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Path Integral Approach to Coulomb's Law

Vivek Chhetri
Assistant Professor
Department of Physics
Sikkim Government College, Tadong, Sikkim, India

Abstract:
The Path integral or sum over histories approach to Quantum mechanics was a new approach to Quantum mechanics developed by Richard Feynman, when applied it produces magical results. In this paper I will show that Coulomb's Law can be derived by the spacetime path Integral approach of Quantum mechanics.

Keywords: Path Integral, Spin one particle, virtual particle, spacetime.

I. INTRODUCTION

In Feynman's path integral [4] [5] also called the "sum over histories" approach to Quantum mechanics a particle travels along every possible path through spacetime with each trajectory Feynman associated two numbers the amplitude and phase. The probability of a particle going from A to B is found by adding up the waves associated with every possible path from A to B .This was the picture in the Quantum world for large objects the phase of all the paths cancel except one so we get a single classical path. When Feynman tried to put this picture in a Mathematical framework he was guided by a mysterious remark by Dirac[2] [3]

$e^{i}\int_{t1}^{t2} L(x,x')$ corresponds to $<x_{2,}t_2|x_{1,}t_1>$

Trying to make sense of Dirac's remark Feynman developed his path integral or "sum over histories "approach to Quantum mechanics. Feynman's model had the added advantage because spacetime formulation is easy to visualize and this Lagrangian approach is relativistic ally invariant. In my paper I will use the path integral approach to derive Coulomb's law which we study in school as an empirical law.

II. PATH INTEGRAL FOR SPIN ONE FIELD.

The path integral in Scalar Quantum Field can be written as

$$Z=\int D\emptyset\, e^{i\int dxdydzdt\ \pounds(\emptyset)} \qquad (1)$$

Where £ is the Lagrangian density and $dxdydzdt=d^4x$ the invariant four dimensional spacetime.
The Maxwell's Lagrangian for a photon with a small mass can be written as

$$\pounds=-1/4F_{\mu\nu}\, F^{\mu\nu}+1/2m^2A_\mu A^\mu+A_\mu J^\mu \qquad (2)$$

Inserting the values of $F_{\mu\nu}$ and $F^{\mu\nu}$ in equation (2) for a vector field the path integral can be written as

$$Z=\int DA\, e^{iS(A)} \qquad (3)$$

where the action S(A) is

$$S(A)=\int d^4x\pounds=\int d^4x\{1/2A_\mu[(\partial^2+m^2)g^{\mu\nu}-\partial^{\mu}\partial^{\nu}]A_\nu+A_\mu J^\mu\} \qquad (4)$$

Comparing with equation (5)

$$\int dx_1dx_2..\, dx_N\, e^{\frac{i}{2}x.A.x+iJ.x}=E\ e^{-\frac{i}{2}J.\bar{A}.J} \qquad (5)$$

Where E is a constant and $\overline{A}=A^{-1}$
We can write

$$[(\partial^2+m^2)g^{\mu\nu}-\partial^{\mu}\partial^{\nu}]D_{\nu\lambda}(x)=\delta_\lambda{}^\mu\delta^4(x) \qquad (6)$$

We go to the momentum space by defining

$$D_{\nu\lambda}(x)=\int d^4x/(2\pi)^4\, D_{\nu\lambda}(k)e^{ikx} \qquad (7)$$

Plugging in equation (6) we find that

$$[-(k^2-m^2)g^{\mu\nu}+k^\mu k^\nu]\, D_{\nu\lambda}(k)=\delta_\lambda{}^\mu \qquad (8)$$

Thus giving

$$D_{\nu\lambda}(k)=-g_{\nu\lambda}+k_\nu k_\lambda/m^2/k^2-m^2 \qquad (9)$$

$D_{\nu\lambda}(k)$ is called the propagator in Quantum field theory .The propagator plays a very important role in field theory it is an amplitude a particle starts at some point in the past and ends up at another point in the future. It is an alternative to wave functions. The propagators have a neat mathematical property they are Green's functions. Using equation(5) we can write

$$Z(J)=Ce^{i/2}\iint dx^4dy^4J(x)D(x-y)J(y) \qquad (10)$$

$$Z(J)=Ce^{iW(J)} \qquad (11)$$

Where $W(J)=-1/2\iint dx^4dy^4J(x)D(x-y)J(y)$ (12)

D(x-y) is called the free propagator. We will substitute the value of $D_{\nu\lambda}(k)$ from equation(9) in equation(12) to calculate the value of energy and force between two charges.

