < \varepsilon$

Theorem2.6

Every continuous function on a open set is uniform continuous

Proof

Let M and N are two metric spaces with d_M and d_N respectively.
Let f:M→N is continuous function
To prove that: f is uniform continuous
Let f is function, ie, f is open.
By the definition of continuous,
Let $d_M \subseteq d_N$, f : $d_M \rightarrow d_N$ is continuous if ε> 0 there exist $\delta > 0$ such that $d_M(x, y) < \frac{\delta}{2}$
Implies $d_M(f(x),f(c)) < \varepsilon$
Let $d_M(x_i, y) \leq d_M(x_i, y) + d_M(x, y) < \frac{1}{2}\delta_{x_i} + \delta < \delta_{x_i}$
Which implies
$d_N(x_i, y) \leq d_N(f(x_i), f(x)) + d_N(f(x_i), f(y)) < \frac{1}{2}\varepsilon + \frac{1}{2}\varepsilon = \varepsilon$
Clearly which is uniform continuous.

Hence “every continuous function on a open set is uniformly continuous”

3. CONCLUSION

Motivated by the definition of local continuity versus, we have discussed above uniform continuous investigation.

4. REFERENCES

[1].*Bourbaki, Nicolas. “General Topology” Chapters 1–4 [Topologie Générale] ISBN 0-387-19374-X*. Chapter II is a comprehensive reference of uniform spaces.

[2]. *Dieudonne, Jean* (1960), “Foundations of Modern Analysis” Academic Press.

[3]. Robert G.Bartle Donald R.Sherbert,” Introduction to Real Analysis”, Third Edition.