III. ORIGIN OF FORCE

Since we know the value of our propagator our function W(J) which is related to the energy and the path integral.

$$W(J)=-\int d^4k/(2\pi)^4\, J^\mu(k)^*\ -g_{\nu\lambda}+k_\nu k_\lambda/m^2/k^2-m^2+i\varepsilon\ J^\nu(k) \qquad (13)$$

Since current conservation $\partial_{\mu}J^{\mu}=0$ implies that $k_{\mu}J^{\mu}(k)=0$ our equation(13) now can be written as

$$W(J)= 1/2\int d^4k/(2\pi)^4\, J^{\mu}(k)^*1/ k^2-m^2 +i\varepsilon\, J_{\mu}(k) \qquad (14)$$

If $J(x)=J_1(x)+J_2(x)$ where $J_1(x)$ and $J_2(x)$ are concentrated in two local region 1 and 2 of spacetime. W(J) will contain four terms of the form $J^*_1J_1$, $J^*_2J_2$, J^*_1J and $J^*_2J_1$. If we neglect the self interaction we have two terms.

$$W (J) = 1/2 \int d^4x/(2\pi)^4J^*_2(k)\, 1/k^2-m^2+i\varepsilon\;\; J_1(k) \qquad (15)$$

$$W (J) = 1/2 \int d^4x/(2\pi)^4J^*_1(k)\, 1/k^2-m^2+i\varepsilon\;\; J_2(k) \qquad (16)$$

We interpret equation (15) as follows. In region 1 of spacetime there exists a source that sends out disturbance in the field which is later absorbed by a sink in region 2 of spacetime. Experimentalists call this disturbance in the field a particle of mass m.It is easy to see that $i/k^2-m^2+i\varepsilon$ is playing the role of the propagator for a scalar field. Let $J(x)=J_1(x)+J_2(x)$ where $J_a(x)=\delta^3(\bar{x} -\bar{x}_a)$. In other words J(x) is a sum of sources that are time independent infinitely sharp spikes located at x_1 and x_2 in space. As before W(J) contain four terms we neglect self interaction now W(J) can be written as

$$1/2\int d^0xd^0y\iint d^0k/2\pi e^{ik0(x0-y0)} \;\; \int d^3k/(2\pi)^3e^{ik(x1-x2)}/k^2-m^2+i\varepsilon \qquad (17)$$

If we can choose K^0 equal to zero these particles are not on the mass shell these particles are virtual particles our equation (17) becomes

$$W(J) = - \int d^0x\int d^3k/(2\pi)^3e^{ik.(x1-x2)}/\bar{k}^2+m^2 \qquad (18)$$

Since $Z(J) = Ce^{iW(J)}$ for virtual particles this becomes

$$Z=\langle 0|e^{-iHT}|0\rangle=e^{-iET} \qquad (19)$$

We can write

$$E = \int d^3k/(2\pi)^3e^{ik.(x1-x2)}/\bar{k}^2+m^2 \qquad (20)$$

The integral works out to be

$$E= 1/4\pi re^{-mr} \qquad (21)$$

If we put the photon mass to be zero, we get

$$E=1/4\pi r \qquad (22)$$

Since F=-dE/dr our force is repulsive which varies as
$F\sim 1/r^2$ exactly as coulomb's had derived.

IV. CONCLUSION

We have proved from path integral approach that two like charges repel each other with a force which varies as $F\sim 1/r^2$, but our derivation also contains the mechanism of repulsion, it is the exchange of virtual massless spin one particle which is producing the repulsive force which varies as $F\sim 1/r^2$.The exchange of particle can produce a force was a profound conceptual advance in physics in the twentieth century.

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